Ultrametric predictions for small-field chaos in spin glasses: theory and simulations

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We investigate the chaotic response of the Gibbs state in spin glasses to an external magnetic field, focusing on the crossover regime in which the (vanishing) field strength scales as 1/N, where N is the system size. We show that Replica Symmetry Breaking (RSB) theory [1] yields universal predictions for this chaotic behavior: these depend only on the zero-field overlap distribution P(q) and not on any other microscopic details of the system. Using P(q) as the sole input, we obtain quantitative analytical predictions for the statistics of the states in a weak magnetic field.

In the infinite-volume limit, each spin-glass sample is described by an infinite hierarchy of states organized in a tree-like structure. We generate the associated probability distribution efficiently by sampling from a representation based on the Bolthausen–Sznitman coalescent [2]. This allows us to compute quantitatively, in the crossover regime, the properties of the system in a magnetic field, including the overlap distribution in the presence of a weak field and the degree of decorrelation as the field increases.

To test these predictions, we performed simulations of the Bethe-lattice spin glass and the four-dimensional [3] and three-dimensional Edwards–Anderson models [4], finding excellent agreement in both cases with the universal predictions.

References:

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