

**Abstracts**

ΣΤΑΤΙΣΤΙΚΗ ΦΥΣΙΚΗ



STATISTICAL PHYSICS

2014

International Conference on  
**ΣΤΑΤΙΣΤΙΚΗ ΦΥΣΙΚΗ**  
7 - 11 July 2014

Sheraton Rhodes Resort  
Rhodes - Greece

**Editors: G. Kaniadakis and A.M. Scarfone**



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## Topical workshops

### 1. Anomalous Diffusion

Workshop organized by: S. Abe and J.P. Boon

### 2. Kappa Plasmas

Workshop organized by: M. Lazar and V. Pierrard

### 3. Econophysics

Workshop organized by: L. Pietronero “Economic Complexity” and  
T. Aste “Finance, Risk”

### 4. Sociophysics

Workshop organized by: B. Szymanski

I) G. Korniss, C. Lim “Human behavior, Social networks”

II) V. Constantoudis “Linguistics”

### 5. Complex Networks

Workshop organized by: A. Scala “Interacting networks” and

B. Kahng “Phase transitions in networks”

### 6. Biophysics

Workshop organized by: P. Paradisi

### 7. Environmental Statistical Physics

Workshop organized by: D. Hristopoulos “Environment and Data Analysis” and

C. Lim “Atmospheres and Oceans”

### 8. Quantum Computation and Quantum Information

Workshop organized by: D. Ellinas and J. Pachos

## Preface

The present volume contains the abstracts of the invited talks and the selected contributed oral or poster presentations submitted to the International Conference on ΣΤΑΤΙΣΤΙΚΗ ΦΥΣΙΚΗ held at Rhodes in Greece, from July 7 - 11, 2014.

The Conference is organized in the following three Areas to cover all the Topics of Statistical Physics:

Area A: Foundations and Theoretical aspects of classical, quantum and relativistic statistical physics and thermodynamics. Mathematical aspects and methods, formalism, rigorous results, exact solutions, connections with the methods of high energy physics, string theory, mathematical statistics and information theory, classical, quantum and relativistic transport theory, Boltzmann and Fokker-Planck kinetics, nonlinear kinetics, dynamical systems, relaxation phenomena, random systems, pattern formation, fractal systems, solitons, chaotic systems, strongly correlated electrons, soft quantum matter, mesoscopic quantum phenomena, fractional quantum Hall effect, low dimensional quantum field theory, quantum phase transitions, quantum information and entanglement, power laws, etc.

Area B: Applications to Physical Systems: quantum systems, soft condensed matter, liquid crystals, plasmas, fluids, surfaces and interfaces, disordered and glassy systems, percolation, spin glasses, structural glasses, jamming, critical phenomena and phase transitions, fluids and interfacial phenomena, molecular and ionic fluids, metastable liquids, hydrodynamic instabilities, turbulence, growth processes, wetting, surface effects, films, crystals, confined systems, surfaces and interfaces, chemical reactions, cold atoms, etc.

Area C: Applications to non-Physical Systems: Interdisciplinary applications of statistical physics, networks and graphs, applied networks, biophysics, genomics, environments, climate and earth models, seismology, linguistics, econophysics, social systems, traffic flow, algorithmic problems, complex systems, etc.

The Conference is organized in Symposia/Sessions dealing with general aspects and applications.

Furthermore topical workshops on Anomalous Diffusion, Kappa Plasmas, Econophysics, Sociophysics, Complex Networks, Biophysics, Environmental Statistical Physics, Quantum Computation and Quantum Information have been organized as parallel events.

G. Kaniadakis and A.M. Scarfone  
(Editors of the Abstract Booklet)

## Geometrical characterization of blocks in fractured media

R. Rosenzweig<sup>1</sup>, V.V. Mourzenko<sup>2</sup>, J.-F. Thovert<sup>2</sup>, P.M. Adler<sup>2</sup>

<sup>1</sup>Geological survey of Israel, Malkhe Israel, Jerusalem, Israel

<sup>2</sup>IUPMC Metis, Paris cedex and Institut Prime - CNRS, Futuroscope Chasseneuil Cedex, France

The problem of the division of the three dimensional space by randomly distributed flat polygons is of interest in random geometry. It has also some practical interest since when fractures are sufficiently dense, they partition fractured media into blocks. The determination of the statistical properties of these blocks is important for the oil industry, for instance ; the oil which is initially located in the porous medium, may flow into the fractures and this transfer is controlled by the block geometry. This problem has been so far only addressed by double porosity models. The major purpose of the present contribution is to determine the geometrical properties of these blocks as a preliminary work to the transfer problem just described. This problem has not been addressed in the literature, except for infinite fractures which correspond to the case of finite fractures with a very large density. Fractures of various shapes and of various densities are generated isotropically with random positions. The blocks are determined after the fractures are triangulated ; the triangles are oriented in a consistent way. Therefore, each solid block is limited by several triangulated plane faces. Then, the neighbors of a given triangle are identified. When this is done for all the triangles, the independent connected components of the triangles are identified by a pseudo-diffusion algorithm. Each independent component corresponds to a block. The block density corresponds to the number of blocks per unit volume. Then, measurements are performed on each block such as the volume, the surface and the number of faces. The dimensionless density  $\rho$  which is equal to the average number of intersections of a fracture with other fractures, varies between 1 to 150, for three shapes, namely squares, rectangles with an aspect ratio of 4 and 20-gons which are very close to disks. Some of the results can be summarized as follows. The block density is a power law of  $\rho$  which is independent of the fracture shape with an exponent equal to 4. The fraction of volume occupied by blocks follows a power law as well until it gets close to 1. Somewhat unexpectedly the mean block volume and the mean surface area of the blocks start increasing with  $\rho$  and decrease for large  $\rho$  where they follow the predictions for infinite fractures. The average number of faces of a block increases until it reaches 6 as predicted for infinite fractures.

[1] P.M. Adler, J.-F. Thovert, V.V. Mourzenko, *Fractured porous media*, (Oxford University Press, 2012).

## Transport of molecular fluids through three-dimensional porous media

P.M. Adler, A. Pazdniakou

UPMC Metis, Paris cedex, France

The main purpose of this study is to extend the analysis which has been made for the double layer theory (summarized by [1]) to situations where the distance between the solid

walls is of the order of several molecular diameters. This is of a large interest from a scientific viewpoint and for various engineering applications. The main challenge is that on the nano-scale the molecular structure of the fluid must be taken into account since the flow is affected by the molecular interactions which cause fluid density variations in the vicinity of the solid walls for instance. It is worth mentioning that the Navier-Stokes equation is still able to describe fluid flow correctly for geometries where the distance between the walls is intermolecular interactions at the nano-scale. The intermolecular forces and their influence on fluid structure and dynamics can be taken into account by using the mesoscopic scale models based on the Boltzmann equation [2]. The numerical methods derived from these models are less demanding in computational resources than conventional molecular dynamics methods and therefore long time evolution of large samples can be considered. Three types of fluid particles are considered, namely the anions, the cations and the solvent. They possess a finite diameter which should be at least a few lattice units. The collision frequency between particles is increased by the pair correlation function for hard spheres. The lattice Boltzmann model is built in three dimensions with 19 velocities; it involves two relaxation times. The particle distribution functions are discretized over a basis of Hermite polynomial tensors. Electric forces are included and a Poisson equation is simultaneously solved by a successive over-relaxation method. The numerical algorithm is detailed; it is devised in order to be able to address any three-dimensional porous media. It involves the determination of the densities of each particle species, of the overall density and of the equilibrium distribution function. Then, the electric forces are determined. Collision operators are applied as well as the boundary conditions. Finally, the propagation step is performed and the algorithm starts a new loop. The influence of parameters can be illustrated by systematic calculations in a plane Poiseuille configuration. The drastic influence of the ratio between the channel width and the particle sizes on the local densities and the velocities is systematically shown. The differences with the classical double layer theory are illustrated. Finally, the results of systematic calculations on three-dimensional porous media are presented and discussed.

[1] A. Gupta, D. Coelho and P.M. Adler, *J. Colloid Interf. Science* **319**, 549 (2008).

[2] U.M.B. Marconi and S. Melchionna, *J. Chem. Phys.* **131**, 014105 (2009).

## Optimal High Dimensional M-estimation

M. Advani, S. Ganguli

Stanford University

Modern Statistics has entered the era of high dimensional data, meaning the dimensionality  $P$  of the data may be of the same order as the number of samples  $N$ . Examples of this modern regime span the gamut from bio-informatics to marketing. Unfortunately, high dimensional data carries with it the curse of dimensionality: it is exceedingly difficult to analyze high dimensional data by estimating large probabilistic models. We develop analytical methods and algorithms to combat this curse for a large class of regularized regression problems. Maximum likelihood (ML) and maximum a posteriori (MAP)

estimation are ubiquitous methods for solving this regression problem. We consider instead the optimal tractable estimator. Intriguingly, we find the optimal function is neither ML, nor MAP, but involves a smoothed version of ML and MAP, where the degree of smoothing depends on the ratio  $P/N$ . The optimal loss function enjoys substantial improvements in squared error relative to ML and MAP. These results indicate that widely cherished procedures for analyzing data must be modified in the high dimensional setting, and the analytic theory we develop allows explicit calculation of this modification for a large class of problems.

Specifically, we consider the regression problem of estimating an unknown vector  $w^0$ . The data obeys  $y_i = X_i^T w^0 + \epsilon_i$ , where  $y_i$  are outputs,  $X$  is a design matrix of inputs,  $\epsilon_i$ ,  $w_j^0$  are noise and coefficients drawn from arbitrary distributions  $f(\epsilon)$  and  $g(w^0)$  respectively. We consider the class of M-estimators, which have the form

$$\hat{w} = \arg \min_w \left[ \sum_{i=1}^N \rho(y_i - \sum_j X_{ij} w_j) + \sum_{j=1}^P \sigma(w_j) \right]$$

where  $\rho$  is the loss function and  $\sigma$  is a regularizer. Note that  $\rho = -\ln f$ ,  $\sigma \propto -\ln g$  corresponds to MAP estimation.

We recast this problem using Statistical Physics and apply the replica method to derive the asymptotic means squared error  $\|\hat{w} - w^0\|^2$  as  $N, P \rightarrow \infty$  for arbitrary convex M-estimators and arbitrary noise and co-efficient distributions. Intriguingly, we apply variational calculus to optimize over all possible M-estimators for a large class of noise and co-efficient distributions, deriving  $\rho_{\text{opt}}$  and  $\sigma_{\text{opt}}$  and find they are systematically modified and smoothed in a manner depending on the ratio  $P/N$ : higher dimensionality necessitates more smoothing.

- [1] D. Bean et. al., PNAS **110**, 14563 (2013).
- [2] S. Ganguli and H. Sompolinski, Phys. Rev. Lett. **104**, 188701 (2010).
- [3] D. Donoho and A. Montanari, arXiv:1310.7320 (2013).

## Universal behavior of the Shannon and Rényi mutual information of quantum critical chains

F.C. Alcaraz

Univ. S. Paulo, IFSC, São Carlos, Sp, Brazil

Quantum entanglement measures have been frequently used recently to detect quantum phase transition in many body quantum systems. Measures like von Neumann and Rényi entanglement entropy, concurrence and quantum discord are among the most frequently used ones, see for example. One of the important reasons for the success of these measures in detecting quantum phase transition and ultimately finding the universality class of the system is the simplicity of calculating them with numerical techniques such as density matrix renormalization group (DMRG). Since at the critical point one can usually describe the system with a conformal field theory (CFT) it is natural to look for observables that can be related to the important quantities in CFT. This program has been carried out in one dimension with significant detail by relating the von Neumann and Rényi entanglement entropy of a bipartite system to the central charge of the underlying field theory, see for example. Although these quantities can be calculated relatively easily by numerical calculations they have been out of reach from experimental point of views. Recently another measure, the Shannon entropy, has been

introduced to quantify quantum phase transition which is based on specific measurements in the system. We report on this talk our results that indicate that the Shannon and the Rényi mutual information shared by subsystems of quantum critical chains at their ground states are also measures that could detect the universality class of critical behaviour of the quantum chain. Although this quantity depends on the basis the ground state is expressed our results indicate that there is special basis that we call conformal basis in which its leading behavior is basis independent. On these basis, as happens with the entanglement entropy, its finite-size behavior has depends on the conformal anomaly  $c$  of the underlying conformal field theory governing the long distance physics of the quantum chain. We studied analytically a chain of coupled harmonic oscillators and numerically the  $Q$ -state Potts models ( $Q = 2, 3$  and  $4$ ), the XXZ quantum chain and the spin-1 Fateev-Zamolodchikov model. The Shannon mutual information is a quantity easily computed, and our results indicate that for relatively small lattice sizes its finite-size behavior already detects the universality class of quantum critical behavior.

## Binary versus non-binary information in real time series: empirical results and maximum-entropy matrix models

A. Almog, D. Garlaschelli

Instituut-Lorentz for Theoretical Physics, Leiden Institute of Physics, University of Leiden, The Netherlands

The dynamics of complex systems, from financial markets to the brain, can be monitored in terms of time series of activity of their fundamental elements (such as stocks or neurons respectively). While the main focus of time series analysis is on the magnitude of temporal increments, a significant piece of information is encoded into the binary projection (i.e. the sign) of such increments. Previous analyses, mainly in the field of finance, have indeed documented various forms of statistical dependency between the sign and the absolute value of fluctuations, e.g. sign-volume correlations and the leverage effect. Other studies have also documented that the binary projections of various financial and neural time series exhibit nontrivial dynamical features that resemble those of the original data. All these results suggest that binary projections indeed retain a non-trivial piece of information about the original time series, and call for a deeper analysis of the problem.

In this paper we first provide a robust empirical evidence of novel relationships between binary and non-binary properties of real financial time series. To this end, we use daily closing prices of S&P500 stocks over the period 2001-2011. We show that the average daily increment and average daily coupling of an empirical set of multiple time series are strongly and non-linearly related to the corresponding average increment of the binary projections of the same time series. Building on this evidence, we then introduce a formalism to analytically characterize random ensembles of single and multiple time series with desired constraints.

Our approach leads to a family of analytically solved null models that allow us to quantify the amount of information encoded in the chosen constraints, i.e. the selected observed properties of the binary projections of real time series. Different choices of the constraints lead to different stochastic

processes, a result that allows us to relate known stochastic processes to the corresponding ‘target’ empirical properties defining the ensemble of time series spanned by the process itself. After applying the approach to the financial time series in our analysis, we compare the informativeness of various measured properties and show that different properties are more relevant for different time series and temporal windows. Finally, and most importantly, we show that our approach is able to reproduce and mathematically characterize the observed non-linear relationships between binary and non-binary properties of real time series.

