Superstatistical methods for complex systems

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The superstatistics concept was introduced some 23 years ago [1], yet it still finds new applications in a variety of complex systems that exhibit time scale separation in their dynamical variables. Typically, in a superstatistical system there is a parameter (or several such parameters) that fluctuates on a long time scale, long in comparison to the local relaxation time of the system. This parameter can, for example, be the local inverse temperature in a spatially extended system, or the energy dissipation rate in a turbulent flow, or a locally varying variance parameter in a given measured time series. As there is a slowly varying temperature-like variable this is a non-equilibrium situation, which leads to superpositions of different statistics (hence the name 'superstatistics'). These types of superpositions naturally generate heavy tails in the marginal probability densities of the complex system under consideration.

After a brief introduction to the basic concepts I will discuss as a typical example superstatistical behaviour in Lagrangian turbulence [2]. I will then proceed to more recent work [3] dealing with anomalous velocity distributions observed in slow quantum-tunneling chemical reactions. Here one observes q-Gaussian probability distributions where the entropic index q depends on the density n of the reactants. I will discuss a theory based on superstatistics that explains this density dependence q=q(n). The theory also yields predictions for the statistics of temperature fluctuations in small ion trap experimental systems which are experimental realizations. If time remains I will also talk about simple 1-d maps (generalizations of the continued fraction map) that exhibit q-Gaussian behaviour at a critical point [4]. In this case the heavy tails are produced by a different mechanism – strong correlations at the critical point.

References:

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