The quantum first detection problem: From the energy spectrum to the detection probabilities

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We consider the question of when a quantum system initially prepared in state A first arrives" in state B, i.e. the first arrival problem in quantum physics. To determine the arrival, the observer attempts to detect the system stroboscopically with fixed period via a projective measurement.The time of the first successful detection attempt is the first detection time. The corresponding probability of the event is the first detection probability. Formulated in amplitudes rather than in probabilities, the problem becomes very similar to the classical first passage theory of random walks.

It is possible to derive a renewal equation that connects the first detection amplitudes with the free evolution operator. The equation is solved via the classical technique of generating functions.

Due to its quantum nature, the problem exhibits many non-classical phenomena. E.g. due to the quantum Zeno effect, the dynamics of a quantum system becomes locked if it is observed to frequently. This prevents the system to ever be detected. Furthermore, interference effects lead to a non-monotonous decay of the first detection probabilities.

For systems with a continuous energy spectrum, e.g. a freely moving particle, the first detection probability can be expressed in terms of the spectral measure of the evolution operator (which is related to the density of energy states). This allows us to present an exact formula for the total probability of detection in terms of the spectral measures and their Hilbert transforms. The total probability of detection is always less than unity, indicating that the quantum particle is highly likely to either silently pass the detector or to be reflected off the device.

Furthermore, we discuss long-time asymptotic behaviour of the first detection probabilities. It is shown that the latter decay like a power-law with superimposed oscillations. The exponent of the power law is determined by the spectral (or fracton) dimension of the spectral measures alone. The amplitude, phase, and frequency of the oscillation are determined by critical points of the energy-momentum surface and depend in general on the energy spectrum in its full detail.

[l] Friedman, Kessler and Barkai, J. Phys. A **50**, 04LT01 (2017).

[2] Friedman, Kessler and Barkai, arxiv1611.05676 (2016).