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### Study of ion acoustic dressed solitons within the theoretical framework of the Tsallis statistical mechanics

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Over the last few years, a great deal of attention has been paid to nonextensive statistic mechanics based on the deviations of Boltzmann-Gibbs-Shannon (BGS) entropic measure. A suitable nonextensive generalization of the BGS entropy for statistical equilibrium was first recognized by Renyi and subsequently proposed by Tsallis [1], suitably extending the standard additivity of the entropies to the nonlinear, nonextensive case where one particular parameter, the entropic index  $q$ , characterizes the degree of nonextensivity of the system considered ( $q=1$  corresponds to the standard, extensive, BGS statistics). This nonadditivity entropy of Tsallis and the ensuing generalized statistics have been employed with success in plasma physics [2]-[4]. The aim of this communication is to study the ion acoustic dressed solitons within the theoretical framework of the Tsallis statistical mechanics. For this, we have considered a plasma consisting of cold ions, electrons obeying of the Tsallis distribution and positrons. We have then investigated the effect of nonextensive parameter (a parameter quantifying the degree of nonextensivity of electrons) on small amplitude ion acoustic dressed wave and compared the result with the KdV soliton and exact soliton solution of the fourth order pseudo-potential. For this purpose, we have included the effect of fourth-order nonlinearity of electric potential in the expansion of the Sagdeev potential of our model of plasma and we have integrated the resulting energy equation with proper boundary condition required for a soliton structure. This procedure gives soliton solution in terms of coefficients of nonlinearities of second order, third order and fourth order of electric potential in the expansion of the Sagdeev potential. Using the Taylors series expansion of Mach number in terms of soliton velocity and keeping the terms up to second order of velocity, this exact solution reduces to nonsecular dressed soliton solution [5]. Our result reveal that the amplitude and the width of the core and cloud structures, KdV soliton, dressed soliton and the small amplitude exact soliton solution

are significantly modified by the nonextensive effects. In particular, it is noted that as the electrons deviate from their thermodynamic equilibrium, the amplitude and width of all structures increase. The spatial patterns of the dressed soliton are significantly modified by the nonextensive effects. As the nonextensive parameter  $q$  decrease in our model of plasma, the amplitude and width of dressed soliton increase. It is also found that for a given value of nonextensive parameter, the amplitude of the KdV as well as dressed soliton increases.

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### Predator population depending on lemming cycles

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In this work we present a model to describe the time evolution of a predator population in some region (for example, Arctic fox in the Canadian tundra), whose diet is based on small rodents called lemmings. Lemmings are known for the wide fluctuations in the number of individuals, reaching peak abundance in some areas every three to five years. The causes of population fluctuations are unclear, although some combination of predation, food quantity and quality, weather, migrations, or genetic change in individuals which belong to the population, is probably involved.

The main purpose of our work is to describe how the mean number of predators and the probability of their extinction depend on the periodicity of lemmings cycles.

Lemming density can be described, from the mathematical point of view, as a pulse sequence (anomalous increase). Pulses occur only at years of peak population density. This pulse process is random, but it can present some regularities because of the presence of memory. There is some delay after each pulse, during which the next anomalous increase of population is impossible, for example, because of the lack of important resources (for example, food). After this period there is some constant probability that a new peak occurs. As a result cycles with random period take place, and lemming density can be described with the help of dead-time-distorted Poisson processes. In our model amplitudes of pulses are not constant, following the Gaussian distribution (only positive values make sense).

To describe the changes in predator population we use a Langevin equation with a term of multiplicative noise

$$\dot{N}(t) = a + b\sqrt{N(t)}\xi(t) \quad (1)$$

where  $N(t)$  is the predator density,  $\xi(t)$  is the pulse noise, i.e. random variations in lemming density,  $a$  is a negative constant, which account for the predator decrease in the absence of lemmings, and  $b$  is a positive constant, i.e. effect of lemming variations on the predator.

We solve Eq.1 analytically for long times (asymptotic solution) and compare results with those obtained numerically. We

extend our numerical analysis to short and intermediate times, comparing our findings with results obtained in the presence of pulse noise with fixed amplitude.

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### Inference of optimality in constrained thermodynamic processes with prior information

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Bayesian methods have been used in many different areas of research. The choice of an appropriate prior distribution has been a crucial issue in Bayesian analysis. The uncertainty in the system parameters due to incomplete information is captured by deriving a prior probability distribution, or, simply called a prior. Bayesian approach in heat engines was first proposed in [1] and then further applied in quantum and classical models of heat engines [2,3]. We used this Bayesian approach to infer the optimal characteristics for a constrained thermodynamic process in which entropy remains conserved [4]. The inferred characteristics for the reversible model of heat engine with incomplete information show correspondence with the optimal behavior of the system with complete information. The universal feature of efficiency beyond the linear term,  $\eta \approx \eta_c/2 + \eta_c^2/8$ , is also inferred within this approach, where  $\eta_c$  is Carnot efficiency. What will be the form of prior if the constraint equation cannot be solved analytically? To answer this question, we perform the inference using spin-1/2 systems as finite reservoirs [5]. A general form of prior is derived for the entropy conserving process as well as for energy conserving process. The analytical as well as numerical estimates of thermodynamic quantities for this model show good agreement with the corresponding optimal values. The significance of taking into account the prior information can be seen from the results with uniform prior in far from equilibrium regime, where informative prior (based on prior information) gives better results than uniform prior which involves minimal information. However, near-equilibrium, both types of priors are equally good. Another thermodynamic process i.e pure thermal interaction is also analysed within this inference based approach. The results for estimated entropy production are in good agreement with optimal case. Further in this work, we find an intuitive interpretation of the prior such that the particular choice of prior implies the uniform density for the quantity being conserved in the process. Thus we have proposed that certain thermodynamic processes can be estimated quite accurately by a subjective inference procedure. It seems striking that upon quantifying ignorance of thermodynamic control parameters in a process, one obtains estimates which are very close to the observed behavior, which is actually the optimal one and seen in the case of complete information.

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### Clausius inequality and H-theorems for Ulam's redistribution problem

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We discuss the possibility of deriving an H-theorem for the nonlinear discrete time evolution known as Ulam's redistribution of energy problem [1]. In this model particles are paired at random and then their total energy is redistributed between them according to some probability law. This dynamics is actually the same as in random exchange models, studied in econophysics, when transactions are assumed to occur by binary 'collisions' between agents, who e.g. exchange money in the same way as particles in a gas exchange their energy. The evolution for Ulam's problem with arbitrary redistribution law is given by a nonlinear equation for the distribution function, similar to one used in [2], but it appears more useful to represent this evolution as a combination of two processes in terms of a two-particle distribution function [3]. The first process is a linear evolution of two-particle distribution during the collision (with factorised initial condition) while the second one is just its reduction back to a factorized form, which corresponds to new random pairing. Then information theory approach leads to a general inequality for the Ulam's problem, which has the form of Clausius inequality for the entropy change during one step of evolution with some additional 'heat' transfer term. This inequality suggests that it is possible to view the system as an open one, interacting with some external agent, or device, similar to Maxwell's demon, that actually performs the redistribution. Clausius inequality then indicates that this interaction may be interpreted as some special kind of thermalization. However, only for a special set of redistribution laws, given by symmetric beta distributions, for which equilibrium solutions are given by gamma distributions [4], the inequality obtained leads to a proper H-theorem. The H-functional in this case differs from the usual entropy by an additional term that vanishes only for the uniform redistribution law. The relative entropy (Kullback-Leibler distance to equilibrium) for this kind of redistribution decreases monotonically just as for an ordinary stationary Markov process, despite the nonlinearity of the evolution equation (compare with [5]). But for other redistribution laws our approach suggests that the final state of evolution has some features of a non-equilibrium steady state, with constant production of some kind of 'heat' in each irreversible act of redistribution. This probably makes the search for a possible general H-theorem much more difficult.

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### Memory improves precision of cell sensing in fluctuating environments

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The survival and function of cells and organisms crucially depend on precise sensing of the environment. When searching for nutrients or avoiding toxins the bacterium *Escherichia coli* can detect differences in concentration as low as 3nM [1], amounting to approximately 3 molecules per cell volume. T cells can detect single copies of foreign antigen [2] to quickly launch an immune response, while axonal growth cones accurately detect very few molecules of guidance cues (e.g. netrins, slits and ephrins) to follow molecular gradients while seeking their synaptic target [3]. Such high precision appears to be remarkable since sensing and signaling in a cell is affected by many sources of noise [1]. However, what level of precision do we expect from theory? In this work, we analyze the effect of cell memory, e.g. from slow biochemical processes, on the precision of sensing by cell-surface receptors. We derive analytical formulas, which show that memory significantly improves sensing in weakly fluctuating environments. However, surprisingly when memory is adjusted dynamically, the precision is always improved, even in strongly fluctuating environments. In support of this prediction we quantify the directional biases in chemotactic *Dictyostelium discoideum* cells in a flow chamber with alternating chemical gradients. The precision of directional cell sensing was found to be determined by a combination of external cAMP sensing and internal bias of unidentified origin. Since internal biases not aligned with the external gradient are expected to reduce the sensing precision, as confirmed by theoretical modeling [5], the purpose of internal biases remained unclear. While increasing the heterogeneity at the population level (at the expense of precision) is a possibility, our work suggests that in a dynamic environment the internal bias represents memory for filtering, leading to an actual increase in sensing precision. Wolpert's "no free lunch theorem", used as a metaphor here, states that to optimally estimate, search, etc., prior assumptions are necessary [4,1]. Indeed, our two active sensing strategies are based on such priors (memory), updated dynamically. Active sensing strategies are widespread in biology, including integral feedback control in bacterial chemotaxis, olfactory- and phototransduction, as well as kinetic proofreading. The latter is a form of error correction mechanism believed to significantly enhance the specificity in DNA replication, protein synthesis, homologous recombination and T-cell signaling. The striking similarity between our proposed strategies, macroscopic engineering solutions, and learning by Bayesian inference in humans hint toward universal problem solving strategies in nature.

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### Key player countries in FP7 collaboration network

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The Framework Programmes promote and fund collaboration projects in which European research institutions and companies participate. It is a well-known strategy developed by the European Commission to enhance research and technology advances in its member countries, while aiming for social, economic and technological convergence. In past work, several research groups attempted to assess the FP5 and FP6 effectiveness, using largely statistical methods, and network analysis. Here, we study the data of FP7, which is the recently concluded programme, following the network approach. Specifically, we first construct the collaboration network of the institutions (universities, research facilities, companies etc.) of the FP7 accepted proposals. Each institution is a network node and each collaboration between two institutions is a network link. We aggregate the data at the level of institute, city, region and country and thus we get a total of four scales, which reveal a more comprehensive picture of the network. We first determine the basic network properties in all four scales, and compare them to those of the accepted FP6 proposals network that were studied in previous studies. The comparison of the two snapshots of the same network provides useful information on how this network of collaborations evolved in time. Furthermore, we notice that all previous publications focused exclusively on the set of the accepted proposals, which were implemented as projects, out of the set of all submitted proposals. The set of rejected proposals, however, may involve potentially valuable information complementary to that of the accepted ones. Therefore, it would be interesting to examine the network of collaborations formed by the rejected proposals, too, and subsequently compare it to that formed by the accepted ones. Any differences, or lack thereof, between these two networks in the four scales, could be useful in further assessing the effectiveness of the FP Programmes. Focusing on the country scale, we study a selection of structural properties of these networks and compare them to find any possible differences. We use Minimum Spanning Trees and a variety of centrality indices on the country scale of the two networks to determine the countries that serve as the most influential and significant nodes, in each case. Our results show that there are indeed some interesting differences between the two networks, especially on the country scale.

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## Variational principle in the theory of partial distributions and matrices.

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We suggest to present thermodynamic potential as a functional of partial densities and correlations instead of that of energy of particles in an external field and their interactions. It allows one to solve the problem of the limited convergence radius and low accuracy of a virial expansion.

As the partial densities are functional derivatives of thermodynamic potential with respect to potentials of interaction, the transition from the old functional variables to the conjugate (new) ones is carried out using the Legendre transformation. As the result of extremal properties of this transformation, the thermodynamic potential and the correlation functions are determined simultaneously by solving an uniform variational task. This allows one to use direct variational methods and provides the thermodynamic consistency of used models.

The existence of several minima of the thermodynamic potential implies the existence of several phases. The global minimum is absolutely stable. Other minima correspond to metastable phases. A phase transition occurs when two or more minimal values of the functional of a variational problem (the thermodynamic potential) are equal. Local stability of a minimum is expressed by a system of the integrated conditions having the meaning of positivity of generalized susceptibilities. As a simple example of application of the variational method, the surface energy of a liquid as a functional of the unary density of a transitive layer is found. We obtain nonlocal expansion of surface energy in density deviations at distances determined by the radius of direct correlations of volumetric phases.

Another applications are a model of crystal, a quasicrystal model of liquid and a model of the liquid-crystal transition.

We also suggest a method of consecutive approximations for the definition of kernels of the integral transformation that provides transition to new variables.

Thermodynamic potential of a quantum system is functional of the effective interaction potential determined by the Matsubara operator. The conjugate variables are partial density matrices. Thus, the suggested method is applicable to both classical and quantum systems.

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## Geographic location, network patterns and population distribution of rural settlements in Greece

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Our work addresses the problem of how social networks are embedded in space, by studying the spread of human population over complex geo-morphological terrain. In recent statistical research, there is considerable effort to project human communication characteristics onto inter-city mobility and migration patterns or on the growth dynamics

of individual cities. Introduction of geo-reference terrain data to the theoretical models could elaborate more general issues on how networks grow on the background of spatially variable fields.

We report for the first time approximate scaling relations on the spatial distribution of human population in the northern Mediterranean land terrain. We focus on human population with occupations that strongly relate to the geomorphology of the landscape, and thus consider rural communities with economies based on agriculture and livestock. We restrict our research to villages or small cities up to a few thousand inhabitants, in mountainous areas in Greece, to avoid large discrepancies in the land use or in the demographic characteristics that arise from the proximity to the sea or to an extended metropolitan area.

Regarding the location of the communities, we study terrain parameters like altitude, slope, curvature and orientation. The mountainous terrain presents a familiar tree-like structure, derived from the water routes (streams and rivers) that define valleys and land plateaus. It then appears that the population generally avoids flat land plateaus and river beds, preferring locations slightly uphill, away from the plateau edge.

The geographical distribution of the communities is quantified in reference to the road network. We find that the cumulative distribution of the number of communities  $N$  increases as  $N \approx d^{1.4}$ , where  $d$  is the road distance from the regional prefecture capital, or as  $N \approx \ell^{0.85}$  with  $\ell$  the road distance away from the regional road network that connects the prefecture capitals. The population distribution can be also quantified: The road distance per resident decreases with the community population  $P$  he resides as  $D \approx 1/P^{1.4}$ .

The above statistical results can be used in both applied and theoretical studies. We routinely use them in telecommunications planning, to expand broadband services via optical fiber technologies and serve population networks in rural communities. More generally, such quantitative results regarding the structure of population networks could relate human occupations and communication needs to the characteristics of the geographical fractal profile that underlies the corresponding population settlements.

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## Information filtering networks

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Filtering information out of big complex data has become one of the major tasks and a crucial bottleneck in present scientific and industrial endeavors. Indeed, the continuous increase in the capability of automatic data acquisition and storage is providing an unprecedented potential for science. However, the ready accessibility of these technologies is posing new challenges concerning the necessity to reduce data- dimensionality by filtering out the most relevant and meaningful information with the aid of automated systems. In complex datasets information is often hidden by a large

degree of redundancy and grouping the data into clusters of elements with similar features is essential in order to reduce complexity [1]. In this task we are facing the problem of catching simultaneously two complementary aspects:

- i) reduce the complexity and the dimensionality of the data by identifying clusters which are associated with local features;
- ii) keep the information about the emerging global organization that is responsible for cross-scale activity. It is therefore essential to detect clusters together with the different hierarchical gatherings above and below the cluster levels.

We have proposed a method [1] that provides both clustering subdivision and hierarchical organization without the need of any prior information, without demanding supervision and without requiring thresholding. The method is based on the construction of a networks associated with relevant information shared among the variables. This network can be used to reduce the complex interweaved dependency structure between variables unwinding the backbone network of relevant relations [1,2,3]. Network-theoretic tools can be then used to analyze the overall data-structure extracting content and reducing complexity.

