Statistical mechanics approach to coevolving spin system

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Studies of the network structure of real systems, like financial markets, social and biological structures, networks of information, transportation networks, showed that their topology has numerous non-trivial properties. Characteristics of analyzed empirical networks could have not been explained with the classical random graph model, what gave birth to new network models, able to recreate some of the observed phenomena. Initially, most of these models focused on the graph evolution, often growth in the number of nodes and edges, with an arbitrary type of dynamics. On the other hand, we could observe the development of a different approach considering the statistical ensemble of graphs [1], also referred to as exponential random graphs. This formalism, borrowed from statistical physics, proved successful in equilibrium description of uncorrelated graphs [2] and networks with simple structural interactions [3]. It also led to a phenomenological theory of topological phase transitions in evolving networks [4]. Newly discovered concept of networks with a complex structure quickly echoed around spin models community, especially spin glass and agent-based models. One of the motivations for using complex networks in agent-based modeling was their much higher resemblance to the real-world structures, than regular lattices or Poissonian graphs.

We propose a statistical mechanics approach to a coevolving spin system with adaptive network of interactions. The dynamics of nodes states and network connections is driven by both spin configuration and network topology. We consider a hamiltonian that merges the classical Ising model and the statistical theory of correlated random networks. As a result, we obtain rich phase diagrams with different phase transitions both in the state of nodes and in the graph topology. Adjusting values of two parameters describing ratio between the topological and the spin part of the hamiltonian we can observe a whole variety of different effects including: multiple-star configurations, high degree clustering, network disintegration and recombination, continuous and discontinuous phase transitions in magnetization, energy, and the largest degree. We argue that the coupling between the spin dynamics and the structure of the network is crucial in understanding complex behavior of the real-world systems, and omitting one of the approaches renders the description incomplete.

[l] Z. Burda, J. D. Correia, A. Krzywicki, Phys. Rev. E **64**, 046118 (2001).

[2] Z. Burda, A. Krzywicki, Phys. Rev. E **67**, 046118 (2003).

[3] J. Berg, M. Lassig, Phys. Rev. Lett. **89**, 228701 (2002).

[4] G. Palla, I. Derenyi, I. Farka, Phys. Rev. E **69**, 046117 (2004).