The square lattice Ising model on the rectangle

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The partition function of the square lattice Ising model on the rectangle, with open boundary conditions in both directions, is calculated exactly for arbitrary system size $L \times M$ and temperature. We start with the dimer method of Kasteleyn, McCoy & Wu, construct a highly symmetric block transfer matrix and derive a factorization of the involved determinant, effectively decomposing the free energy of the system into two parts, $F(L,M) = F_{\rm strip}(L,M) + F^{\rm res}_{\rm strip}(L,M)$, where the residual part $F^{\rm res}_{\rm strip}(L,M)$ contains the nontrivial finite-L contributions for fixed M [1]. It is given by the determinant of a $M/2 \times M/2$ matrix and can be mapped onto an effective spin model with M Ising spins and long-range interactions. While $F^{\rm res}_{\rm strip}(L,M)$ becomes exponentially small for large L/M or off-critical temperatures, it leads to important finite-size effects such as the critical Casimir force near criticality. In the finite-size scaling limit $L,M \to \infty$, $T \to T_{\rm c}$, with fixed temperature scaling variable $x \propto (T/T_{\rm c}-1)M$ and fixed aspect ratio $\rho \propto L/M$, we derive exponentially fast converging series for the related universal Casimir potential and Casimir force scaling functions [2]. At the critical point $T = T_{\rm c}$ we confirm predictions from conformal field theory. The presence of corners and the related corner free energy has dramatic impact on the Casimir scaling functions and leads to a logarithmic divergence of the Casimir potential scaling function at criticality.

[l] A. Hucht, J. Phys. A: Math. Theo. **50**, 065201 (2017).

[2] A. Hucht, J. Phys. A: Math. Theo. **50** (2017), arXiv:1701.08722