From Local Instability to Global Chaos: How Hamiltonian Dynamics Builds Statistical Mechanics

Chris Antonopoulos[1]
[1]University of Essex, Colchester, United Kingdom

How does the rich statistical behaviour of many body systems arise from the bare bones of deterministic Hamiltonian dynamics? This talk tackles that fundamental question by tracing, step by step, how microscopic instability grows, spreads, and ultimately fuels macroscopic ergodicity in nonlinear lattices and high dimensional Hamiltonian models.

We begin with low dimensional exemplars such as the Hénon-Heiles system, where ordered islands and chaotic seas coexist and simple periodic orbits (SPOs) act as the geometric skeleton of phase space. Building on this intuition, we move to large scale systems, including Fermi-Pasta-Ulam chains and discretised Gross-Pitaevskii (BEC) Hamiltonians, where symmetry arguments and linear stability analysis reveal entire families of SPOs and pinpoint their destabilisation thresholds. These thresholds scale predictably with system size and energy density, marking the onset of weak chaos.

Pushing to higher energies, we show how chaotic layers associated with different SPOs interact, merge, and eventually generate strong, system wide chaos. In this regime, Lyapunov spectra converge and the Kolmogorov-Sinai entropy grows linearly with the number of degrees of freedom, demonstrating its extensivity and signalling the emergence of ergodic, thermodynamic like behaviour.

To navigate these vast phase spaces efficiently, we employ the Smaller Alignment Index (SALI), whose sensitivity and speed make it an ideal tool for detecting chaos in high dimensions.

Altogether, the results build a unified dynamical picture: from local instabilities around specific periodic structures to global statistical behaviour in the thermodynamic limit, offering insight into how chaos underpins the foundations of statistical mechanics.