Exploring quantum search algorithms with the renormalization group

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We explore Grover's quantum search algorithm on fractal networks with the renormalization group (RG) and determine its computational complexity in terms of the random walk exponent d_w and the spatial (fractal) dimension d_f. We obtain exact – and likely general – results for the complexity which grows as N^α with $\alpha = 3d_w/(2d_f) - 1$, which the Tulsa-Method can optimize to $\alpha = 1/\min\{d_s, 2\}$, referring to the spectral dimension $d_s = 2d_f/d_w$ of the network [1]. To this end, we develop the real-space renormalization group analysis of the quantum walk equation and analyze the probability to overlap with the sought-after site asymptotically in time and system size N. We establish a large universality class of search algorithms by considering entire families of quantum coin and search operators. The derivation of generalized unitarity conditions allows us to chose highly efficient implementations with coin-dimensions smaller than the site-degrees of the network.

We proceed by describing Grover's quantum search algorithm, which can provide a significant speed-up over classical computation. Grover (in 1997) has shown that a quantum walk, starting from a uniform initial state, can locate an entry in an unordered list of N elements (i.e., sites in some network) almost certainly in a time that scales as $\sim \sqrt{N}$, a quadratic speed-up over classical search algorithms. However, that finding was based on a list in which all elements are interconnected with each other, thus, raising the question regarding the impact of geometry on this result. We have to split the search-task into two questions: (1) How fast can the weight of the wave function be moved to the desired location? And (2), how likely are we going to discover the desired element when a measurement is taken?

We then describe how to apply RG to answer those questions [2-3]. Regarding (1), the RG predicts that the weight of the wave function can be accumulated in a time $t_{\rm walk} = O\left(N^{d_w/(2d_f)}\right)$ via Grover's amplitude amplification, which competes with the fundamental Grover-limit of taking $t_{\rm opt} = O\left(\sqrt{N}\right)$ steps to rotate in Hilbert space the initial (uniform) state into the eigenstate of the sought-after site. For instance, in a regular lattice, where $d_w = 2$, the Grover-limit can always be achieved in dimensions $d_f = d > 2$. Regarding (2), our RG finds that the probability to locate the desired element in a measurement at the optimal time falls off as $O\left(N^{1-d_w/d_f}\right)$ when $d_w/d_f > 1$. The combination of these results leads to the above prediction for α.

[l] Boettcher, Li, Portugal, Fernandez, in preparation (2017).

[2] Boettcher and Li, arXiv:1704.04692.

[3] Boettcher, Falkner, Portugal, Phys. Rev. A **91**, 052330 (2015).