

Cyclic Periodic Synchronization In Ring-star network of Piecewise Linear Discontinuous Maps

Vismaya V. S.¹, Sishu Shankar Muni

¹Digital University Kerala, India

In this study, we investigated the emergence and persistence of cyclic periodic synchronization patterns in networks of coupled piecewise-linear discontinuous (PLD) maps. These maps, characterized by inherent discontinuities in their dynamical rules, exhibit a wide range of complex nonlinear behaviors, including multistability, strong sensitivity to initial conditions, and intricate bifurcation structures. Notably, the considered PLD map serves as a discrete-time analogue of the well-known differential Chay neuron model, which is widely used in computational neuroscience. As such, it successfully reproduces bifurcation scenarios similar to those observed in experimental studies of neural inter-spike intervals, thereby providing both mathematical richness and physical relevance. We began by analyzing the dynamics of an isolated PLD map using standard nonlinear analysis tools, including bifurcation diagrams, Lyapunov exponent spectra, and border-collision bifurcation analysis. The results reveal the presence of period-adding structures, a hallmark of discontinuous dynamical systems. These structures are organized by border-collision bifurcations, which define sharp transitions between dynamical regimes and provide a clear mechanism for the onset of complex temporal behavior. Such features form the foundation for understanding more intricate collective dynamics in coupled systems. Building on the single-map analysis, we then explored the collective behavior of networks of coupled PLD maps arranged in different topologies, namely ring, star, and hybrid ring–star configurations. The introduction of coupling leads to a rich variety of emergent spatiotemporal patterns. In addition to conventional states such as full synchronization, complete desynchronization, traveling waves, and clustered dynamics, we observed the formation of higher-order cyclic periodic synchronization patterns, including cyclic period-3, period-4, and period-5 states. These patterns represent organized yet nontrivial collective dynamics arising from the interplay between local nonlinearities and network interactions. A key finding of this work is that the emergence, stability, and persistence of these cyclic periodic states are strongly governed by the coupling strengths associated with the ring and star components, denoted by ϵ_r and ϵ_s , respectively. These parameters regulate mechanisms such as cluster competition and cluster collision, which in turn determine whether cyclic synchronization patterns are sustained or destabilized. Furthermore, detailed regime maps in both parameter space and coupling space reveal clear boundaries between different dynamical behaviours. These maps highlight the critical role of interface width and basin boundary structures in mediating transitions between synchronization patterns.