In this presentation, several possible ways to build financial networks with different topological properties are reviewed and the corresponding different kinds and levels of information content filtered out by the networks are discussed and compared. A new computationally-efficient method [4] that improves the planar maximum filtered graph method [1,2] is also introduced. Clustering techniques that make use of planarity properties are discussed [1] and applications to drug repurposing [5] and quantification of risk and strategies for portfolio selection are presented.

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### From correlations between assets prices to modeling of economic interactions and back

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Financial data are complex and multidimensional with several variables - returns of stock prices, for instance - evolving simultaneously in an inter-dependent manner. In order to model, forecast and value quantities in this complex system the underlying network of interactions between variables must be extracted. When the process is modeled with multivariate (log)normal distributions, in principle, we should have an exact way to generate a interaction model from the measured covariance matrix  $\mathbf{C}$ . Indeed, the inverse of the covariance matrix  $\mathbf{J} = \mathbf{C}^{-1}$  should provide the interaction structure that generates such correlations. However, in practice, the measured covariance  $\mathbf{C}$  is imprecise and therefore its inversion produces noisy, unreliable results [1]. It is therefore essential

to filter out the most reliable and relevant part of the covariance matrix  $\mathbf{C}$ , discarding the unreliable part, operating the inversion on the relevant part only. This is the essential idea around a novel method, we named 'LoGo [2], that inverts only the relevant part of the covariance matrix producing in this way an interaction structure  $\hat{\mathbf{J}}$  which is less affected by noise (but still keeps the essential property of being positively defined). When combined with network-filtering techniques [3,4] this method shows a powerful capability to reveal the relevant economical interactions underneath the collective fluctuations of financial market variables.

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### Dimensional Transition Point in Thermodynamic Properties of Maxwell-Boltzmann Gases

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Considering the developments in nanoscience and nanotechnologies in recent years, studying thermodynamic properties of low-dimensional systems and nano structures becomes an important necessity. In such systems, thermodynamic limit is not valid since the domain is confined. Thus, proper way to study thermodynamics at nano scale is to consider finite-size effects.

Thermodynamic properties are represented by summations over momentum states by definition. In the thermodynamics of macro systems, it is conventional to replace summations by integrals for several reasons like obtaining analytical expressions and decreasing the calculation burden. However, reducing domain size to nano scale causes an ascending divergence from continuum approximation in thermodynamic functions.

When domain size is not much larger than the most probable de Broglie wavelength of particles, wave nature of particles makes thermodynamic properties size and shape dependent [?, ?, ?]. Therefore, behaviors of thermodynamic properties are considerably different than the classical picture. For this kind of systems, it is not appropriate to calculate thermodynamic properties by using integrals based on continuum approximation. One should either calculate thermodynamic state sums numerically or search for a better analytical approximation than continuum one.

To represent the confinement magnitudes of nano domains, dimensionless inverse scale factor  $\alpha$  is defined as half of the most probable de Broglie wavelength divided by domain size. In this study, for rectangular domains having three and lower dimensions, single particle partition functions are derived analytically for Maxwell-Boltzmann statistics by using Poisson summation formula (PSF). It is seen that there is a critical value of  $\alpha$  for which there is a transition from high dimensional representation to the lower one. This is called dimensional transition here. This critical point has a unique value of  $\alpha^* = 1.23547$ . For  $\alpha < \alpha^*$ , the first two

terms of PSF give sufficient representation for the behavior of thermodynamic state functions. Conversely, for  $\alpha > \alpha^*$  the first term of summation (ground state mode) represents the behavior properly. Error of the analytical equations is under 1% even at the critical point.

Thermodynamic properties such as free energy, entropy, internal energy, chemical potential, heat capacity and pressure of a Maxwell-Boltzmann gas are derived by considering dimensional transitions. It is seen that the derived expressions give very precise results for thermodynamic properties of Maxwell-Boltzmann gases at nano scale in comparison with those of exact summations.

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### Current and efficiency of Brownian particles in entropic barriers

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The particle transport phenomena in entropic barrier systems, induced by unbiased non-equilibrium fluctuations, play a crucial role in many physical and biological systems. Therefore, recently, there has been an increasing interest in the transport properties of entropic barrier systems because of potential applications. In order to reveal the physical mechanism of transportation in the barrier systems, the energetics of these systems have been investigated in a few seminal theoretical studies. Firstly, Magnasco [1] showed that a Brownian particle which is subjected to external fluctuations can undergo a nonzero drift while moving under the influence of an asymmetric potential. The basic idea of Magnasco is that efficiency can be optimized at a finite temperature. Based on energetic analysis of Magnasco, energy conversion and particle current for energetic ratchet and entropic barrier systems have been investigated in detail [2-7]. Reguera et al. [8-10] used mesoscopic non-equilibrium thermodynamic theory to derive the general kinetic equation of a system and analyze in detail the case of diffusion in a domain irregular geometry in which the boundaries induces entropic barrier when approaching the dynamics by coarsening of the description. Ai et al. [11] showed that the convenient coupling of thermal noise and asymmetry in the boundaries of a tube can leads to a net current under asymmetric unbiased forces and in the presence of symmetric unbiased periodic force accompanied with a low external load such as friction [12]. The previous works on efficiency and current of Brownian particles are based on the consideration of the entropic barriers under temporal forces. However zero mean oscillating forces in entropic barrier systems may also induce efficiency and current in which Brownian particles can be rectified by thermal noise as well as temporal forces. In this study, considering the temporarily unbiased force and different forms of oscillating forces, we investigated current and efficiency of Brownian particles in an entropic tube structure and presented numerically obtained results. We showed that different force forms raise to different current and efficiency profiles in different optimized parameter intervals. We found that an unbiased oscillating force as well as an

unbiased temporal force leads to current and efficiency which are depending on these parameters. We also observed that the current and efficiency caused by temporal and different oscillating forces have maximum and minimum values at different parameter intervals.

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### Casimir Force of an Ideal Bose Gas in Bose-Einstein Condensation Phase for Different Trapped Potentials

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It is known that the confinement of the vacuum fluctuation of the electromagnetic field between two plates gives rise to a long-ranged Casimir force [1]. The similar effect was found quantum particle systems confined to restricted geometries. Martin and Zagrebnev [2] showed that the Casimir effect caused by thermal fluctuation of a perfect Bose gas in slab geometry and they also obtained the asymptotic expression of the grand potential with a universal Casimir term. This study was extended by Gambassi and Dietrich [3], who obtained the relationship between the thermodynamics Casimir effect in the Bose gas slabs and the critical Casimir forces, and scaling function for the Dirichlet boundary condition. Recently, there has been an increasing interest in the Casimir effect because of potential applications. The Casimir effect of an ideal Bose gas was also studied at finite temperatures with or without traps for the Dirichlet boundary condition. The Casimir force of a weakly interacting dilute Bose-Einstein condensation (BEC) confined by a pair of parallel plates with Dirichlet and periodic boundary conditions was investigated at zero and finite temperature by using the field-theoretical method. In addition, it has been noted in these studies that the quasiparticle vacuum in a zero-temperature dilute weakly interacting BEC should give rise to a measurable Casimir force. With the help of the steepest descent method, the Casimir effect induced by the fluctuations of an imperfect Bose gas contained between two infinite parallel plane was investigated by Napiorkowski and Piasecki. They obtained Casimir force of the imperfect Bose gas in the case of different boundary condition. Casimir effect and scaling function of a perfect Bose gas trapped in a generic power-law potential and confined between two slabs have been obtained based on grand potential of a perfect Bose gas for various boundary conditions. Biswas et al. presented an analytical theory for the Casimir force for interacting Bose gas. In a recent study, Napiorkowski et al. obtained Casimir force and discussed critical behavior of the  $d$ -dimensional imperfect (mean-field) Bose gas confined in a slit-like geometry. In this study I will give analytical solution of Casimir force for ideal Bose gas in Bose-Einstein condensation phase for quartic and magneto-optical potentials [4,5].

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### Statistics of erosion events in rocky coasts and percolation theory

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This work describes a new approach to the statistics of rocky coasts erosion events. In general two quantities measured on the field may define large erosion events: the horizontal eroded area at the cliff top and the maximum local retreat, i.e. the eroded depth of penetration.

Measures from the field [Marques, 2008] suggest that both these quantities are distributed as power laws, for large sizes, defining two different decay exponents. Moreover, a power law relation, characterized by a third exponent, is observed between the two quantities. This observation points to a geometrical property of cliff failures, indicating that their shape is statistically similar for different failure magnitudes. Power laws are familiar in the physics of critical systems. The corresponding exponents satisfy precise relations and they are proven to be universal features, common to many different class of systems. Following the approach typical of statistical physics, we propose a scaling hypothesis resulting in a relation between the three exponents under study. It means that the three exponents are not independent quantities and that there is a constraint between the decay of the frequency as a function of the magnitude of the erosion events and their geometry. Given the difficulties to obtain reliable data on the field, the existence of such a relation could be important for the analysis of field data.

From a different point of view, we develop a numerical model [Sapoval, 2004] of marine erosion, based on a simple feedback mechanism between erosion and damping, that yields numerical values for the exponents. Despite the minimal definition of the model, its behavior is strikingly close to the real statistics, and the exponents resulting from extensive numerical simulations fairly agree with the ones measured on the field.

These results suggest that the mathematical theory of percolation, which lies behind our simple model, can possibly be used as a guide to decipher the physics of rocky coast erosion and could provide precise predictions to the statistics of cliff collapses.

Quite interestingly, our model seems to relate percolation, a model at the basis of the physics of wetting and infiltration of fluids in materials, to the large scale erosive action of the oceans on continents. Percolation theory has been previously invoked for the description of the irregular large scale geometry of rocky coasts, in terms of their fractal or conformal properties [Boffetta, 2008]. Here, we show how the coastal erosive dynamics could represent an other, important hook to corroborate such a link.

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### Growth in the global products network

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Recent works based on exportation database [1,2] have been able to establish the structure of networks of products proximity and complexity. We study evolution dynamics on such networks in order to characterize global parameters capable to influence economical growth rates. We discuss conditions which favor optimal growing rates under different economic scenarios.

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### Taming Complexity: Controlling Networks

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The ultimate proof of our understanding of biological or technological systems is reflected in our ability to control them. While control theory offers mathematical tools to steer engineered and natural systems towards a desired state, we lack a framework to control complex self-organized systems. Here we develop analytical tools to study the controllability of an arbitrary complex directed network, identifying the set of driver nodes whose time-dependent control can guide the systems entire dynamics. We apply these tools to several real networks, finding that the number of driver nodes is determined mainly by the networks degree distribution. We show that sparse inhomogeneous networks, which emerge in many real complex systems, are the most difficult to control, but dense and homogeneous networks can be controlled via a few driver nodes. Counter-intuitively, we find that in both model and real systems the driver nodes tend to avoid the hubs.

## Stability Dynamics in Pattern formation for a nonlocal population dynamics

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The processes of self-organization in populations are currently of great importance to understand the patterns of growth, migration, interaction and evolution in ecological systems. A fundamental understanding of these processes is the dynamic nature of the pattern-no-pattern transition, which needs to be better investigated. In this paper we propose a most general equation to study pattern formation for one-species population and their limit domains in systems of length  $L$ . We derive an iterative method to resolve the population equation. Based on this we have established a precision control function and the total number of individuals that shows as the system can dynamically reach a self-organized state or not. We use a nonlocal equation for one-species to study the evolution of a population density  $U(x, t)$  in the form:

$$\begin{aligned} \frac{\partial U(x, t)}{\partial t} &= a \int_{\Omega} g_{\alpha}(x - x') U(x', t) dx' \\ &- b U(x, t) \int_{\Omega} g_{\beta}(x - x') U(x', t) dx'. \end{aligned} \quad (1)$$

Where  $a$  and  $b$  are the growth and competition constants, respectively. To accomplish this we include nonlocality in both the growth ( $g_{\alpha}(x - x')$ ) and competition terms ( $g_{\beta}(x - x')$ ) where the integral kernels are now depend on characteristic length parameters  $\alpha$  and  $\beta$ . We say that a pattern exists if the stationary solution  $U(x, t \rightarrow \infty)$  is nonuniform. We introduce a new order parameter  $\rho$  which measures the pattern-no-pattern transition for the population system. This order parameter can be defined from the numerical or analytical knowledge of the population density function  $U(x, t)$ . We show that this new parameter has an analogy with classical order parameter in thermodynamics and critical phenomena physics (Eg. the order parameter density of the liquid-vapor phase transition). We show that for the population density the number of peak, its position and intensity can be determined in the parameter space  $(\alpha, \beta)$ . For a number of maxima  $n \geq 3$  the density can be approximated by wave functions in space which are very similar to memory effects in optical phenomena. In this way, the nonlocality in space [1] is similar to nonlocality in time [2]. Consequently, it is possible to make an analogy with the phenomena of peak-adding in self-pulsation of laser with feedback [3].

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## 'LoGo' - From local correlations to a sparse estimate of the global inverse covariance matrix

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Modern data are often complex: multidimensional and highly interdependent. The modelling of these complex systems requires the extraction of the underlying interaction graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ . Assuming that the data is describable by a stationary probability distribution we identify the multivariate Gaussian distribution as one particular result of a maximum Entropy ansatz. In theory for these Gaussian distributions the key to the interaction graph is the inverse of the covariance matrix  $\mathbf{J} = \mathbf{\Sigma}^{-1}$  pointing to the area of Gaussian Markov Random Fields [1] where zero entries in  $\mathbf{J}$  correspond to non-adjacent nodes in the interaction graph which both correspond to conditional independence. However, in practice we have no access to the real covariance matrix  $\mathbf{\Sigma}$  and consequently have to work with its maximum likelihood estimate, the sample covariance matrix  $\mathbf{S}$ . But due to noise and the finiteness of time series  $\mathbf{S}$  is inhering errors we need to address. Furthermore, from the principle of parsimony we want to introduce parameters (i.e. non-zero entries of the inverse covariance matrix  $\mathbf{J}$ ) to our model only if the data requires them [2]. We summarise these challenges with the following two sub-problems: a) Choosing the set of edges  $\mathcal{E}$  of the underlying graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$  and the zero/non-zero pattern of  $\mathbf{J}$ , respectively. b) Assigning values to the chosen non-zero entries of  $\mathbf{J}$ . We propose the following approach to deal with these problems: 1) starting from filtering techniques [3] applied to the covariance matrix, we filter out the most reliable information resulting in a set of edges  $\mathcal{E}$  that form a decomposable graph; 2) we exploit the fact that for Gaussian Markov Random Fields the associated probability function factorises in a neat and handy way and use the circumstance that marginal Gaussian distributions have covariance matrices which are sub-matrices of the global covariance matrix; 3) this allows us to invert the sample covariance matrix locally and compose these local parts to a sparse global estimate of  $\mathbf{J}$  [4]. This method of inverting  $\mathbf{S}$  is exact given that the underlying graph  $\mathcal{G}$  is decomposable which is advantageous because it will not introduce new errors while at the same time it uses less information of  $\mathbf{S}$  than the standard inversion process which makes it more stable to initial errors of  $\mathbf{S}$ . Finally, we use these theoretical findings to construct improved filtering algorithms

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### Nonequilibrium quantum phase transitions in the XY model: comparison of unitary time evolution and reduced density matrix approaches

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We study non-equilibrium quantum phase transitions in XY spin 1/2 chain in the framework of C star algebras developed by Ruelle, Aschbacher, Pillet, etc. We show that the well-known equilibrium quantum phase transition at magnetic field  $h = 1$  persists also in the non equilibrium setting as long as one of the reservoirs is set to absolute zero temperature. In addition, we find non-equilibrium phase transitions associated to discontinuities in the imaginary part of the correlation matrix for any two different temperatures of the reservoirs at  $h = 1$  and  $h = h_c \equiv |1 - \gamma^2|$ , where  $\gamma$  is the anisotropy and  $h$  the magnetic field strength. In particular, two non-equilibrium quantum phase transitions coexist at  $h = 1$ .

