Cascades, Lvy-Clifford algebra of generators of vector multifractals to analyse and simulate intermittent dynamical systems

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In the 1980s, multifractals were a major breakthrough in the key question of intermittency that thus became understood as resulting from an infinite hierarchy of fractal supports of the singularities of the underlying equations. This opened an alternative to numerical, scale truncated simulations of these equations (e.g. Royer et al., 2008, Lovejoy and Schertzer, 2013 for climate).
However this was mostly done for scalar fields, whereas the fields of interest, e.g., the velocity for turbulence, are generally vector fields. The gap between arbitrary large dimensions of the domain and the restriction to 1D codomain has prevented many developments. In particular, it prevented to investigate the pivotal question of interactions between vector components and their non trivial symmetries. The latter are unfortunately indispensable for most applications and challenging issues such as the climatology of (exo-) planets based on first principles (Pierrehumbert, 2013) or to fully address the question of the relevance of quasi-geostrophic turbulence and to define an effective, fractal dimension of the atmospheric motions (Schertzer et al., 2012).
Fortunately, considering the Lie algebra of stochastic generators of cascade processes enabled to generalise multifractals to arbitrarily large codomains, e.g. large dimensional manifolds. Simple considerations on spherical and hyperbolic rotations have led to investigate the neat example of stable Levy generators on Clifford algebra. Both provide a number of seductive properties, respectively universal statistical and robust algebraic properties that define the basic symmetries of the corresponding fields (Schertzer and Tchiguirinskaia, 2015).
These properties provide first elements of a convenient multifractal calculus and should help to overcome current obstacles to the use of multifractal analysis and simulation at their full extent. This will be illustrated with the help of atmospheric turbulence simulations for two practical applications: wind energy and rainfall nowcasting.

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