The Legendre transform and non-additive entropies

N. Kalogeropoulos
TBA

Our goal is to elucidate the properties and the underlying dynamics of systems described by non-additive entropies. We expect that, on the way, we should also get a better understanding that may allow us to choose which entropic functional should be most appropriate for which system. In this work, we argue that the conventional Legendre transform that is extensively used in Classical Mechanics, Statistical Mechanics, Thermodynamics, Quantum Field Theory etc, has to be modified In the case of systems described by non-additive entropies.

We use a combination of statement from the theory of optimal transportation on the one hand, and convex geometry/analysis on the other hand. To be concrete in our proposal, we apply the underlying argument to the case of the Havrda-Charvat / Vajda / Daroczy / Lindhard-Nielsen / Cressie-Read / Tsallis entropy. We point out its pertinent convexity displacement properties that naturally lead us to the use and significance of s-concave functions. We combine this with some relatively recent results of Artstein-Avidan, Milman and Klartag which study the Legendre transform in such spaces of functions, propose a modification that preserves its essential properties and eventually prove its uniqueness for the class of s-concave functions.

If true, the present proposal may have far-reaching consequences for the use of Legendre transforms in Physics. Such transforms cease to have a universal form in Statistical Mechanics and Thermodynamics and their use should largely depend on the convexity properties of the corresponding entropic functional. As a result, the thermodynamic formalism based on the non-additive entropies may have to be reworked to reflect the current suggestion.

Naturally the ultimate test of such a suggestion is the confrontation of its predictions with experimental data. Due to the difficulty in performing explicit analytic computations for most models conjecturally pertinent to non-additive entropies, the present work can also be partly checked by performing numerical computations on such models.

More details about this proposal can be found in the preprint by the author The Legendre Transform in Non-additive Thermodynamics and Complexity available as arXiv:1704.08508 and references therein.

[l] N. Kalogeropoulos, The Legendre transform in Non-additive thermodynamics and Complexity, arXiv:1704.08508

[2] C. Villani, (Springer-Verlag, Berlin, Germany 2009).

[3] S. Artstein-Avidan, B. Klartag, V. Milman, Mathematika **51**, 33 (2004).