Parameter estimation via reluctant discrete quantum walks: an operational approach

P. Jarvis

University of Tasmania, School of Physical Sciences

We consider the applicability of the maximum likelihood formalism for inference and parameter estimation, in the context of measurement on quantum simulations. We set up the likelihood function, and consider the maximum likelihood estimation of a free parameter which controls the behaviour of a quantum system. Specifically, we reformulate such problems in terms of an equivalent quantum random walk scenario: the estimation, via position measurements on the walker, of the parameters of the reshuffling operator in the coin space of the walker.

In the simplest case, this parameter is the angle of an orthogonal rotation in a two dimensional coin space. We investigate the case of the 'reluctant' quantum walker, which, after k steps, is always measured to be displaced from its initial position by a fixed number d of lattice units. As the number of evolution cycles increases, the likelihood function exhibits ever sharper maxima at certain rotation angles θ^*, governed by the 'reluctance index' $r = k/d$, which gives rise to the possibility of reliable and accurate parameter estimation.

A quantum channel formulation of the original quantum walk provides for an operational approach to the determination of likelihood-maximizing intervals, and its optimal value therein. For the periodic case, and with a unitary reshuffling matrix operating on the compound coin-walker system, the maximum likelihood analysis is shown to lead to a measurement problem for a novel quantum system, describing circularly coupled quantum walks on a ring lattice.

For this work we develop novel analytical expressions for the probability distribution function and likelihood function of the walker, in the case of a two dimensional coin space, for the random walker on the line, or on a discrete interval (including periodic boundary conditions). The technical details rely on well-known properties of generalized Laguerre polynomials, arising from traces of powers of the 2×2 coin reshuffling matrix. For example for the 'very reluctant' walker which always returns to the origin with $d = 0$, the outcome is a ${}_2F_1$ polynomial, sharply-peaked around $\theta^* = \frac{1}{2}\pi$.