Observation-Time-Induced Crossover in Diffusion with Fluctuating Diffusivity

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Diffusion in heterogeneous systems is often characterized by fluctuating diffusivity, giving rise to rich stochastic behavior beyond simple Brownian motion. While the temperature dependence of diffusion is usually regarded as well understood, typically assuming a linear proportionality between diffusivity and temperature, we show that this apparent scaling can exhibit nontrivial crossover behavior depending on the observation time.

Within a general framework of diffusion with fluctuating diffusivity, we demonstrate that the proportionality coefficient linking diffusivity to temperature is not necessarily a fixed quantity, but can become observation-time dependent. In particular, short-time measurements predominantly reflect local fluctuations of the diffusivity, capturing the intrinsic heterogeneity of the underlying medium and the variability of local transport environments. In contrast, long-time observations probe an effectively coarse-grained dynamics, where temporal averaging leads to a renormalized, effective diffusivity that masks microscopic fluctuations. As a consequence, distinct scaling regimes emerge, resulting in an observation-time-induced crossover in the apparent temperature dependence of diffusion.

We analyze the mechanism underlying this crossover by explicitly considering the competition between the intrinsic timescales of diffusivity fluctuations and the observation time. When the observation time is shorter than the characteristic timescale of diffusivity variation, the system exhibits locally heterogeneous transport properties, and the measured diffusivity reflects instantaneous fluctuations. Conversely, when the observation time is sufficiently long, the system approaches an effectively homogeneous regime governed by averaged dynamics. This transition between regimes leads to a qualitative change in the observed temperature scaling, rather than a simple quantitative correction, and thus requires careful interpretation when comparing experimental and theoretical results.

Although non-Gaussian features are often emphasized in heterogeneous diffusion processes, our results show that even coarse-grained observables such as diffusion coefficient retain clear signatures of underlying heterogeneity through their observation-time dependence. In this sense, the present framework provides a complementary perspective to stochastic dynamics beyond Gaussianity, focusing instead on effective transport coefficients, their scaling behavior, and the role of temporal coarse-graining.

These findings provide a unified interpretation of time-dependent transport in a wide range of complex systems, including soft matter, biological environments, and disordered materials, where fluctuating environments are ubiquitous. More broadly, they highlight the fundamental role of observation protocols in determining experimentally inferred transport coefficients, and suggest that care must be taken when interpreting temperature scaling from finite-time measurements. The present results thus offer a new viewpoint on how apparent physical laws can emerge from, and be modified by, underlying stochastic heterogeneity.