A matrix function approach with analysis of fractional random walks on regular n-dimensional lattices

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The study of random walks on networks has become a rapidly growing interdisciplinary research field within the last two decades. Networks are omnipresent structures in nature occuring in biology, human cities, transportation networks, electricity networks, the internet, social networks, crystalline lattices, and many further examples can be found. Networks appear as structures of communicating subsystems (nodes) which are somehow connected. A very successful approach to describe the stochastic dynamics of information exchange between the nodes in a network is the *random walk* concept [1]. Random walks appear naturally in a diversity of problems such as in the development of search strategies on networks such as the world wide web (google), but also in the context of a diffusing interstitial atom in a crystalline lattice.
In this presentation we develop an approach for time-discrete and continuous random walks on undirected regular networks with some explicit expressions for cubic periodic lattices in $n = 1, 2, 3, ..$ dimensions in the framework of Markovian process. First we invoke necessary conditions for 'good' Laplacian matrices to describe random walks. To this end we consider first the random walk on a regurlar network having only next neighbor connections [1]. Then we analyse random walks generated by matrix functions of the above mentioned simple Laplacian matrix. We consider a series of examples for admissible matrix functions to define various kinds of random walks. We demonstrate that the frational Laplacian matrix $L^{\frac{\alpha}{2}}$ defined by a power of an admissible Laplacian matrix L is admissible in the range $0 < \alpha \leq 2$. The associated fractional random walk dynamics is governed by a master equation involving *fractional powers of Laplacian matrices* $L^{\frac{\alpha}{2}}$ where $\alpha = 2$ recovers the normal random walk. We derive for regular networks, in particular n-dimensional cubic lattices key quantities such as the transition matrix, fundamental matrix and closely related generating functions for the fractional random walk. We obtain in this way for the fractional random walk the mean relaxation time (Kemeny constant), return probabilities and first passage probabilities. We show that the transition matrix exihibits for large cubic n-dimensional lattices a power law decay and for large times characteristic $t^{-\frac{n}{\alpha}}$ behavoir, indicating emergence of levy flights. It follows from our results that the efficiency to explore the network is increased when instead of a normal random walk ($\alpha = 2$) a fractional random walk ($0 < \alpha < 2$) is chosen.

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