Rare Events, Big Jumps, and Redundancy in Random Walkers Target Reaching

Alessandro Vezzani[1,2,3], Elisabetta Ellettari[3], Giacomo Nasuti[3], Alberto Bassanoni[2,3], Raffaella Burioni[2,3]

[1]CNR - IMEM, Parma, Italy, [2]INFN , Parma Group, Italy, [3]Università di Parma, Parma, Italy

Reaching a target in a complex environment is a fundamental challenge across natural systems, from chemical reactions to animal foraging and sperm fertilization.

Lévy walks, characterized by broadly distributed step lengths, have been widely used as a model for effective dynamics in target-reaching problems (e.g., foraging). In such settings, rare events dominated by long relocations govern the dynamics, as described by the so-called big jump principle.

On the other hand, in processes such as fertilization, a well-known strategy to reduce search times is redundancy: deploying many independent searchers increases the probability that at least one succeeds. Here, we show that redundancy, when combined with rare-event statistics, can efficiently organize target-search processes in the presence of big jumps.

In particular, we study the mean first-passage time for a system of N independent walkers performing jumps drawn from a power-law distribution. We show that the mean first-passage time of the fastest walker scales as $\langle T\rangle \sim 1/N$, representing a dramatic speed-up compared to classical Brownian motion, where the decay follows $1/\log N$. This improvement arises because, for sufficiently large N, target reaching is governed by extreme-event statistics, i.e., by the big jumps.

We derive a scaling law linking the number of walkers N to the size X of the search region. This relation identifies a crossover between a regime dominated by a single large fluctuation ("big jump") and a regime characterized by Gaussian extreme-value statistics.

Finally, we extend the model to include random velocities. As a prototypical example, we consider mammalian fertilization and derive, within a coarse-grained framework, a cross-species scaling relation between the number of spermatozoa and the typical uterine size.