Adaptive Orthogonal Bases for Non-Equilibrium Velocity Distributions from a Superstatistical Framework

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Non-thermal velocity distributions are ubiquitous in many complex systems, including space and astrophysical plasmas, where departures from equilibrium lead to heavy tails and strong deviations from the Maxwellian paradigm. Standard spectral representations of velocity distribution functions—such as Hermite or Laguerre expansions—are typically constructed around Maxwellian equilibria. As a consequence, they often require a large number of modes to accurately represent non-thermal features, particularly suprathermal tails.

In this work, we introduce a framework for constructing adaptive orthogonal bases tailored to non-equilibrium distributions, motivated by superstatistical models. Starting from a prescribed stationary distribution obtained from a superstatistical mixture of Maxwellians, we systematically derive a family of orthogonal polynomials associated with this reference state. These polynomials provide a natural basis for expanding velocity distribution functions around non-thermal equilibria.

We demonstrate the advantages of this approach through synthetic experiments in which perturbed non-equilibrium distributions are expanded using both traditional Maxwellian-based bases and the proposed adaptive basis. Our results show that when the basis is matched to the underlying stationary distribution, the dominant information of the system is captured by the lowest-order modes, leading to significantly faster spectral convergence. In contrast, standard Hermite or Laguerre expansions require a substantially larger number of modes to represent the same distribution, particularly in the presence of suprathermal tails.

This framework provides a systematic bridge between superstatistical descriptions of non-equilibrium states and spectral representations of kinetic distributions, opening new possibilities for reduced modeling and analysis of complex systems far from equilibrium.