How does a cloud of random walkers with exclusion relax to equilibrium?

Christophe Deroulers[1,2], Stéphane Plaszczynski[2], Alexandre Souto Martinez[3]

[1]Université Paris Cité, Paris, France, [2]IJCLab (CNRS, univ. Paris Saclay & Paris Cité), Orsay, France, [3]Universidade de São Paulo, Ribeirão Preto, Brazil

Exclusion, or hardcore repulsion (the impossibility for two finite-size particles to share the same position in space), is probably the simplest local interaction, yet it is a key phenomenon in many contexts or models. It is present in almost all physical systems of active or condensed matter to take into account the excluded volume of agents, atoms, or molecules. E.g. motility-induced phase separation of active matter doesn't exist without exclusion. In biology, collective behaviour of living cells and tissues is also partly conditioned by the steric interactions of cells. In machine learning applications, like the promising technique of particle swarm optimisation, or discrete diffusive generative artificial intelligence, there are clouds of agents moving in an abstract space, and some algorithms employ short-range repulsive behaviour of these agents to improve their performance.

Here, we investigate how a set of discernible or indiscernible particles performing continuous time random walks with exclusion relaxes to equilibrium on regular graphs (i.e. where all nodes have the same degree) and in particular on hypercubic lattices of finite size. Even if the particles do not perform pure random walks (e.g. if they do run-and-tumble motion), our results will apply on space and time scales where the ballistic component of their motion becomes negligible.

When the particles are indiscernible, this model is the simple exclusion process, and has been the subject of numerous studies. In particular, in the last twenty years, remarkable rigorous results about the so-called cutoff phenomenon (an abrupt drop of the distance which separates the system from the uniform, equilibrium distribution of particles during the relaxation) have been obtained in different geometries. However, like most interactions in many-body systems, exclusion makes analytical predictions of the collective behaviour of agents difficult, and there are still many open questions, especially for discernible particles, on non-simple geometries, or going beyond the one-site occupation probability.

We study the structure of the spectrum of the evolution matrix of this stochastic process. We compute several of the lowest-magnitude eigenvalues (those which govern the large-time relaxation) perturbatively, both for discernible and indiscernible particles. This allows us to give asymptotic analytic expressions for the global relaxation time of the cloud of particles, but also for the relaxation time of its radius, which is much smaller, as functions of the lattice size and shape and of the number of particles. We compare them to exact numerical results obtained in 1D and 2D for modest system sizes and numbers of particles.

Finally, we give some results on regular random graphs and on non-regular graphs. This may be useful e.g. to provide information about the efficiency and speed of convergence of recently developed frugal exploration strategies of space by swarms of robots.