Doubly autoparallelism on the space of probability distributions

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One of the important feature of information geometry studied in [1,2] is a pair of mutually dual affine connections with respect to Riemannian metric. A manifold with such geometric structure is called a statistical manifold.

In statistical manifold there exists a submanifold that is simultaneously autoparallel in terms of both of the affine connections. Such submanifolds, which we call *doubly autoparallel* (DA), play important roles in several applications, e.g., MLE of structured covariance matrices, semidefinite program (SDP) [3,4], the self-similar solutions to the porous medium equation [5] and so on.

In this presentation, we consider doubly autoparallelism on a parametric family of probability distributions with the Fisher information as a Riemannian metric, which is an important and familiar example of statistical manifold.

Consequently, we give a characterization of DA submanifolds (, i.e., statistical models of probability distributions) in a algebraic way, and discuss its interesting properties. In particular one of them is that DA submanifolds admit the unique minimizers with respect to the alpha-divergences [1,2](Tsallis relative entropies [6]) for all alpha, which implies that there uniquely exist the maximum entropy distributions with respect to not only the Boltzmann-Gibbs entropy but also Tsallis entropy with constraints of the normalized q-expectations. Finally, we show examples of DA submanifolds. The obtained results would provide us with information and insights to consider statistical models in statistical physics.

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