Inferring Weighted Networks Across Scales from Coarse-Grained Data

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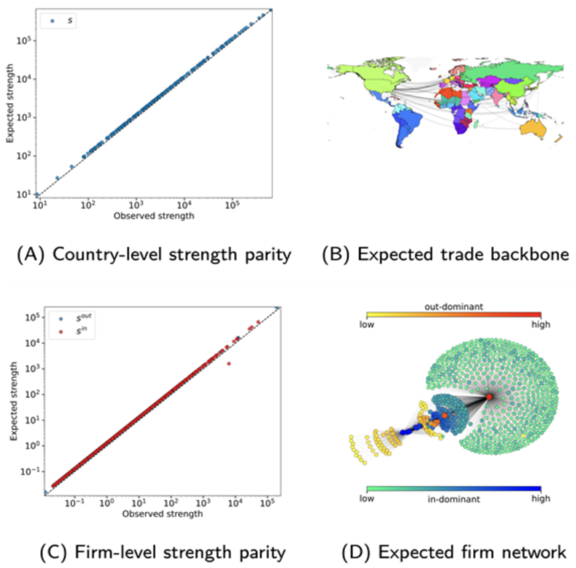

(A) Country-level strength parity (B) Expected trade backbone

(C) Firm-level strength parity (D) Expected firm network

Many economic systems can be modeled as networks, where nodes represent firms, sectors or countries and links encode production, trade or financial flows. In practice, however, these systems are often observed only in aggregated form: firms are grouped into sectors, countries into macro-regions, and only aggregate flows are available, while the underlying microscopic links remain hidden. This mismatch between the scale of interactions and that of observation is especially severe in production systems, where firm-to-firm relations are confidential and only sector-level information is accessible. Recent work introduced the first geometry-free scale-invariant framework for binary network reconstruction under node aggregation [1]. Here we extend that idea to the weighted case and introduce the weighted Scale-Invariant Gravity Model (wSIGM), to our knowledge the first weighted reconstruction model explicitly designed to preserve its functional form under aggregation, enabling calibration on an observed coarse layer and consistent propagation to finer resolutions without refitting. The framework applies to both undirected and directed networks. We validate it on two systems with known microscopic structure: the undirected International Trade Network (ITN), where we calibrate wSIGM on macro-regions and reconstruct countries, and the directed Dutch production network, reconstructed from an exclusive 2021 administrative dataset made available through our collaboration with Statistics Netherlands (CBS), where we calibrate on 3-digit Dutch industry classes and reconstruct both 4-digit classes and individual firms.

We benchmark wSIGM against CReMB, a state-of-the-art weighted reconstruction method that uses essentially the same local marginals but is calibrated directly on the target fine-grained layer [2,3]. On the ITN, averaged over 1991–2000, wSIGM improves country-level precision from 0.788 to 0.822 (+4.3%), specificity from 0.681 to 0.749 (+9.9%), and accuracy from 0.746 to 0.765 (+2.5%). At the weighted level, it reduces the root-mean-square error from 1304.8 to 1295.4 (−0.7%), while increasing the coefficient of determination from 0.505 to 0.511 and the weighted Jaccard similarity from 0.428 to 0.433, despite using only the aggregated density constraint. On the Dutch benchmark, across all available goods groups, wSIGM systematically improves precision, specificity and accuracy at both 4-digit industry and firm level; at firm level, mean precision nearly doubles, increasing from 0.074 to 0.140. For the representative goods group shown here, it also yields a more accurate firm-level weighted reconstruction, reducing the root-mean-square error from 4.15 to 3.31 (−20.4%), increasing the coefficient of determination from 0.937 to 0.960, and raising the weighted Jaccard similarity from 0.504 to 0.555. These results show that detailed weighted economic networks can be inferred reliably from coarse-grained information alone.

References:

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