

Synchronization and finite size scaling on hierarchical lattices

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We study the transition to synchronisation in hierarchical lattices[1] using the evolution of Chate Manneville maps placed on a triangular lattice. Connections are generated between the layers of the triangular lattice assuming that each site is connected to its neighbours on the layer above with probability half. The maps are diffusively coupled, and the map parameters increase hierarchically, depending on the map parameters at the sites they are coupled to in the previous layer. The system shows a transition to synchronisation which is second order in nature, with associated critical exponents. However, the V-lattice, which is a special realisation of this lattice shows a transition to synchronisation which is weakly discontinuous [2,3]. This transition can thus be said to belong to the class of explosive synchronisation with the explosive nature depending on the nature of the substrate. A finite time, finite size analysis was carried out for the order parameter for different lattice sizes for synchronization on the branching hierarchical lattice, where the order parameter is defined to be the fraction of synchronized sites of the maximal cluster on the lattice. In the case of the V -lattice, the data for lattices of sizes for lattice sizes of order 20×20 , 30×30 , 50×50 , and 100×100 was seen to collapse nicely on the same smooth scaling surface. Two dimensional slices have been used to find the scaling exponents. In the case of the branching hierarchical lattice, scaling analysis carried out the order parameter for the synchronizing realizations. The order parameter plotted as a function of the coupling parameter shows a dip at a certain value of the parameter, indicating that the tendency to synchronize decreases at certain parameter values. We discuss the implications of our results, and draw parallels with avalanche statistics on branching hierarchical lattices[1].

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