Spectral Reorientation and Wick Rotation of the Riemann Zeta Function: A Theoretical Framework for the Hilbert-Pólya Conjecture

Murty Uppuluri[1,2]

[1]Golden Sparks Capital, Fairfax, United States, [2]IIT, Kanpur, India

A foundational challenge in quantum statistical physics is the identification of a self-adjoint operator whose spectrum corresponds to the non-trivial zeros of the Riemann zeta function.

This paper proposes a formal theoretical reorientation of the complex-plane via the transformation effectively executing a 90-degree Wick rotation and a translation to the critical axis.

Under this transform, the vertical manifold of the critical line is mapped onto the real axis unfolding the complex strip into a one-dimensional spectral representation. We demonstrate that in this frame, the Dirichlet series is reformulated as a superposition of plane-wave propagators, where the non-trivial zeros manifest as real resonance frequencies. This re-centering provides a natural geometric basis for applying Gaussian Unitary Ensemble (GUE) statistics, treating the zeros as eigenvalues of a chaotic quantum system. Furthermore, we derive the von Mangoldt explicit formula within this spectral coordinate system, representing the distribution of prime numbers as a harmonic synthesis of sine waves on a real-valued timeline. By aligning the analytic structure of with the foundational principles of statistical mechanics and spectral theory, this transformation offers a clarified pathway toward verifying the Hilbert-Pólya conjecture and understanding the emergent thermodynamics of the prime number distribution

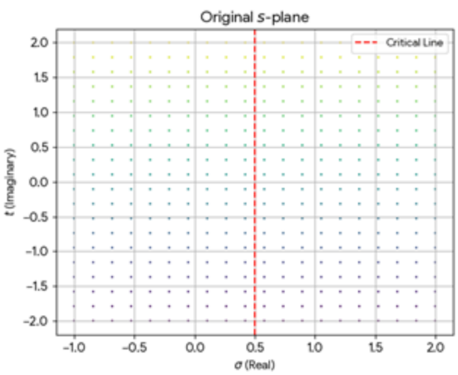
To visualize the Euler Product in new coordinate system, it is most effective to plot the logarithm of each prime term. Taking the logarithm converts the product into a sum of individual "prime waves.".

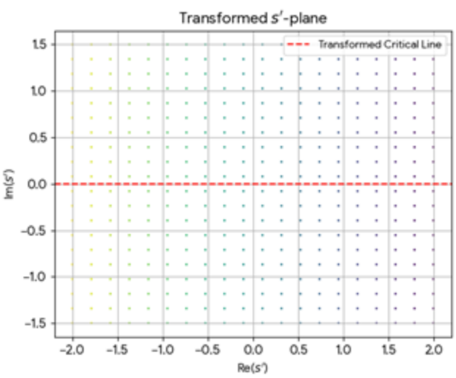
Oscillation Frequency: Each prime p creates a wave - $\cos(s' x \ln(p)) + i \times \sin(s' x \ln(p))$. [frequency $\ln(p)$]

The amplitude of each wave is scaled by $1/\sqrt{p}$.

The interference of these "prime harmonics" generates the spectral landscape where the Riemann zeros reside.

At the zeroes, both the Real and Imaginary combined signals often show high volatility or specific geometric patterns. This is the Arithmetic Quantum Chaos at work.





Feature	Standard s-plane	Your s'-plane ($i(s-1/2)$)
Critical Line	Vertical ($\mathrm{Re}(s) = 1/2$)	**Horizontal** ($\mathrm{Im}(s') = 0$)
Symmetry	$\Lambda(s) = \Lambda(1-s)$	$\Lambda(s') = \Lambda(\bar{s'})$
Oscillations	Spirals in complex space	Pure real-valued waves (Z-function)
Zeros	Expected on $\mathrm{Re}(s) = 1/2$	Expected on **Real Axis**

We can visualize how the $Z(s')$ function oscillates and hits zero more frequently as we move along your real axis.