Methods of traveling waves in solutions of hyperolic phase field equation

I. Nizovtseva, P. Galenko

Ural Federal University, Friedrich-Schiller-Universitat Jena

Fast transitions exist when the phase interface moves with the velocity comparable or even larger than the characteristic speeds of transport processes. In the present work, we use the phase field theory of fast transitions which is described by the system of hyperbolic equations for the heat and mass transport and dynamics of interphase propagation [1]. Traveling wave solutions for the interface propagation described by the hyperbolic CahnAllen equation are found. To obtain the solution for the transition from an unstable state with the Landau potential, we use the first integral method, which directly follows from the well-known HilbertNullstellensatz theorem. The obtained complete class of traveling waves consists of continual and singular solutions. Continual solutions are represented by tanh-profiles and singular solutions exhibit unbounded discontinuity at the origin of coordinate system. With the neglecting inertia of the dynamical system, the obtained traveling waves include the previous solutions for the parabolic CahnAllen equation. The proven existence of traveling waves in a form of hyperbolic tangent function for the Cahn-Allen hyperbolic equation provides the ability to construct more complicated and much more rigorous analytical solutions of problems having an essential scientific merit and practical significance for the phase field crystal model. In particular, using the amplitude wave representation, one may reduce the phase field crystal equation, which is sixth order in space, to the hyperbolic PDE, which has a form of advanced CahnAllen equation [2]. To obtain the solution for the transition from a metastable state with the Landau de Gennes potential, we use the tanh-method which self-consistently defines the amplitude of the traveling wave, correlation length within it and its characteristic velocity. Qualitative analysis the phase interface velocity and correlation length for the given driving force of phase transition is given in comparison with the previous traveling wave solutions obtained for the crystalline front invading liquid metastable states [3].

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