

# Nonlinear Fokker-Planck Equation for Strongly Pseudo-additive Entropies

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In accordance with Jaynes' maximum entropy principle, the system equilibrium distribution can be obtained as a solution to the entropy maximization subject to the complex system internal energy constraint. In the case of Shannon entropy maximization, the resulting distribution is the Boltzmann-Gibbs one, and the system equilibrium is described by Boltzmann-Gibbs thermodynamics. However, once the Shannon entropy is substituted by an alternative entropy form, the resulting equilibrium distribution is not the Boltzmann-Gibbs one. This is particularly relevant for the interpretation of the nonlinear Fokker-Planck equation (NLFPE), which is a widely accepted model for the analysis of time series fluctuations. On the other hand, various alternatives to Shannon entropy have been proposed in information theory and statistical mechanics. The most general class that satisfies the generalized Shannon-Khinchin axioms is the class of Strongly Pseudo-Additive (SPA) entropies, which have been widely used in the modeling of complex systems that are characterized by heavy tails.

In this talk, following the method by Frank and Daffertshofer, we provide the general explicit form of the NLFPE drift and diffusion terms that correspond to SPA entropies, by requiring that the steady-state solution of the NLFPE is the same as the equilibrium solution computed by the maximum entropy principle. The result includes, as special cases, previously derived forms corresponding to Sharma-Mittal, Tsallis, Rényi, and Gaussian entropies. In addition, we present new results that are related to alternative entropy measures such as Nojiri-Odintsov-Faraoni entropy and logarithmically corrected Shannon entropy, which were previously considered in black hole thermodynamics. The resulting drift and diffusion terms are analyzed from thermodynamic and information theoretic perspectives.

References:

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