Burgulence in random matrix theories

M.A. Nowak

Mark Kac Complex Systems Research Center, Jagiellonian University, Poland

Traditional description of the dynamics of eigenvalues origins from the seminal paper by Dyson, who proposed stochastic description of the evolution of eigenvalues.

One can, hovever, rephrase this problem in another language, similar to the toy-model of turbulence formulated by Burgers. The basic objects in this formalism are the characteristic determinant and the inverse characteristics determinant. In this picture one can identify spectral viscosity as the inverse of the dimension of the matrix and spectral shock waves in the limit when the size tends to infinity. The limiting produre determine critical exponents of the emerging universal behavior, which can be verified in lattice simulations. This phenomenon of spectral shock waves is a generic phenomenon and appears in Hermitian, Laguerre, Jacobi and Fourier (circular) random matrix models. It explains also the origin of Airy-like, Bessel-like, Pearcey-like and Bessoid-like type of spectral oscillations in the vicinity of the critical points. One can also point at several analogies to other systems, e.g. to caustics phenomena in diffractive optics. The same concept of shock waves appears also naturally in non-hermitian random systems, but the shocks are linked to critical behavior of certain left-right eigenvector correlators (Petermann factors). Stochastic evolution of non-hermitian systems requires therefore a subtle entanglement of eigenvalues and eigenvectors, in contrast to the stochastic evolution of hermitian systems, where eigenvectors decouple.

We propose the formalism, which takes this simultaneous evolution into account. We also point out , why standard approaches to non-normal random systems have missed that coevolution. This result leads to the paradigm shift, pointing at the crucial role of eigenvectors for non-normal systems. As a particular and important application, we augment so-called single ring theorem with the new result for squared mean condition numbers. We give several examples and confirm positively our predictions by large scale numerical simulations. Finally, we analyze the consequences of this new result for some popular matricial models of neuronal networks.

[l] Z. Burda et al., Phys. Rev. Lett. **113**, 104102 (2014).

[2] S. Belinschi et al., J.Phys.A **50**, 105204 (2016).

[3] J.P. Blaizot et al., JSTAT **5**, 54037 (2016).