We show that that the mathematical setup corresponds physically to the picture advocated by Landauer as a proper description for the non-equilibrium steady state of mesoscopic systems. We use these non-equilibrium phase transitions to test the utility of two models of reduced density operator, namely Lindblad meso-reservoir and modified Redfield equation. The first method considers that there is zone of the boundaries where the interaction with the environment occurs. The environment is modeled through Lindblad operators and the system density matrix evolves according to the Lindblad equation. We show that the non equilibrium quantum phase transition at  $h = 1$  related to the divergence of magnetic susceptibility is recovered in the meso-reservoir approach, whereas it is not recovered using the Redfield master equation formalism. However none of the reduced density operator approaches could recover all the transitions observed by the  $C^*$  algebra. We show in a simpler example, namely a periodic chain of non-interacting fermions coupled to a source and a drain, that although some observables such as occupation per site and particle current are correctly predicted, the none-equilibrium steady states derived in the meso-reservoir approach differ from the one obtained by the Landauer picture in the interaction zone. In addition we also study the quantum mutual information in all regimes and find a logarithmic correction of the area law in the non-equilibrium steady state independent of the system parameters. We also study thermalization properties of the meso-reservoir approach.

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### Credit Default Swaps networks and systemic risk

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The financial crisis has made clear that we need empirical measures of systemic risk that are able to capture some pre-

cursors of systemic events such as the building-up of financial instability. At the same time, financial networks play an important role in financial stability. The network of derivative contracts, in particular the network of Credit Default Swaps contracts plays an important role in systemic risk. There has been a growing interest in networks constructed from time series of financial indicators building on a stream of works that has already a long tradition. An important question concerns to what extent these networks are suitable candidates to provide measures of the building up of instabilities in financial markets.

In this paper, we start from the time series of Credit Default Swaps (CDS) spreads that are considered to reflect more directly the default probability of the related institutions. In particular, our dataset encompasses a total of 176 top firms in the financial sector in the period from 2002 to 2011. To broaden the robustness of the results, we compare 5 different methods among the most commonly used in the literature to construct networks from time series (e.g., linear and non-linear correlation, Granger causality and -draw-up). We study the evolution of these networks over time and we observe a structural change at the very onset of the crisis, end of 2008, but not before. We then take the networks as a proxy of the possible interdependencies among the institutions over which the CDS are issued and we carry out a stress-test analysis by applying the computation of Group DebtRank in the case of a small shock hitting all the institutions. Group DebtRank is a measure introduced in a previous work that estimates the impact on the whole system due to a shock hitting several nodes of a financial network. Similarly to the topological analysis, we observe a clear increase of Group DebtRank from the end of 2008, but not before. We finally couple the shock used in Group DebtRank to a macroeconomic indicator reflecting the potential loss of financial products depending on house prices in the US. In this case, we find a clear build up of the values of Group DebtRank during the several years before 2008.

Overall, the results suggest that networks constructed from CDS time series are not able to capture, alone, the building up of systemic risk in the financial system. The results also suggest that incorporating appropriate macroeconomic indicators in the network measures could be a promising way to detect instabilities.

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### Transient states and congestion in simple dynamical models on networks

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The complex networks have been extensively used to cope with the problem of extracting relevant information from the big data-base that new technologies allow to collect on complex systems. Biological systems, social systems and economics are the main application research fields. Statistical Physics has

developed suitable methods to characterize the equilibrium or states and the existence of phase transitions. Recently the data base have been enriched by dynamical properties of the considered systems opening the possibility of studying the transient states and the transitions to critical states. As a consequence there has been new motivations to study stochastic dynamical systems on network-like interactions structures. Being inspired by the traffic dynamics we propose a class of models defined on a network topology whose nodes are characterized by a state  $x \in [0, 1]$  which evolves according

$$\dot{x}_k = \sum_j \pi_{kj}(t)\Phi(x_j, x_k) - \sum_k \pi_{jk}(t)\Phi(x_k, x_j)$$

where  $\pi_{kj} \in [0, 1]$  is a stochastic random matrix which defines the weights of the network links (i.e.  $\pi_{kj}$  weights the directed link between the nodes  $j$  and  $k$  and  $\sum_k \pi_{kj} = 1$ ). The functions  $\Phi(n_j, n_k)$  define the 'particles' flows on the link  $j \rightarrow k$  which is a nonlinear function of the node states (and possible of the node features) and satisfies the conditions

$$\phi(0, x_k) = 0 \quad \lim_{x_k \rightarrow 1} \phi(x_j, x_k) = 0 \quad \lim_{x_j \rightarrow 1} \phi(x_j, x_k) \geq 0$$

and for any value  $x_k$  there exists a critical point such that  $\partial\phi/\partial x_j = 0$ , which corresponds to a local maximum for the  $x_j$  state. According to our assumptions when the state of a node tends to 1, the node is congested and it prevents other particles from entering in the node. This gives rise to a congestion spreading in the network following a backward cascade. We have studied the existence of equilibrium points for the average dynamics and their stability, the transition probabilities to congested states and the statistics of the first congestion time and the effect of a delay time for the propagation of the information on the network on the dynamics in congested states. We have also considered possible local strategies to control the congestion development and the congestion spreading in the system. These results have been applied to the study of traffic congestion formation on a road network using the traffic data recorded by 300 magnetic spires in the road network of Emilia Romagna region in Italy. The theoretical model suggests the existence of new observables that may forecast the development of traffic congestion.

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## Tension-induced binding of semiflexible biopolymers

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Many cellular processes, such as cell motion, adhesion or mytosis, require the reorganisation of the cytoskeleton. This remodeling is achieved with transient or reversible cross-linkers, which bind and unbind stochastically with

characteristic on- and off-rates. Besides thermal unbinding, which in vitro can be controlled by temperature, mechanical forces are known to affect cross-link unbinding. C. Heussinger in Ref. (1) studied the bending of a bundle of actin filaments and showed that unbinding is a cooperative phenomenon, characterised by an energy barrier. The goal of our present work is to investigate theoretically the effect of the polymer tension on the collective behaviour of the reversible cross-links. We use a model of two parallel-aligned, weakly-bending wormlike chains with a regularly spaced sequence of binding sites subjected to a tensile force. Our model is a generalisation of the model with permanent cross-links which was presented in Ref. (2). Reversible cross-links attach and detach at the binding sites with an affinity controlled by a chemical potential. In a mean-field approach, we calculate the free energy of the system and we show the emergence of a free energy barrier which controls the reversible (un)binding. The tension affects the conformational entropy of the chains which competes with the binding energy of the cross-links. This competition gives rise to a sudden increase in the fraction of bound sites as the polymer tension increases. In the strong stretching limit, we show that this transition is related to the cross-over between weak and strong localisation of a directed polymer interacting with an effective square potential well in the transverse direction. The observed scaling of stretching force with cross-link strength and chemical potential can be explained with that simple model of a directed polymer in an effective confining potential. In Ref. (2) we show that the tensile stiffness of the pair of polymers increases with the number of cross-links. Our new finding of a force-induced first-order transition in the number of cross-links implies a sudden force-induced stiffening of the polymers. Apart from its relevance to cytoskeletal bundles, our model also bears some resemblance to the denaturation of DNA which is known to be affected by external stretching forces as discussed in Ref. (3).

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## Fractional entropy and space distillable metric of nonlocal states

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Field of algebraic topology has undergone an essential evolution in attaching algebraic objects to topological spaces starting from simple graph models associated to vertices and edges in terms of Laplacian matrices, leading to higher-order dimensional structures modified through simplices or cliques, finalizing with an elegant mature representation of nonlocal hypergraph states. Speaking about quantum entropy aspect, the properties of Laplacian graph matrices when restricted to quantum states are recently elucidated [Braunstein et al. (2006), Berc (2014)]. However, the exact relation between given extremes of a hypergraph nonlocality encoded through system sets of specific oriented structures and entropy properties of ordered structure of a system as a whole is still poorly understood. A direct repercussion was inferring global properties from systems local information which lead to monotonicity of information in central limit

theorem establishing, further, a framework for quantifying complexity from information geometry in the context of a classical probability theory (at the same time providing that the entropy does not increase on average under stochastic local operations with classical communications). In this work we present new families of stochastic complex information modules encoded in the hypergraph states which are defined by the fractional entropy descriptors. To each hypergraph state it is assigned a specific fractional entropy statistics, derived from its probability of occurrence. Combinatorial optimization is utilized via covering and matching number of an appropriate hypergraph. To further illustrate the divergence in the complexity of classical and quantum representation of a hypergraph nonlocality, we outlined specified measure realized via Bures metric [Braunstein et al. (1994)], used to quantify degree of purity and the extent to which information is altered by changing the topological space of hypergraph substructures, which we call space-distillable metric (assignable to  $n$ -dimensional hypergraph). Mapping the properties of different hypergraph states to  $n$ -dimensional Hilbert space we have investigated application of space-distillable metric of complexity to tensor - product dependent networks. We further illustrate our model with applications in the design of approximation algorithms for scalable quantum information system.

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### Resonant hybrid quantum bit processing with proton channeling

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Research in the field of induction-generation of mutual quantum correlations - entanglement, implies accurate control, manipulation and transfer of quantum information units qubits addressable for modeling and implementation in quantum computers. On the other hand, particles, such as photons, electrons (spin qubits) that coherently interact with each other, can retain their interaction features in the scope of entanglement pairs (so-called bipartite entanglement).

Here we present a new method for controlling an hybrid electron-nuclear spin system using the superfocused proton beam in MeV energy regime [1]. The features and benefits of hybrid quantum bits (qubits) based on the coupling of electron spin to nuclear spins in the solid state environment of isotopic pure  $^{29}\text{Si}$  nanocrystal are investigated and exploited for production of a new type of memory unit for quantum processing in silicon. The nuclear spins system in case is locked via the hyperchanneled proton beam in a desired state or precisely manipulated to oscillate between  $m_I = +1/2$  and  $m_I = 1/2$  state on the nanosecond time scale. This approach can provide a fast and efficient way of controlling nuclear spin qubits and also could enable the generation of switchable spin-based quantum gates by addressing isolated electron spin. The anisotropic hyperfine coupling which allows strong mixing of quantum states is achieved by the control mechanism of spin qubit based on the proton pulse sequence through nanosilicon crystal channel. These analyses are based on direct induction of superfocused proton beam-atom

entanglement and coherent control of qubits quantum state for times which can exceed seconds allowing precise qubit manipulation, and single qubit measurement. The newly proposed control mechanism is based on the anisotropy of the hyperfine interaction induced in the resonance with optimal CP superfocused field [2, 3], allowing generation and manipulation over arbitrary quantum gates while addressing only the electron spin.

Using proton highly precise and coherent channeling regime we have obtained efficient pulse shaping separator technique for engineering quantum gates, introducing a method for control of a nuclear spins coupled via anisotropic hyperfine interactions in isolated electron spin manifold, without radio wave (RW) pulses.

We suggest that presented method which combine field of ion channeling with quantum state engineering can be efficiently implemented in experimental synchrotron light sources with the aim to facilitate preservation and efficient transfer of experimentally observed quantum particles states contributing to the overall background noise reduction.

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### Microscopic models for the study of economic inequality, tax evasion and welfare measures

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We formulate a family of models for the description of the micro-processes of money exchange, taxation and redistribution in a closed market society. These models are expressed by systems of nonlinear differential equations of the kinetic discretized-Boltzmann kind. While traditional treatments of these subjects in mainstream economics rely on the assumption of a representative rational agent, our approach fits in with a complex system perspective. Society is described as an ensemble of individuals divided into a finite number of income classes; the individuals exchange money through binary and ternary interactions, the total wealth being a conserved quantity. The ternary interactions represent taxation and redistribution processes: they express the subtraction, in correspondence to each binary transfer, of an amount whose percentage (tax rate) depends on the income classes of the individuals involved in the interaction; and they define the redistribution (possibly weighted according to a means-tested welfare system) of this amount to all individuals. The frequencies with which the interactions occur as well as other parameters can be tuned to provide a probabilistic representation as realistic as possible. E.g., we can fix the probability that in an encounter between two individuals the one who pays is the rich or the poor, we can postulate that the exchanged amount depends on saving propensity, etc. We show the emergence from the interplay of the individual interactions of observable collective patterns: all computational simulations suggests that after a sufficiently long time the solutions of the equations reach an equilibrium state corresponding to an income distribution, which depends on the total wealth and on the interaction parameters, but not

on the specific initial distribution. Distributions arising from initial conditions for which the majority of individuals belong to lower income classes exhibit fat tails as do the real world ones. An excellent fit for these distributions obtained through computational simulations is provided by the  $\kappa$ -generalized distribution introduced by Kaniadakis. We also consider the occurrence of tax evasion and explore its effect on the income distribution and the inequality index of the society. We also investigate the behavior of the Gini index in dependence on taxation rates gap and welfare means-testing. Concerning tax evasion, our findings support the idea that a fair fiscal policy and individual correctness contribute to the overcoming of economic inequalities. The models under study provide a flexible tool which could lead to the identification of parameters and policies producing desirable trends.

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## Evolution, information and entropy of multiplex networks

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A large variety of complex systems, from the brain to social networks and complex infrastructures, are formed by several networks that coexist, interact and coevolve forming a network of networks. Modeling such multilayer structures and characterizing the information encoded in them is crucial in order to understand their dynamics and predict their evolution. In this talk I will present recent works on statistical mechanics of multiplex networks. Multiplex networks are formed by  $N$  nodes linked in different layers by different networks. I will present models that generate multiplexes with different types of correlations between the layers, and evaluate in the context of weighted multiplex network what is the information present multiplex networks structures that cannot be extract if one consider single layers taken in isolation.

