Exact results for steady-state probability characteristics of Verhulst and Hongler models with multiplicative Poisson white noise

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Many stochastic processes occurring in population dynamics, neurodynamics and ecology exhibit instantaneous discrete jumps and therefore can not be provided by excitation in the form of white Gaussian noise. They must be modeled differently in terms of delta-pulse noise which characteristics are determined by the statistics of pulse amplitudes and intervals between stimuli. The nonlinear dynamical systems perturbed by Poisson white noise can be analyzed in the framework of Markovian theory on the basis of integro-differential Kolmogorov-Feller equation which is a generalization of the Fokker-Planck equation. The analytical treatment of such nonlinear dynamical systems poses more difficulties compared to those driven by white Gaussian noise. As a result, the exact expressions for the stationary probability characteristics of these systems have been obtained only for very limited cases, in particular, when the amplitudes of pulses have one-sided exponential probability distribution.

In this report, for a nonlinear dynamical system, described by the Langevin equation with the multiplicative Poisson white noise having exponentially distributed amplitudes of delta-pulses, new exact analytical results for the steady-state probability density function are derived from the Kolmogorov-Feller equation using the technique of inverse differential operator [1,2]. Specifically, we examine well-known Verhulst equation for the density of isolated population with fluctuating saturation parameter. This model with multiplicative Poisson white noise having pulses of positive polarity can adequately describe the effect of pandemics, natural disasters and other negative phenomena, leading to a significant reduction in the population size within a short time period. Further we consider the stochastic Hongler equation which is a good approximation for the model of genetic selection because it retains some features of the latter [3]. In the case of multiplicative Poisson white noise having bipolar exponentially distributed amplitudes of delta-pulses we find noise-induced transitions in the steady-state probability distributions when changing the noise parameter such as the mean rate of pulse appearing.

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