Dually flat geometries in the state space of statistical models

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The study of the geometry of models of statistical physics started with the work of (Weinhold, 1975) and (Ruppeiner, 1979). A key notion is the thermodynamic length which is a measure for distances in the space of equilibrium states of the model. Appealing is the relation between model interactions and the curvature of the state space. On the other hand, (Amari, 1985) showed that the state space of a statistical model can be equipped with a dual set of flat geometries. The duality between the two geometries is based on Legendre transforms and is the same duality which in thermodynamics relates inverse temperature to energy and Massieu's function to entropy. One of the two geometries is flat in intensive coordinates such as inverse temperature and external magnetic field. The other is flat in the extensive variables energy and total magnetization. Because geodesics are straight lines it is easy to calculate a thermodynamic length in these flat geometries. The ideal gas is discussed as an example.
The relative entropy is the central quantity. It is called the Kullback-Leibler divergence in the mathematics literature. It determines the Fisher information matrix, which in the geometric approach plays the role of the metric tensor. By taking derivatives of the metric tensor one obtains the Christoffel symbols. They determine whether the geometry is flat or curved. The geodesics are calculated from the Euler-Lagrange equations.
The talk is intended for an audience of physicists knowing basics of statistical physics and thermodynamics. The mathematical language of differential geometry is avoided and replaced by familiar notions coming from classical mechanics and relativity theory.

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