## Non conventional fluctuation - dissipation approach in Geophysical Fluid Dynamics

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Here we introduce a dynamic approach for the study of the geophysical fluid dynamics phenomena characterized by a small interaction between the variables of interest and the rest of the system. The approach is similar to the one developed some years ago [M. Bianucci, R. Mannella, P. Grigolini and B.J. West *From dynamics to thermodynamics: Linear Response and statistical mechanics*, *Phys. Rev. E* **51**, 3002 (1995)] to derive statistical mechanics of macroscopic

variables on interest starting from microscopic dynamics with no need of the standard *ad hoc* statistical assumption, as, for example, the fixed given temperature and/or the Gaussian fluctuations of the mean velocity. These assumption in fact implicitly introduce Thermodynamics in the system fixing the value of the temperature as an external imposed parameter. The novelty of the cited approach for deriving statistical mechanics is that the diffusion and the friction terms, and consequently the temperature, are obtained as functions of dynamical properties of the irrelevant system (that plays the role of a thermal bath). Here we apply the same procedure to a completely different context, i.e. geophysical fluid dynamics. For example a particular case could be the complex ocean - atmosphere interaction, where, changing all the scales, the role of the "microscopic" system is played by the atmosphere, while the ocean (or some ocean variables) plays the role of the macroscopic system of interest. Thus the chaotic and divergent features of the fast atmosphere dynamics remains only in the decaying properties of the correlation functions and of the response function of the atmosphere variables, while the exponential separation of the perturbed (or close) single trajectories does not play a direct role. The general theoretical framework is always the Zwanzing projection method, used to obtain a Fokker-Planck Equation for the probability distribution of the variables of interest, under some general assumptions. However it must be stressed that in the cited papers aiming at deriving thermodynamics from microscopic dynamics, the underlying microscopic systems are generally Hamiltonian, while in the present approach the models considered as microscopic systems underlying the large scale geophysical fluid dynamics are usually not Hamiltonian (often dissipative). Remarkable is the fact that the non Hamiltonian starting model leads to non-standard fluctuation-dissipation relations between the irrelevant system (for example the atmosphere), and the system of interest (for example ocean variables). Finally we illustrate an example of application of this approach to the study of the ENSO phenomenon.

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## Statistical Power Law due to Reservoir Fluctuations and the Universal Thermostat Independence Principle

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Energy fluctuations in a subsystem - reservoir couple [1], also expressed in terms of thermodynamical uncertainty relations [2], lead to Tsallis distribution with  $q = 1 - 1/C$  for ideal gas reservoirs, with  $C$  being the heat capacity of the total system [3]. For stable materials  $C > 0$ , and this allows only for  $q < 1$  Tsallis entropy formula.

Particle number fluctuations in the reservoir, either achieved naturally in a huge, inhomogenous heat bath or artificially by averaging the statistics over repeated events in high energy

experiments, lead to further effects. For ideal fermionic and bosonic reservoirs the binomial and the negative binomial distributions of the particle number,  $n$ , in the reservoir combined with the microcanonical ideal gas statistical factor,  $(1 - \omega/E)^n$ , lead exactly to Tsallis power-law behavior with the parameters  $T = E/\langle n \rangle$  and  $q = \langle n(n-1) \rangle / \langle n \rangle^2$ . For the binomial distribution this means  $q = 1 - 1/k$ , for the negative binomial one  $q = 1 + 1/(k+1)$ .

These results also can be viewed as an approximation for arbitrary particle number distributions in the reservoir up to subleading (second) order in the canonical expansion  $\omega \ll E$ . For non-ideal systems the general expansion to second order delivers  $q = 1 - 1/C + \Delta T^2/T^2$ , a combined result with the heat capacity and the variance of the temperature of finite heat bath. These quantities seem to act against each other. Here the variance of the temperature is meant for the estimator  $1/S'(E)$  of the thermodynamical temperature, the latter defined by  $1/T = \langle S'(E) \rangle$ .

Finally we demonstrate that a deformed entropy  $K(S)$  can be constructed and used for demanding  $q_K = 1$  in the same approximation, i.e. using a canonical expansion with vanishing second order term. This requirement we call Universal Thermostat Independence Principle (UTI) [4]. The final entropy formula contains the Tsallis, Renyi and Gibbs-Boltzmann expressions as particular cases. Surprisingly there is another limit, that of huge reservoir fluctuations,  $C\Delta T^2/T^2 \rightarrow \infty$ , when the low-probability tails, formally canonical to this entropy formula, approach the Gompertz distribution,  $\exp(1 - e^x)$ , also met in extreme value statistics.

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### Scaling analysis as a tool study the effects of load on hand tremor movements in essential tremor

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We have used the Wavelet Transform (WT) and the Detrended Fluctuation Analysis (DFA) methods to analyze hand tremor movements in essential tremor (ET), in two different recording conditions (before and after the addition of wrist-cuff load). Essential tremor (ET) is one of the most common movement disorders of neurological type, and has substantial consequences to persons' everyday life. In spite of frequency of the occurrence of this neurological disorder, the mechanisms underlying the tremor in ET are not fully understood. Two major components are believed to play a role in ET development, either alone or in combination - a mechanical-reflex component, or peripheral component, which is influenced by body mechanics, and an oscillatory, or central component, presumably involving neuronal networks in the brain.

The tremor power spectra (PwS) usually display more than one typical peak frequency. It has recently been proposed that this structure manifests the existence of multiple sources of tremor. Following this proposition, we have analyzed time series of tremor movements comprised of peak-to-peak

(PtP) intervals, and extracted from regions around the first three main frequency components of power spectra. We have compared these time series in two different recording conditions. Thus, we have tried to assess the existence of multiple sources of ET by way of examining the effects of load on ET.

Our study confirms that weight loading can help distinguish between different spectral components of ET. Scaling behavior in the region of main frequency component of recorded tremors changes in two ways when the load is added: the calculated WT scalegrams and DFA functions are displaying a change of slope and a shift in a position of crossover region. The observed existence of dynamical separation of tremor components upon the addition of load may indicate the first frequency component as the one affected by peripheral reflex mechanisms. We also conclude that the observed change in scaling, connected with a change of a position of the crossover point, could be associated with the emergence (with load) of a movement control mechanism that interferes with the existent pathology in ET. Future studies should aim at broadening and resolving the question of biological significance of observed WT and DFA results, in comparison with healthy subject behavior, or with other pathological tremor data.

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### Exploring the Applications of Fractional Calculus: Hierarchically-Built-Polymers

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Expressions involving fractional derivatives have a long history in the theory and modelling of viscoelastic materials, going back to the nineteen thirties. Quite early it was realized that dynamical aspects of polymeric materials may often be modeled using fractional operators. In this respect polymers are very challenging [1], since, through changes in their chemical structure, in the medium surrounding them, and in their density in solution they can display vast dynamical domains in which anomalous behaviors occur [2]. The classical approaches to model such behaviors are connected to the theory of generalized Gaussian structures (GGS) [2]. Furthermore, the GGS theory is naturally related to fractional generalized Langevin equations of non-Markovian nature [3]. As an example, we take now the stiffness of polymers into account and extend the GGS formalism to semiflexible tree-like structures; among them are dendrimers and regular hyperbranched structures [4]. Semiflexibility leads to restrictions on the bonds orientations, modeled through constraints on the bonds' lengths and on the angles between adjacent bonds. In fact, we are able to deal with arbitrary, locally defined stiffness parameters  $t_i$ , where each  $t_i$  is related to  $\theta_i$ , the angle between the pertinent bonds at the junction  $i$ . Under these conditions we obtain analytically the distribution functions and the potential energies for arbitrary semiflexible treelike polymers (STP). Now, it turns out that the structure of the potential energy for STP is very simple, given that its corresponding matrix is sparse. This allows us to readily determine the mechanical and dielectric relaxation of semiflexible treelike macromolecules. For this we present

(besides the known results for chains) also our findings for stars, for dendrimers, for dendrimers built from stars, and for Vicsek fractals. Moreover, given that we can fix locally the values of the  $t_i$  at each junction point  $i$ , we can readily treat STP whose stiffness parameters are inhomogeneous and even random. Anomalous behavior manifests itself in the intermediate frequency region of mechanical relaxation forms, which are different for distinct semiflexible architectures. Furthermore, the model [4] can be extended to also include dihedral constraints. Remarkably, also in this, more complex situation one can obtain semianalytic results, as we show for linear chains [5].

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## A Field Theoretic Approach To The Electric Double Layer

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Interactions between charged objects in electrolyte solutions are of fundamental importance in technological and biological systems [1]. These interactions can be heavily altered by the presence of multivalent ions that possess a spatially distributed charge. In the first approximation, we restrict our consideration to rod-like multivalent counterions. Mean field theory is not able to describe such highly charged systems [2], and the correlations within highly charged particles needs to be taken into account. We extend and employ a self-consistent field theory that has been shown to be accurate from the weak to the intermediate through to the strong coupling regimes [3,4,5]. The pressure between equally charged surfaces is calculated. Numerical results show that close to the charged surface, the counterions are oriented parallel to the surface, whereas at distances greater than half of the counterion length, they are randomly oriented. Due to the restriction of the orientations of the rod-like counterions at the surfaces, the counterion density at the charged surface decreases to zero. For large surface charge densities, the force between like charged surfaces becomes attractive, as a result of charge correlations. Zwitterionic counterion systems are also examined [5].

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## Works of art as multilayer networks

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As it is well known, multilayer networks describe complex systems where multiple various links between nodes connect the elements of complex systems. The purpose of our presentation is to show that art works can be described as complex systems of this type.

A literary text consists of words so it is possible to build a network between these words, as the network arises out of the appearance of the words in the text. Rhythmic and rhyme relationships between words of the poem form additional layers in the net. Masucci and Rodgers have built a network of semantic correlations between words in the novel "1984" by George Orwell [1]. Obviously, the character of relationships between personages of the novel has a different nature compared to the character of the connections between words appearing in the text, so the personages' network forms a separate layer. We have calculated the Pearson correlation coefficient for the personages' networks in literary texts: "Anna Karenina" by L. Tolstoy, "The Adventures of Huckleberry Finn" by M. Twain, "David Copperfield" by Charles Dickens, "Les Miserables" by V. Hugo, "The Iliad" by Homer, and The New Testament. It was found that all these networks were disassortative [2], therefore personages' networks of the literary texts are not social networks.

For a random partition of the plane into disjoint contiguous units, it is possible to make a scale-free network where the units become nodes and the common borders between the units become their connections [3]. For the Cubists paintings by Picasso, Braque, Mondrian and others, these units are triangles, squares and rectangles, though it is possible to define other geometric and non-geometric figures as these units. We have calculated the main indicators of the network model for the Portrait by Picasso (1926), as well as for several paintings by Mondrian and by Malevich [3]. The second network layer in these paintings forms a network where nodes are connected if their units are of the same colour. Melody is converted into a network system, when the musical notes of various lengths are taken for the nodes of the network [4]. Music has evolved from ancient single-voice melodies to two-voice lines which appeared in the 10th century, followed by chords and finally by multi-voice melodies of the 20th century music. We have studied Prelude in A major by Chopin to describe correlations between melodic, harmonic and rhythmical structures of the music. In our approach, the same notes in different octaves belong to different network layers. Each of these structures can be represented as a network and the musical piece as a set of interdependent networks.

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## Anomalous diffusion in reaction-diffusion systems

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In many physical, chemical, biological systems particles, molecules, cells, move in a seemingly random sequence of steps, but one often observes that their trajectories are such that their mean squared displacement deviates from the classical linear diffusion law and that the normal Gaussian structure of the dispersion is deformed or replaced by a different distribution, two features that are the signatures of anomalous diffusion. This is also the case when the diffusing particles are subject to reactive processes: the diffusion-reaction (R-D) steady state is most often not exponential as classical R-D equations would predict. Such behaviors have been observed and measured in particular in biological systems where particles diffuse and undergo reactions in complex media.

We consider a Master Equation approach based on a Markovian random walk model generalising Einstein's equation with a reactive term wherefrom we show how the mean field formulation leads to a generalized nonlinear Reaction-Diffusion equation with non-classical solutions. Steady state solutions exhibit either a long range power law behavior showing the relative dominance of sub-diffusion over reaction effects in constrained systems, or conversely a concentration distribution with finite support describing situations where diffusion is slow and extinction is fast. The analytical steady state solutions are compared with experimental data obtained by measurements performed in the *Drosophila* wing disc where morphogens are produced by a subset of cells wherefrom they diffuse and are degraded thereby forming a concentration gradient whose profile shape appears crucial for subsequent cell differentiation.

The sub-diffusive nonlinear reactive steady state profile with infinite support reproduces very well the experimental data indicating slow degradation combined with extended sub-diffusion. Profiles with finite support would exhibit stronger sensitivity to input flux changes, and it seems that such profiles with finite support have not been identified in morphogen gradient formation. This observation may suggest that extreme sensitivity excludes this type of profile in natural morphogen gradient formation because degradation is too fast with respect to diffusion in order to establish the necessary gradient wherefrom cell differentiation eventually emerges.

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## Braided statistical physics with anyons

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In systems with physics constrained to two dimensions, point like particles named anyons can occur which have more general statistics than bosons or fermions. Beyond mere possible existence they were found to be a good description for low lying quasi-particle excitations of topologically ordered phases such as fractional quantum Hall systems, Majorana edge modes of nanowires, and two dimensional spin liquids. Recently there has been experimental progress in preparation

and control of these systems with the goal to observe anyonic statistics. This is further motivated by the discovery that braiding some types of non-Abelian anyons provides for naturally fault tolerant quantum computing. As experiments are not yet able to manipulate anyons individually, it would be beneficial to reveal their exotic braiding properties on a macroscopic scale. In particular, we are interested in the ground states and transport properties of anyons that can have direct observable consequences. We introduce a quasi-1D ladder model to capture some of the essential physics of anyons in and out of equilibrium. First, using a new numerical simulation algorithm for matrix product states with built-in anyonic symmetry we report on ground state phases of itinerate non-Abelian anyons on the ladder. In the 1D case, prior work showed that such systems with Heisenberg like dynamical interactions together with hopping exhibit spin-charge separation. We show additional phases appear when purely topological braiding interactions on a ladder are also allowed.

Second, we investigate the role of disorder on the propagation of both Abelian and non-Abelian anyons. It has been known for more than five decades that randomised local potentials can suppress diffusion of quantum particles a phenomenon known as Anderson localisation. This mechanism is based on randomisation of phases that correspond to individual particle histories and consequent destructive interference. We model this using a discrete time anyonic quantum walk in a disordered topological background. In particular, we consider a walker that braids around islands canonically arranged on a line, where the number of static anyons at a given island assumes a random value. For the Abelian anyons this causes the walker acquiring random discrete phases which, as we demonstrate, leads to localisation. For the non-Abelian ones we resort to numerical treatment. A continuous time anyonic Hubbard model, studied numerically with the matrix product state approach, concludes that Ising non-Abelian anyons do not localise.

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## The asymmetric simple exclusion process on chains with a shortcut revisited

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Novel features of the totally asymmetric simple exclusion process (TASEP) have been found on networks with nontrivial geometry. In the approach advanced in our work [1] each macroscopic segment of the network is considered in a stationary phase determined by its injection and ejection rates. The only mean-field type approximation used neglects the correlations between different chain segments. This allows one to treat the coupling between each two connected segments as coupling to reservoirs with certain effective rates.

We consider TASEP on open networks consisting of three consecutively coupled macroscopic chain segments with a shortcut between the tail of the first segment and the head of the third one. The model describes directed motion of molecular motors along twisted filaments. Two versions of the shortcut are studied. The first one, suggested in [2], describes a zero-length shortcut allowing for a particle at the last site of the first segment to jump with a given probability directly to the first site of the third segment, provided the latter is empty. The second version of the shortcut represents a chain of arbitrary length running in parallel to the second segment. Particles choose with equal probability between the second segment and the shortcut. For the zero-length shortcut we obtain unexpected results which revise the findings of [2] in the case of maximum current through the network [3]. Our theoretical analysis shows that the shunted segment can exist in low-density and high-density phases, as well as in a coexistence (shock) phase. Surprisingly, the main quantitative parameters of the linear density profile in the case of a shock phase are determined by a positive root of a cubic equation, the coefficients of which linearly depend on the probability of choosing the shortcut. Numerical simulations demonstrate that the behavior of the model with a shortcut of arbitrary length is qualitatively similar. In finite-size networks with head and tail chains of equal length, and equal injection and ejection rates at their external ends greater than one half, the local density distribution and the nearest-neighbor correlations in the middle chain correspond to a shock phase with completely delocalized domain wall. Within the domain wall theory, the main parameters of the shock phase can be expressed in a universal way through the shortcut current. Upon moving the shortcut from the central position to the head or tail of the network, these profiles take shapes typical of a high- or low-density phase, respectively [4]. The unexpected conclusion is that a shortcut in the bulk of a single lane may create traffic jams.

