Ruppeiner geometry and thermodynamic ensembles of anti-de Sitter black holes

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We discuss the role of Ruppeiner's geometry in describing restricted thermodynamic fluctuations in the context of anti de Sitter (adS) black holes which have a negative cosmological constant. We explicitly work out the relation between constrained thermodynamic fluctuations in black holes and the instrinsic as well as extrinsic thermodynamic geometry of the relevant codimension one hypersurfaces in the thermodynamic state space. Thus, for example, it turns out that the constrained moments of fluctuations of thermodynamic quantities have a geometrical interpretation as the Lie derivatives of induced thermal metrics on the relevant ensemble hypersurface. We show that Ruppeiner's geometry encodes instabilities and critical points associated with different ensembles. We then discuss the thermodynamic geometry of the extended state space of adS black holes, wherein the cosmological constant is a fluctuating thermodynamic variable. The quantity conjugate to the cosmological constant is the thermodynamic volume while the black hole mass is in fact its enthalpy. A detailed study is undertaken of the curvature contour of the 4D Kerr-AdS black hole. It turns out that geometry is able to shed light on hitherto unforeseen instabilities in various ensembles in the extendded state space of Kerr-AdS black holes. In particular, thermodynamic geometry suggests an instability in the Schwarzschild-AdS limit for all the ensembles except the pressure ensemble, which is equivalent to the unextended state space of the Kerr-AdS black holes.

In addition, we provide our perspective on the physical interpretation of the state space thermodynamic curvature for black holes and further revisit its connection with the singular part of the free energy especially in relation to a non extensive system like the black hole. We further discuss the possible significance of the thermodynamic curvature in the context of the AdS/CFT correspondence in string theory, which is a duality between a bulk gravity theory and a gauge theory living on the boundary. We end the talk with conclusions and some future plans.

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