Universal results for stochastic renewal processes in presence of drift and state dependent noise

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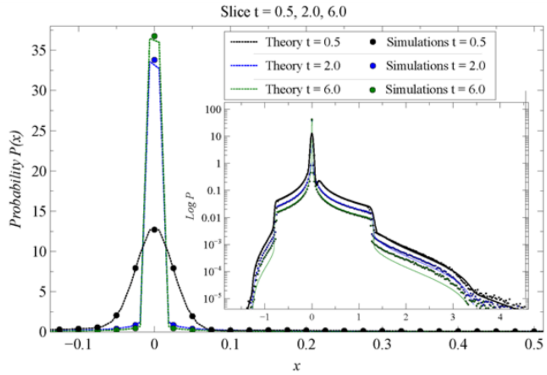

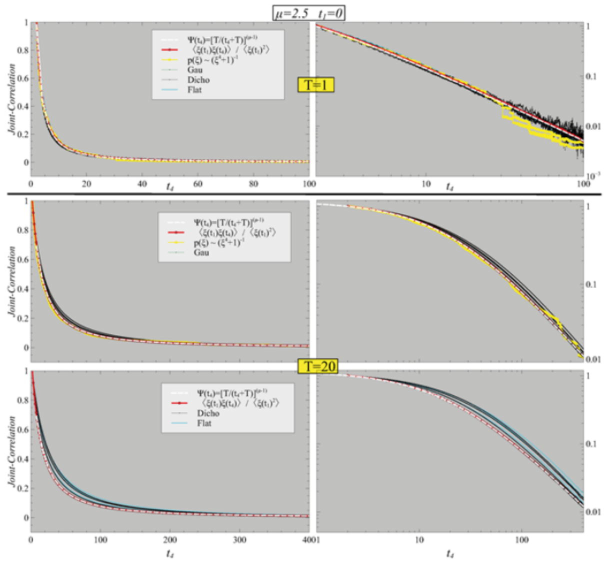

Lévy walks and Lévy flights (or Continuous Time Random Walks, CTRW) with drift and position-dependent jumps provide a general framework for describing a wide range of natural and engineered systems. These correspond to stochastic differential equations (SDEs) with multiplicative step and spike/shot noise, respectively. We analyze these classes of models and derive several exact and asymptotic analytical results.

First, we obtain a closed-form expression for the n-time correlation functions of both noise types (spike and step), expressed as a sum over all 2^{n-1} compositions of the observation times. The connection to the G-cumulant formalism is immediate: this ordered partition structure mirrors the moment-G-cumulant relation, which simplifies the subsequent derivation of the Master Equation (ME).

Second, we derive simple and universal analytical expressions for the aforementioned results. Specifically, for step noise:

1. In the Poissonian case, we observe fast (exponential in time) convergence towards the telegraph process, allowing an exact and simple ME for the corresponding Lévy walk process, with drift and state dependent jumps. From this result we argue that, in many cases, the standard Gaussian-noise hypothesis is neither realistic nor an especially effective route to inferring the statistical properties of the variables of interest. These properties can instead be derived more directly, without invoking the Gaussian assumption, by the fact that the underlying fluctuations can be effectively approximated by Telegraph-like noise.

2. For waiting-time PDFs that decay as a power law with exponents $\mu > 2$ and $1 < \mu < 2$, we show that the multi-time correlation functions decay similarly to the two-time correlation function evaluated at the extreme times. This result, through the use of G-cumulants, allows us to directly obtain the ME for the Lévy walk process, with drift and multiplicative noise.

In the case of spike/shot noise,

1. we obtain an exact, non-local-in-time universal ME for the corresponding CTRW with drift and position-dependent jumps.

2. Under a moderate time-scale separation, this exact ME collapses into a simple local-in-time equation that is formally identical to the Poissonian ME, with the constant rate $1/\tau$ replaced by a time-dependent renewal rate $R(t)$.

From these results, both local and global properties of the PDF can be readily inferred. This framework offers a simple yet realistic representation of stochastic forcing in complex dynamics, with broad applications across physics, biology, climate science, and beyond.