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## Nonlinear Stochastic Dynamics of Quantum and Laser Systems

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We present the results of studying the dynamical chaos regime in dynamics of quantum (atomic ensembles) systems, generation of a laser with absorbing cell and chaotic self-oscillations in the backward-wave tube on the basis of numerical analysis by means a complex of advanced quantum methods and chaos-geometric algorithms (Glushkov et al). We analyze the temporal dependences of the output signal amplitude, phase portraits, statistical quantifiers for a weak chaos arising via period-doubling cascade of self-modulation and for developed chaos at large values of the dimensionless length parameter. Our approach combines together the

non-linear analysis methods to dynamics, such as the wavelet analysis, multi-fractal formalism, mutual information approach, correlation integral analysis, false nearest neighbour algorithm, the Lyapunov exponent's analysis, surrogate data method, memory matrix formalism etc. In table we present our calculation data on the Lyapunovs exponents' ( $\lambda_1$ -  $\lambda_6$ ) for two self-oscillations regimes in the backward-wave tube: i). the weak chaos (normalized length:  $L=4.24$ ); ii) hyper (developed) chaos ( $L=6.1$ ). The correlations dimensions are respectively as 2.9 and 6.2 and the Kolmogorov entropy values are 0.261 and 0.742. Our analysis is in very good agreement with the similar data [3] and confirms a conclusion about realization of the chaotic features in dynamics of the backward-wave tube. The presented example has shown high perspectives of a combined quantum geometry and chaos theory methods to studying a chaotic dynamics of the complicated quantum and laser systems.

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## Statistical physics of charged liquids under nanoconfinement

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Water mediated electrostatic interactions between charged macromolecules and ions are omnipresent in various nanoscale systems. From the charge selectivity of biological ionic channels to energy storage in supercapacitors, these interactions are at the heart of many biological and industrial processes. An accurate formulation of electrostatics at finite temperature is thus necessary to understand and optimize the functioning of these systems. Despite their importance, for several decades, the understanding of electrostatic interactions has been limited to mean-field dielectric continuum models such as the Poisson-Boltzmann (PB) formalism that bypass electrostatic correlations and the charge structure of the water solvent. The talk will focus on newly developed integral equation theories that aims at overcoming these two limitations by incorporating charge correlations and solvent charge structure into the current formulation of electrostatics. The first part of the talk will be devoted to correlation

Regime	$\lambda_1$	$\lambda_2$
Weak chaos	0.261	0.0001
Hyperchaos	0.514	0.228
Regime	$\lambda_3$	$\lambda_4$
Weak chaos	-0.0004	-0.528
Hyperchaos	0.0000	-0.0002
Regime	$\lambda_5$	$\lambda_6$
Weak chaos	-	-
Hyperchaos	-0.084	-0.396

Table 1: Numerical parameters of the chaotic self-oscillations in the backward-wave tube:  $\lambda_1 - \lambda_6$  are the Lyapunov exponents in descending order (our calculation data).

effects in dielectric continuum liquids. I will introduce a self-consistent (SC) calculation scheme that allows to take into account charge fluctuations in confined Coulomb liquids. First, I will characterize electrostatic correlation effects on the ionic selectivity of membrane nanopores. Then, I will couple the SC scheme with hydrodynamic transport equations in order to compute the ionic currents induced by a potential gradient at the extremities of the nanopore. By comparison with electrophoretic ion transport experiments, I will show that correlation effects associated with the confinement of the liquid can solely explain the reduced ionic conductivities of biological alpha-Hemolysin pores. Within the framework of this correlation-corrected ion transport theory, I will illustrate the possibility to control via charge fluctuations the electrophoretic translocation of DNA molecules through membrane nanopores.

In the second part of the talk, I will present a solvent-explicit formulation of electrostatic interactions that overcomes the mean-field and the dielectric continuum approximations. The proper consideration of the discrete charge composition of solvent molecules will be shown to result in two coupled integral equations that incorporates on the same footing non-local electrostatic interactions and charge correlations. By comparison with experimental capacitance data, I will show that the solvent-explicit formalism can explain the particularly low differential capacitances of carbon-based materials in terms of the solvent-electrode correlations, improving the agreement of previous mean-field theories with experiments by an order of magnitude.

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### The rise of China in the international trade network: a community core detection approach

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By viewing the international trade system as an interdependent complex network, this paper uses community detection and community core detection techniques to examine both the global dynamics, i.e., communities disappear or reemerge, and the regional dynamics, i.e., community core changes between community members, in the ITN over the period from 1995 to 2011. We find that the Asia-Oceania community has displayed rich dynamics both internally and externally. That is, the Asia-Oceania community was present during 1995-2001 and was led by Japan, and then it disappeared and was integrated with the America community during 2002-2004, and finally it reemerged during 2005-2011 and was led by China. With a model of weight-driven preferential attachment both inter- and intra-communities, we are able to explain the dynamics observed in the Asia-Oceania community. Each period a node will be selected and by chance it may increase its edge weight with an inter-community node (if the edge already exists;

otherwise a new edge will be established). It will then increase its edge weight with an intra-community node. Outside its own community, the selected node prefers to increase its edge weight with the node with high external strength. Inside its own community, it prefers to increase its edge weight with the node with not only high internal strength, but more importantly, high external strength. Our simulation results show that the global dynamics, i.e., communities disappear or reemerge can be generated by this model setting. In light of the model, the interpretation of the dynamics in the Asia-Oceania community can be that, the community collapsed after China entered the WTO and built strong trade relationships with other communities, especially with the external cores, i.e., the United States and Germany, and China became regionally attractive due to the preference of external strength and restored the Asia-Oceania community and emerged as the community leader. We find some supporting evidence in the trade data. In particular, the behavior of the ratio of the inter-community trade to the intra-community trade between the Asia-Oceania community and the America community coincides with the disappearance and reemergence of the Asia-Oceania community. Within the community, China surpassed Japan after 2003 in terms of both inter- and intra-community trade. In our simulation, the external strength can only be increased by chance. In reality, however, it can be achieved by a series of strategic moves in trade policy.

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### Self healing networks

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We discuss a simple model of self-healing for complex networks; in particular, self-healing capabilities are implemented through distributed communication protocols that exploit redundant links to recover the connectivity of the system. We analyze the effect of the level of redundancy on the resilience to multiple failures. As a metric for the resilience, we measure the fraction of nodes still served after a given levels of network damage. We first resort to numerical simulations to study the effects of redundancy on healing performances under different connectivity patterns: from planar grids, to small-world, up to scale-free networks. Finally, we discuss some recent analytical results allowing to characterise the phase diagram for square grid and for Erdos-Renyi graphs.

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## Discord and information deficit in the XX chain

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The investigation of quantum correlations in mixed states has recently attracting strong attention. While in bipartite pure states such correlations can be identified with entanglement, it was recognized that separable (nonentangled) mixed states may still exhibit useful quantum correlations. It has been shown that for the mixed-state based quantum algorithm of Knill and Laflamme, an exponential speedup over classical algorithms can in fact be achieved without entanglement. This has turned the attention to alternative measures of quantum correlations for mixed states, which are able to capture the quantumness of such mixed states. The aim of this work [1] is to analyze in detail the quantum correlations of spin 1/2 pairs in chains with XX-type first neighbor couplings in a transverse field, at both zero and finite temperature, through the analysis of the behavior of quantum correlations measures such as the quantum discord, the geometric discord, and the one-way information deficit, as well as other generalized measures [1]. Such model is particularly interesting for both quantum information and condensed matter physics, exhibiting distinct features such as eigenstates with definite magnetization along the field axis and a special critical behavior. It is first shown that in contrast with entanglement discord-type measures exhibit common features such as a non-zero value for all separations  $L$  at all temperatures  $T > 0$ . Exact asymptotic expressions for the decay with  $L$  and  $T$  will be provided, on the basis of the exact treatment based on the Jordan-Wigner fermionization. Nonetheless, we will also show that important differences between the quantum discord on the one side, and the geometric discord and information deficit on the other side, do arise in the minimizing local spin measurement. While in the quantum discord the latter is always orthogonal to the transverse field (even at strong fields if  $T > 0$ ), in the geometric discord and information deficit it exhibits a perpendicular to parallel transition as the field increases, at a field lower than the  $T = 0$  critical field  $B_c$ . Such transition in the minimizing measurement is present at all temperatures and separations, and is shown, to be a signature of the transition from a Bell state to a separable aligned state of the dominant eigenstate of the reduced state of the pair. This difference indicates a distinct response of the minimizing measurement in these quantities to the onset of quantum correlations.

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## Entropy production in systems described by nonlinear Fokker-Planck equations

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The H-theorem, and consequently the second law of thermodynamics, which states that the entropy of an isolated system always increases for irreversible processes, leads to the interesting phenomenon of entropy production. Within the statistical definition of entropy, the entropy production depends directly on the time derivative of the corresponding probability; for this purpose one may use, e.g., the Boltzmann, or Fokker-Planck equations in the case of continuous probabilities, or the master equation, when dealing with discrete probabilities. Most investigations in the literature are concerned with the production of Boltzmann-Gibbs entropy [1] and so, one makes use either of the standard master equation, or of the linear Fokker-Planck equation. The applicability of linear differential equations in physics is usually restricted to idealized systems, which are valid for media characterized by specific conditions, like homogeneity, isotropy, and translational invariance, with particles interacting through short-range forces and with a dynamical behavior described by short-time memories. However, it is very common, particularly within the realm of complex systems, to find physical systems that do not fulfill these requirements, e.g., those presenting one (or more) of the following properties: spatial disorder, competing interactions, long-range interactions, long-time memories. In such cases, the associated equations have to be modified, and very frequently, nonlinear terms are considered in order to take into account such effects. Among these equations one could mention the nonlinear Fokker-Planck equations that are intimately related to anomalous diffusion. In the present work we study the entropy time rate of systems described by nonlinear Fokker-Planck equations – which are directly related to generalized entropic forms. Both entropy production, associated with irreversible processes, and entropy flux from the system to its surroundings, are studied [2]. Some examples of known generalized entropic forms are considered, and particularly, the flux and production of the Boltzmann-Gibbs entropy, obtained from the linear Fokker-Planck equation, are recovered as particular cases. Since nonlinear Fokker-Planck equations are appropriate for the dynamical behavior of several physical phenomena in nature, like many within the realm of complex systems, the present analysis should be applicable to irreversible processes in a large class of nonlinear systems, such as those described by Tsallis [3] and Kaniadakis [4] entropies.

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### Temperature inversion in a system with long-range interactions

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Temperature inversion due to velocity filtration, a mechanism originally proposed by Scudder to explain the heating of the solar corona, is demonstrated to occur also in a simple paradigmatic model with long-range interactions, the Hamiltonian mean-field (HMF) model. The model can be seen either as a system of globally coupled point particles moving on a circle or as a system of fully coupled (mean-field)  $XY$  spins. The dynamics of such a model, as of most systems with long-range interactions, is characterized by the presence of quasi-stationary states. The latter are out-of-equilibrium states that occur during relaxation of an isolated long-range interacting system to equilibrium. Due to a fast “violent” relaxation on times of  $O(1)$ , a generic initial condition reaches a QSS. These states persist for very long times, diverging with the number of degrees of freedom  $N$  as  $N^\alpha$  with  $\alpha \geq 1$ . Then, finite- $N$  effects drive the system to equilibrium. The lifetime of a quasi-stationary state is thus effectively infinite in the thermodynamic limit  $N \rightarrow \infty$ .

Using molecular dynamics simulations, we show that if the system is prepared in an initial state where the velocity distribution is given by a kappa distribution, the system typically settles into an inhomogeneous quasi-stationary state in which the velocity distribution has suprathreshold tails (that is, the tails present in the initial condition survive the violent relaxation), and the temperature and density profiles are anticorrelated: denser parts of the system are colder than dilute ones. Since the dynamics of systems with long-range interactions is effectively described by the Vlasov equation, on the timescales corresponding to the lifetimes of the quasi-stationary states, we argue that this phenomenology is explained by the velocity filtration mechanism and that the appearance of temperature inversions may be a generic property of long-range interacting systems.

Moreover, a weaker temperature inversion (i.e., occurring only where the density gradient is larger, but not in the whole system) can be observed also starting from initial conditions that do not have suprathreshold tails, like water-bag and even Maxwellians, provided they are not the true equilibrium ones. Velocity filtration does not seem able to explain this latter phenomenon and a different dynamical mechanism is proposed.

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### Protein translocation in narrow pores: inferring bottlenecks from native-structures

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In voltage driven translocation experiments (Kasianowicz et al. 1996), an applied voltage across two electrolytic cells connected through a nanopore induces the migration of proteins and other macromolecules across the nanopore. The macromolecule engaging the pore produces detectable ion current variations that can be very informative on the physical and chemical properties of the passing species. This kind of analysis is a simple extension of the Coulter method for counting and sizing particles suspended in electrolyte solutions by measures of electrical resistance. While this experimental technique is announced to work for the fast and cheap sequencing of nucleic acids, its effective applicability to protein molecules is still under debate and intensive research. An increasing accumulation of experimental data (see, Nivala et al. 2013, Bayley and coworkers 2013) supports the view that large protein translocation across narrow pores occurs via a multistep process involving a sequence of dynamical bottlenecks (stall events). These events can be considered, to some extent, the fingerprint of the passing molecule. Our computer simulations on a coarse-grained model of the protein-pore system confirm the multistep scenario which results from the tight coupling between the simultaneous unfolding process and translocation dynamics. Moreover our results strongly indicate a tight relationship between: i) the stall events of the transport dynamics, ii) the ascending ramps in the free-energy profile  $G(Q)$  of a translocation reaction coordinate  $Q$ , and iii) the regions of the protein richer in “backward native-contacts” (i.e. native non-bonded interactions among those aminoacids that have not yet entered the pore). We thus can argue that the sequence and nature of such bottlenecks might have a simple and univocal interpretation in terms of the structural properties of the protein native-state. Therefore, inference on the presence of a multistep translocation dynamics of proteins can be done from the knowledge of their native-state topology (Bacci et al 2013). In a possible inverse scenario, we guess that the straightforward correlation between stalls and protein structural elements would allow to distinguish protein motives and domains from the detection of stalls in the translocation dynamics.

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## Percolation in multiplex networks with overlap

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Multilayer, interdependent and multiplex networks have gained a lot of attention in recent years for their applications in social sciences, large-scale infrastructures, neuroscience, etc. In particular, it has been shown that such networks can describe the resilience of large coupled infrastructures (power grids, Internet, water systems, ) to failures, by studying percolation properties under random damage. A feature common to many empirical cases, such as in transportation networks, social networks and epidemiology, is the high occurrence of edge overlap, i.e. node pairs connected on different layers. In spite of this important observation, however, the theory has been lacking a formalism to deal with overlap. Here we give a general theoretical approach that includes edge overlap in percolation of multiplex networks. Percolation is not only a simple model of robustness and stability, but it also constitutes a fundamental step in dealing with more complex models and even dynamical processes occurring on the network. We apply this formalism to the case of 2 and 3-layer Poisson graphs and investigate the resulting critical phenomena. Numerical simulations show excellent agreement with the theory. We show that the presence of a critical value of edge overlap in a duplex can both change the order of the phase transition from hybrid first order to second order, and improve the robustness of the system. We also show that in multiplexes with more than two layers the observed critical phenomena become remarkably complex, including the presence of high order multicritical points. On one hand, there may occur first order phase transitions between percolating phases with different strengths. On the other, it emerges that overlap of edges which does not involve all the layers can also change a continuous phase transition to a discontinuous one, making system reliability less predictable. This is a feature that may be relevant in the design of large-scale infrastructures.

