Pareto-optimal protocols for active particles in moving traps

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Optimal control theory is often framed around a single objective: minimizing the average work, entropy production, or fluctuations required to drive a system between two states. This single-objective perspective has yielded remarkable results, including bang–bang protocols, geometric bounds on dissipation, and connections to optimal transport theory. In realistic physical and biological systems, however, multiple performance measures compete simultaneously, and improving one typically comes at a cost to another. Protocols that minimize average work, for instance, often lead to large fluctuations, while protocols that suppress fluctuations tend to require greater energetic input. Optimizing a single objective therefore provides only a partial view of the full landscape of achievable control strategies, and the trade-offs between competing objectives remain largely unexplored in the context of thermodynamic optimal control.

In this talk, I introduce a general framework for Pareto-optimal thermodynamic control that makes these trade-offs explicit and systematic. Rather than targeting a single predefined objective, the approach characterizes the complete family of driving protocols that realize optimal compromises between competing thermodynamic costs. This family of solutions forms a Pareto front in objective space, where each point corresponds to a protocol that minimizes a particular weighted combination of the relevant objectives. The framework reduces to a generalized Euler–Lagrange equation for an effective Lagrangian, so that the full landscape of optimal trade-offs is accessible through a unified variational principle. Well-known single-objective results emerge naturally as limiting cases at the boundaries of the front.

As a concrete and analytically tractable example, I study an active particle dragged through a fluid by a time-dependent harmonic trap. The two competing objectives are the mean work input and its fluctuations, quantified by the variance of the work. I show that this problem admits an exact closed-form solution valid across all driving regimes, yielding a one-parameter family of optimal protocols that smoothly interpolates between the two extremes of minimizing average work or suppressing fluctuations entirely. The optimal protocols take a universal functional form governed by a single characteristic timescale, which itself depends on a modified effective Péclet number encoding the relative importance of the two competing objectives. This effective Péclet number reveals that systems with different activity strengths can realize identical optimal control architectures at different points on their respective Pareto fronts, defining equivalence classes of optimal control problems. The degeneracy is broken only at the level of thermodynamic costs, where more active systems pay a higher fluctuation penalty for the same mean work.