Mixed-order phase transition in a minimal, diffusion based spin model

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We present and analyze a minimal exactly solved model that exhibits mixed-order phase transition [**?**, **?**]. Mixed-order or hybrid transitions do not fit into the modest classification of thermodynamic transitions and as such, they used to be overlooked or incorrectly identified in the past. The recent series of observations of such transitions in many diverse systems suggest that a new taxonomy of phase transitions is needed.

The model is defined as follows: We study N distinguishable and noninteracting spins, which can have two states (+1 and -1). With a probability q, we randomly select one of positive spins and change its state to the opposite. With a probability $1-q$ we perform the reverse action, i.e. a randomly selected negative spin is flipped.

We call the model "diffusion-based" because its hamiltonian can be recovered from a simple dynamic procedure, which can be seen as an equilibrium statistical mechanics representation of a biased random walk.

We analyze the model within both: canonical and grand canonical ensembles. In the canonical ensemble, the model exhibits first order transition with power-law fluctuations. In the grand-canonical approach, phase diagram of the model is much more complicated. We outline derivation of the phase diagram, in which the triple point has the hallmarks of the hybrid transition: discontinuity in the average magnetization and algebraically diverging susceptibilities. At this point, two second-order transition curves meet in equilibrium with the first-order curve, resulting in a prototypical mixed-order behavior.

Finally, we show that, given fixed system size, our model can be seen as a highly simplified version of the dynamic social network model discussed in [**?**]. In the network model mentioned, nodes are separated into two groups representing opposing interests. Members of the first group (introverts) seek to get rid of their connections, whereas these who belong to the second group (extroverts) want to accumulate their highest possible number. It was suggested that the model exhibits the extreme Thouless effect in which the density of connections between introverts and extroverts jumps from a value which is close to zero, to a value close to unity, when the number of extroverts becomes larger than the number of introverts. Results obtained for the minimal spin model suggest that the phenomenon observed in these networks is a discontinuous phase transition with power-law fluctuations.

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