Cluster-based Message-Passing Optimization in Rugged Energy Landscapes

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Spin systems with competing interactions defined on sparse and heterogeneous graphs constitute the backbone of many QUBO and Ising optimization formulations [1,2], with applications spanning statistical physics and real-world industrial optimization [3]. Yet, even with modern computational tools and specialized hardware, reliably locating low-energy states in these models remains difficult. Algorithms based on single-spin updates frequently stall in metastable configurations, especially in regimes where frustration and network topology give rise to spatially extended correlations [4].

We propose CLuMP (CLuster-based Message-Passing), an algorithmic framework that combines belief propagation (BP) [5] with collective spin rearrangements. Instead of relying on purely local moves, CLuMP constructs connected subgraphs and performs coordinated updates informed by BP messages. By explicitly limiting the amount of frustration allowed inside each cluster, the method ensures stable BP convergence on comparatively large subgraphs, enabling coherent transformations that may involve hundreds of spins simultaneously. In this way, CLuMP extends cluster-based Monte Carlo concepts [6,7] to frustrated sparse systems, where traditional cluster techniques are generally ineffective.

The method is assessed on spin-glass models across several underlying graph structures, including random regular graphs as well as two- and three-dimensional lattice topologies. Across all tested settings, the proposed cluster updates consistently escape metastable trapping more effectively than state-of-the-art local heuristics, achieving lower energies with fewer effective computational steps. These findings demonstrate that large-scale, frustration-aware collective updates can be implemented efficiently even in sparse disordered systems.

More broadly, CLuMP offers a scalable computational strategy for combinatorial optimization and probabilistic inference tasks characterized by rugged energy landscapes. By harnessing medium- and long-range correlations through message passing, it provides a practical route beyond the intrinsic limitations of strictly local dynamics.

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