

Information geometry on the thermal probability distributions for a weakly confining potential

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Information geometry is a useful framework for studying some families of probability distributions by identifying the space of probability distributions with a differentiable manifold endowed with a Riemannian metric and an affine connection which is not Levi-Civita connection. It is known that early developments of information geometry are mainly based on well-known exponential family of probability distributions. We studied some information geometric structures on the κ - and/or q -deformed-exponential families of probability distributions, which are non-Gaussians and with heavy-tails. On these deformed exponential families, we constructed the suitable statistical manifolds and showed some information geometric structures such as generalized Fisher metrics, dually-flat structures, generalized divergence functions, and so on. In this way we now know that information geometric structures exists for not only standard exponential families but also for deformed exponential families. However, if the information geometric structures for the standard exponential families and those for deformed exponential families exist separately and there had no relation each to other, one would have no special interest on them. In this contribution we consider the thermal probability distributions for a weakly confining potential in the basic framework of statistical physics. In contrast to the well-known standard case of Gaussian (or Boltzmann-Gibbs) distributions for strongly confining potential, e.g., a parabolic potential, the quasi-equilibrium thermal probability distributions for a special type of weakly confining potentials become non-Gaussian distributions and with heavy-tails, which describe anomalous diffusions and transports. We study the information geometric structures on the thermal probability distributions for a special type of weakly confining potential and show that the corresponding quasi-equilibrium distribution is indeed the κ -deformed exponential probability distribution, which is introduced by Kaniadakis and Scarfone. In addition we relate the Boltzmann-Gibbs distributions for this weakly confining potential and the κ -exponential distributions for the strongly confining potential, and discuss on the associated information geometric structures.

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