

Thermalization scaling in monolayer graphene: wave turbulence and beyond

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Understanding thermalization dynamics is a fundamental goal of statistical physics. Recently, the wave turbulence (WT) theory has been applied widely to predict the scaling of thermalization times in ideal nonlinear lattices, yet its relevance to real materials is unclear. In this work, we investigate the thermalization dynamics of out-of-plane flexural modes in monolayer graphene with periodic boundary condition, assessing the applicability of the WT theory to this realistic lattice with complex interatomic interactions. The WT theory predicts that, within the weakly nonlinear regime, the thermalization time T_{eq} scales with the specific energy ε as $T_{eq} \propto \varepsilon^{-2}$ in the thermodynamic limit, governed by four-wave resonances. Simulations of large graphene sheets confirm this scaling and show data collapse when the time is rescaled by ε^2 . For small systems, the sparsity of discrete wave vectors in the Brillouin zone suppresses low-order resonances, leading to steeper scaling consistent with higher-order multi-wave processes. In the strong nonlinear regime, when frequency broadening exceeds the spacing between neighboring modes, the Chirikov resonance mechanism sets in, and an alternative scaling law for thermalization emerges. The critical energy for Chirikov overlap decreases with increasing system size, exhibiting a power-law trend. These results extend the WT theory to a structurally complex, experimentally relevant material and clarify how the order of resonant wave-wave interaction, finite-size constraints, and the effects of strong nonlinearity control thermalization dynamics.

