Properties of superdiffusive processes arising from compounded random walks

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The development of random walk models for non-Gaussian processes has been of particular interest due to their frequent occurrence in experiments. These new models give useful insight into the underlying physical mechanisms of the process and are capable of explaining deviations in asymptotic behaviour from existing models. One particular type of a non-Gaussian process is superdiffusion, characterised by a mean-square displacement which is proportional to t^μ where t is the time and $\mu > 1$ is the superdiffusive exponent. This behaviour has been observed in the way animals move, the price of stocks and the dispersion of physical media.

Typically, such processes are modelled using Levy flights, which are derived from a random walk consisting of jump lengths with divergent mean. However, it is not obvious how such models should be modified to accommodate barriers and potentials as their discontinuous trajectory of the process allows the random walk to completely ignore them. Additionally, derivations of modified models for the bounded domain do not retain the same Boltzmann distribution as the standard Gaussian process. It has recently been demonstrated that models which exhibit superdiffusivity on unbounded domains while also being well defined in the bounded domain may be derived via a compounded random walk [1]. This model is able to remedy both of the issues arising from Levy flight models.

In this talk, we formulate the governing equations, which include a space-fractional spectral Fokker–Planck operator, a generalisation of the spectral fractional Laplacian from a compounded random walk in a space dependent potential. Due to the incorporation of compounding in the stochastic formulation, many of the unusual phenomena which arise cannot be obtained from existing models. Of particular note, is the first passage times, which indicates the first time when a random walk could reach a location to trigger an event. Here, we examine the first-passage properties of compounded random walks and compare them with the existing results for Levy flights.

Reference:

[1] C. N. Angstmann, D. S. Han, B. I. Henry, B. Z. Huang, and Z. Xu, Compounded random walk for space-fractional diffusion on finite domains. Phys. Rev. E 111, 024136 (2025).