

Feedback control: Markovian description and information bounds for the work

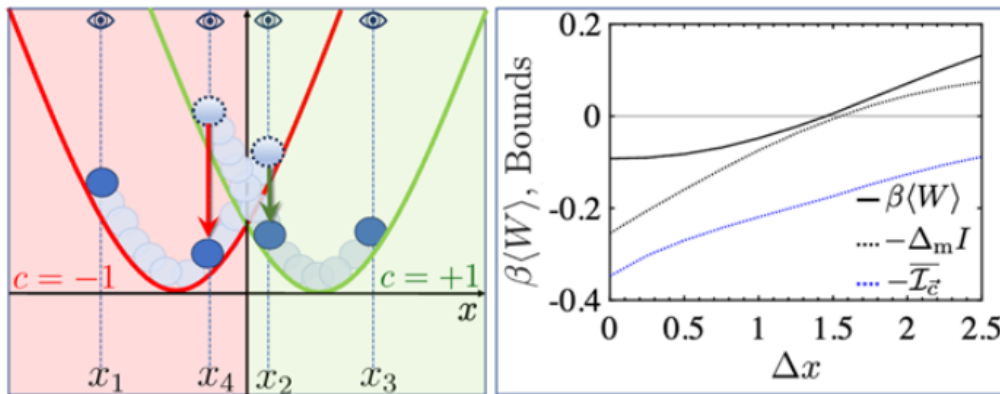
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In feedback-controlled systems, a particle evolves in a potential $V(x, c)$ that is tuned by an external controller c . This controller measures a variable related to the particle state and acts as a demon, using this information to change its own state and modify the potential experienced by the particle. In this way, work can be extracted from a single thermal reservoir by treating information as a thermodynamic resource. This leads to an apparent violation of the second law, which is resolved once the information contribution is explicitly included. The form of this correction, and therefore the corresponding extension of the second law, depends on how the demon, that is, the feedback device, is modeled.

Here, we focus on a broad class of feedback-controlled systems in which an overdamped Brownian particle evolves in a fluctuating potential that is periodically updated every Δt_m , when the controller measures the particle position and changes its internal state. For these systems, two descriptions of the demon can be considered. In the first one, the demon keeps memory of past information, and the full system is described by the instantaneous particle position together with the complete sequence of control actions [1]. In the second one, the demon retains only its current state, so the system is described by (x, c) [2], which forms a Markovian state vector [3]. In the first description, the correction to the second law is given by the transfer entropy I_c , which depends on the probability of the control sequence. In the second description, the correction is given by the change in Markovian information at the measurement, $\Delta_m I$.

By comparing both frameworks, we show that the work bound based on mutual information is always tighter than the one derived from transfer entropy [2].



(Left) Sketch of the harmonic information engine. An overdamped Brownian particle evolves in a harmonic trap with a centre set by the measurement outcome y_n . This outcome corresponds to a noisy measurement of the particle position x_n at times $t_n = n\Delta t_m$, with a uniform error $y_n \in (x_n - \Delta x, x_n + \Delta x)$. If $y_n < 0$, the trap centre is switched to $-L$; otherwise, it is switched to $+L$. (Right) Average work and bounds as a function of the measurement uncertainty Δx .

References:

- [1] Sagawa, T. and Ueda, M., Nonequilibrium thermodynamics of feedback control, *Physical Review E* 85 (2012) 021104.
- [2] Ruiz-Pino, N. and Prados, A., Entropic balance with feedback control: Information equalities and tight inequalities, *Physical Review Letters* 136 (2026) 067101.
- [3] Ruiz-Pino, N. and Prados, A., Markovian description of a wide class of feedback-controlled systems: Application to the feedback flashing ratchet, *Journal of Statistical Mechanics: Theory and Experiment* 2024 (2024) 083204.