State equation of a relativistic ideal gas

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The hydrostatic pressure of an ideal gas is defined as being two thirds of the average kinetic energy of the gas: $p = 2 < K > /3$, where $< K >$ is the average kinetic energy. To calculate this average the kinetic energy of the molecules is to be weighed with the equilibrium distribution function of the gas. For the purpose of this work the relativistic equilibrium distribution function will be considered, both for the classic (i.e., non-quantum) case, to wit the relativistic maxwellian, and for the quantum relativistic equilibrium distribution function, i.e., the Fermi-Dirac and Bose-Einstein distribution functions. In both cases, the relativistic equilibrium distribution function contains in turn the relativistic hamiltonian.

As is well known, starting from the relativistic maxwellian and resorting to ad hoc changes of variables, integration of the former can be performed and the normalization constant of the equilibrium distribution function can then be obtained as a function of number density and temperature: the normalization constant contains a modified Bessel function [1].

In the present work, starting from this well established result the average kinetic energy is calculated through further integration, yielding a result in closed form containing again modified Bessel functions. A large-argument approximation is then taken, and it is then shown how the equation of state can be broken into the classic part plus a correction term accounting for relativistic effects.

The same procedure is then applied starting from the quantum relativistic equilibrium distribution function developed by the same authors [2], and the cases of bosons and weakly degenerate fermions are considered. Here too, further integration leads to an expression for the average kinetic energy containing a number of modified Bessel functions: once again, large argument considerations are applied to underline the effect of the quantum equilibrium distribution functions writing the equation of state as a zeroth order term plus a quantum correction.

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[3] V. Molinari, D. Mostacci, B.D., J. Comp. **45(3)**, 212 (2016).