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## Discontinuous phase transition in a core contact process

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To dynamically understand the resilience of a complex network under random damage, a model for core contact process on complex networks is suggested. From a mean-field rate equation, it is analytically shown that the infection in the model depends both on infection rate  $\lambda$  and initial density

$\rho_o$  of infected nodes. To describe the transition between absorbing phase and active phase, density of infected nodes at time  $t \rightarrow \infty$ ,  $\rho_s(\lambda, \rho_o) \equiv \rho(t \rightarrow \infty, \lambda, \rho_o)$ , is used as the order parameter. In the absorbing phase  $\rho_s = 0$  and in the active phase  $\rho_s > 0$ . The phase transitions in the model are also shown to be discontinuous from the mean-field theory. It is shown that there are three types of transition natures in mean-field level. The first type, which occurs as  $\lambda$  increases with a given  $\rho_o$ , is a discontinuous hybrid transition with exponent  $\beta = 1/2$ , i.e.,  $\rho_s(\lambda) - \rho^* \propto (\lambda - \lambda^*)^{1/2}$  for  $\lambda \rightarrow \lambda^{*+}$ . Here  $\lambda^*$  is a threshold rate and  $\rho^*$  is the density of infected nodes for  $\lambda \rightarrow \lambda^{*+}$ . The second type, which occurs as  $\lambda$  increases with a given  $\rho_o$ , is also a discontinuous hybrid transition with exponents  $\beta = 1$ , i.e.,  $\rho_s(\lambda) - \rho_o^* \propto (\lambda - \lambda_o^*)$  for  $\lambda \rightarrow \lambda_o^{*+}$ . Here  $\lambda_o^*$  is a threshold rate and  $\rho_o^*$  is the density of infected nodes for  $\lambda \rightarrow \lambda_o^{*+}$ . The third type which occurs as  $\rho_o$  increases with a given  $\lambda$  is a discontinuous transition. Furthermore, by using a heterogeneous mean-field theory, the model on annealed networks is shown to have the same physical properties as those from the mean-field theory. From the numerical simulations we also show that the model on quenched networks has the same properties. Since the cluster structure in the active phase of the model is nearly the same as those in the  $k$ -core percolation, the model is a dynamical version of the  $k$ -core percolation as the SIR model is a dynamical version of ordinary percolation. From these results, we also discuss the stability of various networks under core infection processes.

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## Scaling analysis of negative differential thermal resistance

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Negative differential thermal resistance (NDTR), the counterintuitive phenomenon of decreasing heat flux for increasing temperature difference, is the operational mechanism of various model thermal devices such as the thermal transistor and the thermal-memory device, where those devices are made up of one-dimensional nonlinear lattices such as the Frenkel-Kontorova (FK) model, the  $\phi^4$  model and the Fermi-Pasta-Ulam (FPU) model. Here we show, via a system-independent scaling analysis, that negative differential thermal resistance can be artificially generated for any one-dimensional heat flow with a temperature-dependent thermal conductivity, where the general condition for the occurrence of negative differential thermal resistance is given by an inequality with three scaling exponents:  $n_1 n_2 < -(1 + n_3)$ . In this inequality, the scaling exponent  $n_1 \in (-\infty, +\infty)$  describes a particular way of varying the temperature difference, and the scaling exponents  $n_2$  and  $n_3$  describe, respectively, the dependence of the thermal conductivity on an average temperature  $\bar{T} \equiv (T_+ + T_-)/2$  and on the temperature difference  $\Delta T \equiv T_+ - T_- > 0$ , where  $T_+ > 0$  and  $T_- > 0$  are respectively the boundary temperatures at the higher-temperature and

lower-temperature ends of an one-dimensional system. The scaling analysis to be presented here is general in a way that it is applicable to any possible temperature dependence of a system's thermal conductivity, even applicable to cases in the nonlinear response regime where the thermal conductivity could also be a function of the applied temperature difference. For cases with a temperature-dependent thermal conductivity ( $n_2 \neq 0$ ), negative differential thermal resistance can always be generated with a suitable choice of the scaling exponent  $n_1$  such that this inequality is satisfied. The results explain the illusory absence of a regime of negative differential thermal resistance in certain nonlinear lattices (for example, the Fermi-Pasta-Ulam model) and predict new ways of generating negative differential thermal resistance, where such predictions have been verified numerically. The analysis will provide insights for a designing of thermal devices, and for a manipulation of heat flow in experimental systems, such as nanotubes.

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### Isothermal collapse of self-gravitating Brownian particles

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Systems with long-range interactions are numerous in Nature. Some examples include self-gravitating systems and two-dimensional vortices in geophysical and astrophysical flows. Systems with long-range interactions display a very intriguing thermodynamics marked by the inequivalence of statistical ensembles and a great diversity of phase transitions. If the system is isolated, one must use the microcanonical ensemble. This is the correct description of stellar systems such as elliptical galaxies and globular clusters for which the energy is approximately conserved. However, if the system is in contact with a thermal bath imposing its temperature, one must use the canonical ensemble. This is the correct description of a system of self-gravitating Brownian particles. These particles are subjected to a friction force, a stochastic force, and they also interact gravitationally. This could model the dynamics of dust particles in the solar nebula. In that case, the particles experience, in addition to gravity, a friction with the gas and a stochastic force due to ordinary Brownian motion or to turbulence. To simplify the problem we consider a limit of strong friction  $\xi \rightarrow +\infty$  in which the dynamics of the particles is overdamped so that inertial effects can be neglected. In the mean field approximation, which becomes exact in a proper thermodynamic limit  $N \rightarrow +\infty$ , the evolution of the spatial density of particles  $\rho(\mathbf{r}, t)$  is governed by the Smoluchowski-Poisson system. If the system is enclosed within a "box" in order to prevent its evaporation, it can be in equilibrium at sufficiently high temperatures. However, below a critical temperature  $k_B T_c = GMm/(2.52R)$  (where  $R$  is the radius of the box,  $m$  the mass of the particles,  $M$  the total mass of the system and  $G$  the constant of gravity), there

is no statistical equilibrium state anymore and the system undergoes gravitational collapse. This is a two-stages process. There is first a self-similar collapse leading to a finite time singularity: the central density becomes infinite in a finite time but remains integrable at  $r = 0$ . Then, in a post-collapse regime, the system forms a Dirac peak (a concentration of mass) at the center of the system. We show that these regimes of pre- and post-collapse can be described analytically. We also discuss the analogies between self-gravitating Brownian particles, the chemotaxis of bacteria in biology, and the Bose-Einstein condensation.

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### Statistical mechanics of two-dimensional Euler flows and minimum enstrophy states

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The formation of large-scale vortices in geophysical and astrophysical flows (e.g. Jupiter's Great Red Spot) in a remarkable property of two-dimensional turbulence. These coherent structures can be understood in terms of a statistical mechanics of the incompressible 2D Euler equation. In the Miller-Robert-Sommeria (MRS) theory, the statistical equilibrium state is obtained by maximizing a mixing entropy while conserving all the constraints of the inviscid dynamics, namely the energy, the circulation, and an infinite number of Casimir integrals equivalent to the moments of the fine-grained vorticity. Of course, the conservation of an infinite number of constraints leads to technical difficulties. It is also physically abusive since we expect that forcing and dissipation will destroy some of the inviscid invariants. We consider a simplified thermodynamic approach of the incompressible 2D Euler equation based on the conservation of energy, circulation, and microscopic enstrophy. Statistical equilibrium states are obtained by maximizing the Miller-Robert-Sommeria (MRS) entropy under these sole constraints. We assume that these constraints are selected by properties of forcing and dissipation. We find that the vorticity fluctuations are Gaussian while the mean flow is characterized by a linear  $\omega - \psi$  relationship. Furthermore, we prove that the maximization of entropy at fixed energy, circulation, and microscopic enstrophy is equivalent to the minimization of macroscopic enstrophy at fixed energy and circulation. This provides a justification of the minimum enstrophy principle from statistical mechanics when only the microscopic enstrophy is conserved among the infinite class of Casimir constraints. Relaxation equations towards the statistical equilibrium state are derived. These equations can serve as numerical algorithms to determine maximum entropy or minimum enstrophy states. We use these relaxation equations to study geometry induced phase transitions in rectangular domains. In particular, we illustrate with the relaxation equations the transition between monopoles and

dipoles predicted by Chavanis and Sommeria (1996). We argue that saddle points of entropy can be long-lived and play a role in the dynamics because the system may not spontaneously generate the perturbations that destabilize them.

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## Photonic Simulation Of The Dirac Equation In Metamaterials

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The Dirac equation provides a description of relativistic quantum mechanics for an elementary spin-1/2 particle, which predates the discovery of positron, an anti-particle of electron in high energy physics, and also has extensive applications in condensed matter systems such as graphene and topological insulators [1]. It is one of the milestones in the development of modern physics. Recent years it is realized that it is also a key to understand the topological phases from one- to three-dimensional systems and from insulators to superconductors or superfluids. On the other hand, the Maxwells equations form the foundations of classical electrodynamics and modern optics. Modern techniques and material sciences make it possible to precisely control photonic transport in artificial optical materials or metamaterials [2], which attracts great interests in the field of optics. Photonic technology available now is reaching the stage to illustrate unusual ways for photon manipulation such as negative refraction, electromagnetic cloaking, structure-induced coherence, and mimicking photonic black holes. Here we propose to link the metamaterials to topological phenomena in condensed matter systems and the relativistic quantum mechanics [3]. We find that the one-dimensional Maxwells equations can be written in the compact form which has the identical mathematical structure of the 1D Dirac equation. We perform a proof-of-principle photonic simulation of 1D Dirac equation in metamaterials by means of the full wave numerical simulation and microwave experiment. For the first time we successfully implement the band inversion of the Dirac equation. It is noted that with the band inversion chirality of electromagnetic wave in metamaterials changes from the right-handed to left-handed triad, which determines a matter-antimatter correspondence. Furthermore we utilize designing metamaterials to observe experimentally the topological phases and excitations in one dimension. This paves the way to investigate the topological phenomena in condensed matter systems and the Dirac-like particles in high-energy physics in a photonic simulator with controlled experimental parameters. Meanwhile we can also make use of the solutions of the Dirac equation to understand exotic phenomena observed in metamaterials.

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## Subsidization of Production and Financialization of Trade in Agricultural Commodities: two symptoms of the same disease?

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Agricultural subsidies represent a major cost in agricultural policy. Their world-wide annual impact is as large as 365 billions USD, [1]. These government programs are often criticized as an example of distortionary policies which limit the beneficial effects of competition on the selection of good investment opportunities and support inefficient industry protection [2]. Simultaneously, a growing body of literature, see e.g. [3], is criticizing the integration of agricultural commodity and financial markets for the opposite reason: the overflow of capital invested in the commodity markets feeds a destabilizing speculation and a tightening of market regulation is then claimed in order to protect "real" producers. While the empirical evidence of international food price spikes and volatility driven by financial markets is robust, its mechanics is still under debate.

This paper builds a stylized equilibrium model of production, trade and consumption of a given agricultural commodity that, among other findings, rationalizes these contradictory perspectives as a spillover of the inability of capital markets to promote efficient liquidity risk-sharing.

Within this framework we rationalize the subsidization of production and the financialization of trade as two effects with a common origin: capital markets are segmented and liquidity risk is not shared between long-term producers, short-term intermediaries and financial investors. In this case, investment flow is biased toward informed short-term trading at the expense of long-term financing of commodity production. A key observation emerging from the model is that the creation of a market for contingent securities, e.g. a future contract, improves production risk-sharing but tightens the liquidity constraints on long-term producers. Within our model, financialization of commodity markets can be understood as the growth in the volatility of aggregate production driven by an increase in the investment flow toward the short-term future market. The dynamic version of the model accounts for the observed large negative autocorrelation of the commodity price returns. The subsidization of production could then be rationalized as the public transfer forced by the scarcity of long-term investments. Our results suggest that the "time to produce" should be considered a fundamental order parameter in designing more efficient market regulations aimed at improving liquidity risk sharing.

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## Bootstrapping topological properties of complex economic networks

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The reconstruction of a network’s statistical properties when only partial information is available represents one of the outstanding and unresolved problems in the field of statistical physics of networks (Clauset2008, Mastromatteo2012). Yet, addressing this issue can bring to many concrete applications. A paramount example is the case of financial networks, where nodes represent financial institutions and edges stand for the various types of financial ties—such as loans or derivative contracts. These ties result in dependencies among institutions and constitute the ground for the propagation of financial distress across the network. The resilience of the system to the default or the distress of one or more nodes depends on the topological structure of the whole network (Battiston2012). In contrast, due to confidentiality issues, the information that regulators are able to collect on mutual exposures is very limited.

Typically, the analysis of systemic risk has been pursued by reconstructing the network using Maximum Entropy algorithms, that however have the drawback of assuming that the network is fully connected—which also leads to systemic risk’s underestimation (Mastromatteo2012). More refined methods (Mastromatteo2012) allow to obtain network reconstructions with arbitrary level of heterogeneity, but the density of connection must be specified *ex ante* and is not recovered by the algorithm. In order to overcome these problems, we propose a novel approach named bootstrapping (BS) method, which uses the limited topological information on the network to generate an ensemble of networks according to the exponential random graph (ERG) model (Park2004)—where the unknown parameters that define it are replaced by known intrinsic node-specific properties that we call fitnesses. The estimation of the network’s topological properties is then carried out within the ERG-induced ensemble.

We then study how the accuracy of the BS method depends upon the size of the subset of nodes for which the information is available, by focusing on those properties that are commonly used to describe a network: density of links, assortativity, hierarchy. To validate our method, we use synthetic networks generated through the fitness model, as well as real instances of economic systems: 1) the network of trades among countries, and 2) the network of loans of the E-mid interbank money market. We find that the method is remarkably robust with respect to the number of nodes used for calibration, leading (in many cases) to estimates of the networks properties within an error of 5% with the knowledge of a percentage as small as 10% of all nodes.

