Kuramoto oscillators in a ring-like topology

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Identical Kuramoto oscillators with nearest neighbor coupling are considered in a 1D ring-like topology. In agreement with previous results obtained for such systems [1-3], we find that the system exhibits nontrivial collective behavior patterns. We interpret these emergent structures as different synchronization modes. One can also consider these dynamically stationary states as rotating waves with a well defined winding number or phase shift between the oscillators.

As a first task we have reproduced all the results known for such systems. By performing a standard linear stability analysis we link the stability of the stationary states to the winding numbers. Our results are in agreement with the stability conditions given in [3]. Starting the dynamics from random initial conditions the probability of appearance for the stable collective modes was computationally studied. We found that these probabilities are well approximated by a Gaussian envelope curve. We also show that variance of the distributions scales linearly with the system size. These results are in agreement with the ones communicated in [2].

Novel and interesting results are also obtained. Using multidimensional geometry we investigate the dynamics of the system and the basin of attractions for different stationary states. The used image for the phase space enables us to take a deeper look on the processes governing the dynamics. In such manner we attempt a theoretical explanation for the observed normal distribution of the stable states and the scaling properties for its variance. We show that the motion of the characteristic point is limited only to a restricted subspace of N-1 dimensional hyperplanes confined in the N dimensional phase space of the system. A series of two dimensional cross sections of the attraction basins suggest that the structure of the attractor domains are complicated, and hence the time-evolution of the system is simple only in the vicinity of the stable states. Empirically we find an interesting restriction for the dynamics of the characteristic point in the used N dimensional hyperspace. Generalizing the Kuramoto order-parameter for the rotating wave-like states we give an empirical estimate for the time-length of the state-selection process.

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