

Compact localized states and ergodicity breaking in a nonlinear lattice model

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We study thermalization in a fully nonlinear lattice model derived from the two-dimensional defocusing nonlinear Schrödinger equation, with particular emphasis on the role of compact localized excitations in the onset of nonergodic behavior [1]. The model was originally introduced as a toy model for energy transfer in the cubic defocusing nonlinear Schrödinger equation [2]. Using energy-based arguments together with long-time numerical simulations, we identify both ergodic and nonergodic regimes, including a broad region in which the dynamics remains ergodic outside the standard Gibbsian framework. At higher energy densities, the system enters a nonergodic regime in which persistent few-site localization emerges and conventional thermalization breaks down. In this regime, one-site and two-site compact states organize the long-time evolution: for $D < 1$, the energetically relevant maximizer is the one-site state, whereas for $D > 1$ the dynamics predominantly selects two-site staggered compactons. The case $D = 2$ is of particular interest, since long-lived three-site localized structures may also arise for suitable initial data, while the dominant robust pattern remains the two-site staggered state. Therefore, the negative-temperature nonergodic regime is governed by energy maximizers rather than minimizers. These findings complement recent results for the same toy-model setting, where Marzuola and Mattingly classified fixed-mass energy minimizers and identified a distinguished three-mode minimizing family relevant to the invariant measure [3]. Altogether, our results indicate that compact localized states are key dynamical structures in the suppression of thermalization in nonlinear discrete systems.

References:

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