Synchronizability in networks with higher-order directed interactions

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Understanding which network structures promote or obstruct coordinated behavior in coupled dynamical systems is a central challenge in network science, with implications across physics, biology, engineering, and the social sciences. Although synchronization in networks with pairwise interactions among identical nodes has been extensively investigated, many real-world systems involve both heterogeneous agents and higher-order interactions that cannot be reduced to simple links between pairs. In such settings, the mechanisms that support or prevent collective coordination remain far less understood.

To address this gap, we introduce a new definition of network synchronizability that is specifically designed for hypergraphs, where interactions can occur among multiple nodes simultaneously. Unlike traditional notions of synchronizability, which often assume identical node dynamics, our framework captures the ability of a hypergraph topology to align trajectories even when nodes differ in their intrinsic parameters. This makes the proposed measure especially relevant for realistic systems in which heterogeneity is unavoidable. Also, we allow the higher-order interactions to be directed, which is a particularly relevant in control applications [1,2].

The synchronizability metric is derived through a mathematical formulation that links structural properties of the hypergraph to the observed collective dynamics of the system [3]. We show that the metric strongly correlates with empirical synchronization behavior, providing a reliable predictor of when coordinated motion is likely to emerge. Importantly, the method also scales efficiently with network size, making it suitable for the analysis of large systems that would otherwise be difficult to study through direct simulation alone.

This scalability allows for a systematic comparison between networks governed by higher-order interactions and those based solely on pairwise couplings. Through this comparison, we identify multi-body structures that most favor the onset of synchronization, as well as hyperedge types that instead hinder coordination in systems of nonidentical nodes. These results clarify the functional role of higher-order connectivity and reveal that its effects are not universally beneficial, but depend on the interplay between topology and node heterogeneity.

Finally, we apply the framework to opinion dynamics, demonstrating how the metric can be used to detect hyperedges that promote consensus within social groups. In this way, the approach not only advances the theory of synchronization in complex systems but also provides a practical tool for identifying interaction patterns that support collective agreement in heterogeneous social networks.

References:

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