A connection between the damped harmonic oscillator in a heat bath and spatial data modeling

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The problem of missing spatiotemporal data is common in signal and image processing applications. Environmental data sets often include gaps, due to incomplete time coverage, sensor failures, or measurement problems. For example, parts of remotely sensed images may be obscured by clouds, aerosols, or heavy precipitation. The continuously increasing volume of spatial information calls for efficient data reconstruction and simulation methods. Such tasks can be performed with geostatistical methods from Spatial Statistics or with Gaussian Process Regression methods from Machine Learning. Both frameworks rely on the use of covariance functions. The latter incorporate the spatial correlations of the studied process and are thus instrumental in the formulation of predictive equations.

In this presentation we focus on a family of so-called Spartan covariance functions which were developed using a Gibbs random field with a specific energy structure [1]. This random field is equivalent to a statistical Gaussian field theory. Spartan covariance functions include an additional parameter compared to standard covariance models (e.g., Gaussian, exponential). The rigidity parameter describes the resistance of field realizations to gradients. We show that the Spartan covariance function is corresponds to a classical, d-dimensional-time, damped harmonic oscillator in a heat bath. This system is described by a stochastic partial differential equation driven by white noise. Only a handful of covariance functions with explicit expressions are solutions of partial differential equations that result from stochastic partial differential state equations (to our knowledge only the Spartan and the Whittle-Matern [2] covariance functions share this property. Covariance functions originating from ordinary differential equations, such as the OrnsteinUhlenbeck process, are more common [3].

We also show that the trajectories of the noise-driven classical damped harmonic oscillator can be expanded within a compact time or space interval as a bi-orthogonal series of eigenfunctions with random coefficients using the Karhunen-Loeve (K-L) expansion [4]. The K-L expansion allows reduction of dimensionality and efficient simulation of time series and random fields with Spartan covariance functions. Variations of the rigidity coefficient of a Spartan random field or process with a given correlation length can further decrease the random dimensions needed in the expansion.

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