Thermalization Speed of Multimode Nonlinear Lattices

Emmanouil Theodoros Kokkinakis[1],Mercedeh Khajavikhan, Demetrios Christodoulides, Konstantinos Makris

[1]University Of Crete, Heraklion, Greece

Multimode nonlinear optical systems are known to thermalize toward a Rayleigh–Jeans (RJ) distribution determined by their conserved quantities. Here, we show that these invariants do not uniquely determine the rate of thermalization. Even when different initial excitations share the same conserved quantities and converge to the same equilibrium state, their thermalization distances can differ significantly.

We demonstrate this behavior in three representative lattice settings: a one-dimensional uniform lattice, a one-dimensional off-diagonal disordered lattice, and a two-dimensional non-Hermitian Hatano–Nelson lattice. In all cases, we find that the approach to equilibrium depends sensitively on the structure of the initial modal distribution, beyond its associated thermodynamic parameters.

To quantify this effect, we introduce a measure of the separation between nearby initial modal configurations that preserve the same conserved quantities. We observe that faster thermalization is associated with a stronger short-distance growth of this separation, indicating a more pronounced chaotic response, while slower thermalization corresponds to weaker growth.

These results show that, the thermalization rate is not fixed by the conserved quantities alone, but also depends on the initial modal distribution. This opens up the prospect of accelerating thermalization toward a target equilibrium state through a suitable choice of initial conditions.