Maximum-entropy models for networks

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Entropy-maximization represents the unifying concept underlying the definition of a number of methods which are now part of the discipline known as "network theory". Examples of paramount importance are provided by null models, employed both for pattern detection and for for inferring the presence of unaccessible connections from partial information. Recently, a novel application of null models has been proposed, which consists in quantifying the information asymmetry of nodes. In other words, entropy can be fruitfully used to quantify the information content that nodes have access to, thus providing a novel indicator of their topological importance.

In this talk, a general method to define constrained maximum-entropy network ensembles will be illustrated and specific examples of the three aforementioned applications provided. In particular, 1) pattern detection techniques will be applied to reveal early-warning signals of the 2007-2008 worldwide crisis. Two real-world systems will be discussed: the Dutch Interbank Network and the World Trade Web; 2) a comparison between different reconstruction algorithms will be carried out, with particular emphasis on their application to economic and financial networks; 3) several, different, networks will be considered and the corresponding nodes will be ranked (according to different measures which will be, then, compared).

As a conclusion, different ways of enforcing constraints (i.e., either exactly or on average) will be proven not to lead to equivalent ensembles, as measured by the entropy functional. In physical terms, the latter statement can be rephrased by saying that the microcanonical and canonical recipes are, in several cases of interest, not equivalent. This difference, quantifiable upon comparing the values of Boltzmann and Gibbs-Shannon entropy, is found to be a function of the imposed constraints: more explicitly, an extensive number of constraints leads to non-equivalence.

The latter remark implies that the approach used to analyse networks indeed makes a difference and should not be underestimated whenever approaching the study of real-world systems.

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