Optimal navigation in stochastic environments: insights from driven Brownian motion in disordered landscapes

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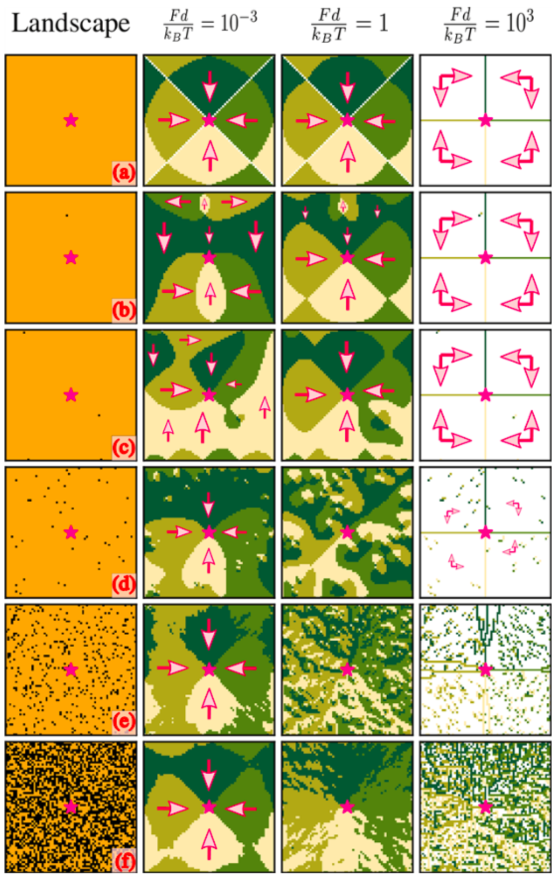

Navigation at the nanoscale, from synthetic nanorobots to living microorganisms, occurs in environments shaped by both thermal fluctuations and structural disorder. In such environments, optimal navigation strategies must reconcile external control with diffusion in landscapes where structural disorder modifies the dynamics. Understanding how environmental disorder reshapes optimal decision-making policies is essential for applications ranging from targeted drug delivery to atomic-scale fabrication processes, as well as for interpreting navigation strategies in biological systems [1–3].

In this work, we investigate the optimal navigation of a Brownian particle driven by a force that biases its motion in a frozen disordered energy landscape, modeled as a lattice with randomly distributed traps [8]. The objective of the particle is to reach a specified target position in minimal mean first-passage time (MFPT). Using dynamic programming techniques, we compute optimal navigation policies that minimize this MFPT and characterize how these policies are modified by the presence of disorder.

We show that the probability for the local policy to change in the presence of traps is directly related to the Kullback–Leibler divergence between the optimal policy distributions in disordered and homogeneous environments. This provides a quantitative measure of how strongly environmental heterogeneity restructures decision-making. Remarkably, this restructuring exhibits a non-monotonic dependence on concentration of traps. We derive an analytical expression for the distribution of optimal policies in the small-force regime where fluctuations become prominent and show that the location of maximum change of the optimal policy occurs at concentration that scales inversely with trap strength.

Importantly, these results do not depend on a specific form of the transition rates and can be generalized to other models, provided they can be linearized with respect to the control parameter. In experimental platforms such as driven colloids, where the applied force is comparable to thermal fluctuations [4–7], this implies that a dilute concentration of traps may produce the strongest modifications of navigation strategies. Our findings therefore extend to a broad class of navigation models, including those used to describe animal foraging and robotic exploration, providing a generic framework to quantify how structural disorder reshapes optimal policies in stochastic navigation problems [8].

References:
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