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## Short and long-range correlations between words in texts: Universality versus cultural effects

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A text consists of words which can be common words like articles, pronouns, conjunctions or less common words that specifies the topic(s) discussed in the text. However, a text is not only a bag of words. To get a meaningful text, the words should be arranged in a particular linear sequence. But, what are the rules governing that sequencing of words in order to have a text with meaning? In more physical terms, what are the forces acting on the words and putting these at order when we are writing a text? One can say that there are two kinds of forces: the first is the grammatical and syntax rules which act mainly at short ranges i.e. at nearby words and the second is the meaning or the topic(s) of the text (meaning field) which influence the word ordering at all scales and induce long-range correlations between words. At the same time, due to the source of the text (the author) the word sequences have strong randomness and unpredictability which are intermixed with the order coming from the above-mentioned correlations. The question addressed in this work regards the detection, quantification and further understanding of these multi-scale correlations in the presence of strong randomness. We are interested in the specific characteristics of these correlations (fractal or multifractal symmetry) and focus on their universality examining the effects of language family. The parallel texts of ten European languages extracted from the proceedings of European Parliament are mapped to word-length series to enable their mathematical analysis. In quantitative linguistics, the length of words has been considered a key feature whose systematic study can reveal important aspects of language structure and text typology. In this work, the short-range correlations in the analysis of word-length sequences are quantified through the *n*-gram relative entropy defined as the subtraction of the *n*-gram entropy of the real sequence from the corresponding entropy of the shuffled sequence. We have found non-zero bigram and trigram relative entropies being larger in Germanic languages w.r.t. Romanic ones. The long-range correlations have been studied through autocorrelation and multifractal analysis. Persistent non-zero values of the autocorrelation function are detected exhibiting a power law deterioration with exponents close to 0.5 for the scale 10 to 100 words. The shuffled autocorrelations fluctuate about zero revealing the dynamical origins of word ordering patterns. In harmony with these findings, fractal analysis of the sequences show multifractal behavior especially at the scales 10-200 words with the Germanic languages indicating more multifractality.

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### Model-aided hybrid metrology and characterization of surface nanoroughness

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The investigation of the surface roughness of films is one of the areas where statistical physics meets nanotechnology. In nanotechnology, the roughening of a film surface at nanoscale attributes to it new properties enabling new applications. At the same time, the surface roughening of films is a typical non-equilibrium statistical process which has been investigated with the whole spectrum of the concepts and methods of the statistical physics of far from equilibrium systems (scaling analysis and exponents, universality classes, dynamic phase transitions, emergence of self-organization, Monte Carlo and molecular dynamics techniques, nonlinear partial differential equations, networks etc.).

A critical step to the proper application of the methods of statistical physics to nanoroughening processes is their comparison with experimental measurements. During the last decades, the explosion of scanning probe/electron microscopy techniques multiplied the experimental roughness data. However, in most cases these measurement data are strongly biased by the applied measurement technique. For example the measurements from Atomic Force Microscopy bear the limitations of the used probe dimensions, whereas the images acquired from Scanning Electron Microscope lack direct height information for the images nanostructures.

To overcome these biases and get more reliable roughness measurements, in this work we propose a model-aided hybrid/holistic approach to the characterization of the nanoroughness of film surfaces, combining in a synergetic manner the measurement data from Atomic Force and Scanning Electron Microscopes (AFM and SEM respectively) along with a model reconstruction of rough surfaces. In particular, the hybrid approach is implemented by obtaining the height distribution function from AFM topographies (given the high accuracy in AFM height measurements) and the autocorrelation function or Fourier transform of the surface morphology from SEM images (given the high spatial resolution in SEM images). These functions are then used as input to an algorithm to generate random surfaces having similar roughness metrics to the real surface. These output surfaces may be finally employed to measure hybrid roughness parameters, such as the active surface area and the statistical parameters of the distributions of surface slopes. The latter parameters may be critical in many applications where rough surfaces are used to control wetting behaviour, bioadhesion or tribological properties. As an example, we apply the hybrid approach to the estimation of the ratio of the real to the nominal surface of a sample of cyclic olefin etched in oxygen plasma.

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### Condensation of large fluctuations in a thermodynamical system

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We consider the large-deviation function (rate function) of the fluctuations of extensive variables in a thermodynamic system. By computing it in some solvable model we show the widespread occurrence of non-analyticities, a feature due to the underlying phenomenon of condensation of fluctuations. Similarly to the more often considered phenomenon of condensation on average (as it is observed, for instance, in the Bose-Einstein condensates) condensation of fluctuations occurs when the contribution of a single mode (usually the one with zero wavevector) to a large fluctuation becomes macroscopic. This interesting issue is however conceptually distinct from condensation on average, and can be observed even in systems with trivial average properties. Indeed, one striking feature is that, contrary to what happens on average, condensation of fluctuations may occur in a classical system even in the absence of interactions or external constraints. We investigate this surprising phenomenon, in and out of equilibrium, in the context of simple models of classical statistical mechanics, like the Gaussian model or the Spherical model, chosen as paradigmatic non-interacting or interacting systems where explicit calculations can be carried over. It is shown that condensation of fluctuations is responsible for the non-analyticity of the large deviation function of extensive observables. The explanation of this phenomenon emerges from the mapping between large deviation events in the given system and typical events in a dual system subjected to an appropriate bias. It is shown that the bias in the companion system induces a mean-field-like effective interaction which, in turn, is responsible for the condensation transition which is mirrored in properties of fluctuations in the original system. Phase diagrams with a condensed and a normal phase, covering both the equilibrium and the off-equilibrium regimes, are derived for observables representative of generic behaviors. These show a rich phenomenology, with the interesting property of re-entrant phase-transitions. The difference in the experimental protocols required to observe the two facets of condensation, in the fluctuation spectra or on average quantities, is highlighted, and possible experimental settings are proposed.

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## Maximum Entropy Principle and Path Dependence

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In spite of its indisputable practical role, it is widely acknowledged that the range of applicability of the Maximum Entropy Principle (MEP) is rather limited. Nonergodic, non-Markovian and non-extensive systems, whose potential configurations grow sub-exponentially with the number of elements or system size, are clearly out of the area of the applicability of the MEP in its classical form. Statistical systems displaying any kind of memory, path dependence or strong internal interactions are candidates to exhibit this non-exponential growing of the phase space behaviour. The question if these systems are describable with a generalised MEP is a hot topic of research. Recent advances on the field of non-extensive statistical dynamics, in particular, the generalisation of the entropy functional ( $(c, d)$ -entropy) provided in (Hanel and Thurner, 2011) offer a clear theoretical framework on how the MEP could be extended to non-ergodic systems.

In this talk we will start reviewing how Boltzmann's fundamental definition of statistical entropy  $S = k \log(W)$  is related to the multiplicities of independent random processes. Crucially, for these kind of systems, this relation provides a straightforward way leading to the usual MEP, where the specific functional to be maximised is the well-known Shannon Entropy. This entropy functional is the only satisfying the four Khinchin's axioms for the entropy. It can be shown that the removal of the 4th Khinchin axiom (assumption of independence) naturally leads to a generalised entropic functional depending on two general parameters,  $(c, d)$ , whose particular values encompass almost all generalised entropies defined in the past. Under this framework, the MEP is perfectly generalisable and consistent (Hanel, Thurner and Gell-Mann, 2014). The generalisation of the principle is performed in an analogous way to the classical derivation of the MEP based on Shannon's entropy from Boltzmann entropy, when independence of random events is assumed. Consistently, this latter case appears as a particular case of the generalised MEP.

We finally apply the generalised MEP above presented to well known simple systems exhibiting path dependence. Indeed, we show that the statistical properties of Dirichlet-type systems can be exactly derived from the generalised MEP. In particular, for the first time we are able to use a generalised MEP to exactly predict the time-dependent distribution functions of simple path-dependent random processes, such as the Chinese Restaurant Process.

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## Country Development as a Complex Dynamical System: Weather-Like Forecasting

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Which will be the growth of the Gross Domestic Product and the competitiveness of China, United States, and Vietnam in next 3, 5 or even 10 years? Despite this kind of question has a large societal impact and an extreme value for economic policy making, how to provide a scientific basis for economic predictability is still a very challenging problem.

Recent results of a new branch – Economic Complexity – set basis for a framework to approach such a challenge and provide new perspectives to cast economic forecast into the framework of forecasting the evolution of a dynamical system as in the case of weather dynamics. Here we argue a recently introduced non-monetary metrics for country competitiveness (fitness) allows for quantifying the hidden growth potential of countries by the comparison of this measure of intangible assets with monetary figures such as GDP *per capita*.

This comparison defines the fitness-income plane where we observe that country dynamics present strongly heterogeneous patterns of evolution. The flow in some zones is found to be laminar while in others a chaotic behavior is instead observed. These two regimes correspond to very different predictability features of the country evolution: in the former regime we find strong predictable pattern while the latter scenario is characterized by a very low predictability. In such a framework, the usual tool used in Economics of regressions is no more the appropriate one to deal with such a heterogeneous scenario and new concepts, borrowed from dynamical systems theory, are needed. We therefore propose a data-driven method - the *selective predictability scheme* - where to assess future evolution of countries we adopt a strategy similar to the *methods of analogues* firstly introduced by Lorenz. In detail we show the existence of at least four different zones in the fitness-income plane, each zone has specific patterns of evolution with precise economic interpretations. This new scheme also draws new perspectives in the assessment of the wealth, development and success of nations. Rather than substituting GDP-based measures with new synthetic and somehow holistic indicators such as *human development index* and *genuine progress indicator*, a more scientific approach would correspond to introduce new dimensions, i.e. variables, to traditional monetary figures as done for fitness to properly grasp those intangible assets a country owns and cannot be merely measured or reduced into income terms.

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## Entropies of deformed binomial distributions

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In most of the realistic models in Physics one must take correlations into account; events which are usually presented

as independent, like in a binomial Bernoulli process, are actually submitted to correlative perturbations. These perturbations lead to deformations of the mathematical independent laws. As a matter of fact, the deformation of the Poisson distribution upon which is based the construction of Glauber coherent states in quantum optics leads to the so-called nonlinear coherent states. In order to study these kind of correlations we have introduced a generalization of the binomial distribution associated with a sequence of positive numbers that can be symmetric or asymmetric, i.e., it preserves or not the symmetry win-loss. The analysis of the nonnegativeness of the formal expressions was a key-point to give them a statistical interpretation in terms of probabilities. We present an approach based on generating functions that solves the previous difficulties: the constraints of nonnegativeness are automatically fulfilled, a complete characterization in terms of generating functions is given and a large number of analytical examples becomes available. We analyze three canonical examples of the symmetric generalization: a) a case based on a so-called  $q$ -exponential function; b) a case based on the Lambert function and Abel polynomials and c) a case based on Hermite polynomials. Interestingly, despite the fact that our symmetric distributions do not in general fulfill the Leibniz triangle rule, in fact, only the example (a) satisfies this rule strictly, we are able to show that for the other non trivial examples the rule is asymptotically verified. We have analyzed numerically these examples and we have also explored analytically the extensivity property of the corresponding BG entropy and other entropic forms, when the number of events tends to infinite. We found numerically and also analytically that for examples (a) and (c) the Boltzmann-Gibbs entropic form is extensive when the number of events tends to infinity but for the example (b) the Boltzmann-Gibbs entropy is not anymore extensive. The extensive entropy in this case is Rnyi entropy, with values of the Rnyi parameter  $q$  between zero and one. In the case (b) the Boltzmann-Gibbs entropy is proportional to the square-root of the number of events when this number tends to infinity.

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## Dynamics of Trust in Networks and Systemic Risk

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The financial crisis of 2007-2008 has motivated a vast amount of research in the realm of systemic risk. The focus has been on creating measures of systemic risk through appropriate network topological properties, which can act as early-warning signals for disaster, as well as on truthfully understanding the underlying mechanisms of contagion in financial markets, which can be amplified by diversification and leverage and mitigated with transaction taxes. In some of these studies, the proposed models and frameworks were tested against historical data, showing for example that reality is not

consistent with models assuming rational expectations.

After Lehman Brothers filed for bankruptcy in September 2008, the worsening of overall trust and confidence in the interbank market drove financial markets towards a credit freeze, corroborating the idea that this sentiment plays a major role in sparking and propagating disruptive events. Moreover, it is highly sensitive to expectations about what other people expect, which is one of the fundamental feedback loops for self-fulfilling prophecies. This notion of trust is already part of a few stylized network models. However, most analyses of the crisis so far have not taken this variable into account. We believe this is mostly due to the difficulty involved in surveying and measuring this sentiment at high frequency.

Based on intuitive assumptions about the confidence one has in each other, we build a network model to capture the features of the aggregate behaviour of the agents in terms of trustworthiness and links. In particular, we look into the propagation and amplification of negative events. We keep our model as simple and tractable as possible, while a number of parameters, such as panic and sensitiveness to difference (homophily), allows for a realistic landscape. In contrast to more static approaches, our model incorporates the dynamics of the trust and inter-agent networks simultaneously, with a mutual feedback between links and agents trustworthiness. Losing trust is a faster phenomena than building trust, triggering panic avalanches which cause sudden disintegrations of the network for specific parameter values. Our findings are supported by numeric simulations and mathematical analysis. This research project is part of the pan-European initiative CRISIS.

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## Statistical analyses of the numerical modelling of geomagnetically induced currents in a Brazilian transmission line.

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Magnetic storms are caused by disturbances in the magnetosphere-ionosphere system, as result of solar activity. Rapid changing currents in this system cause geomagnetic field temporal variations, which induce electric currents in conducting materials at the Earths surface (Pirjola, 2007). These currents, known as geomagnetically induced currents and that propagate in electric power transmission grids, pipelines, telecommunication cables and railway systems might cause problems in transformers. The understanding of GICs effects for different space weather conditions and risk analysis have been a matter of recent concern and debate in the scientific community, especially due to our increasing dependence on electricity and communication systems. The well-known power failure in the Hydro-Quebec system in March 1989, for example, affected human installations over a period of about 30 hours (Bolduc, 2002). The modelling

and prediction of GICs in electric networks may avoid such hazards. The calculation of GICs requires a model of electrical conductivity in the region of study, direct measurements or model of the geomagnetic field, computation of the electric field and network model of the power grid. High-voltage power systems are more vulnerable to GIC flows, especially where they offer low resistance for the electrical current compared to the ground. GICs are usually larger in high latitudes since auroral regions induce large east-west electric fields at the Earth's surface, amplifying these currents up to 100 Ampres. That is the reason why effects of space weather disturbances on high latitudes are more frequently explored. However, there has been a growing perception that GICs might also affect technological systems in low latitude regions. That is the reason why GICs have been recently measured and/or modelled in middle and low latitude countries such as Brazil (Trivedi et al., 2007). Methods of statistical physics could be applied to evaluate possible risks from electric system is subject to large GIC (Langlois, 1996). In this work, we used the plane wave model to calculate the geoelectric field and geomagnetically induced currents during the period 7-10th November 2004. We compare our results with data registered directly in the line (Trivedi et al, 2007). A statistical analysis has been applied to validate our model. We calculated the distribution of the amplitudes of the geomagnetically induced currents obtained from the numerical model and compare it with the distribution of the amplitudes experimentally measured in the line. The probability of the Brazilian power grid be subject to high geomagnetically induced currents is analyzed.

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## Stochastic Processes on interdependent networks

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Modern life in countries of advanced level of development relies on the coordinate functioning of several interdependent infrastructures. Due to their vital role, these infrastructures are often referred to as "critical". The protection of critical infrastructures against natural hazards or antropic attacks represents an important activity of both the European Union and the United States.

Modeling, Analysis and Simulation of such complex system may be achieved by different approaches among which High Level Abstraction Models play an important role due to their promptness in providing results and their capability to outline emergent behaviors. One of the most promizing of such approaches comes from Complexity Science. Standalone Critical Infrastructures can be modelled by that paradigm as networks; consequently, Interdependent Critical Infrastructures need to be treated as "Network of Networks" (NoN's).

Stochastic processes arise naturally on networks as conceptualization of different processes such as the epidemics (for both humans and animals); the gossip propagation; the fault accumulation and propagation on physical infrastructures; diffusion processes and other randomic activities. The same processes need to be modelled for networks of networks. The

present contribution will be focussed on Fault propagation and synchronization stability of interdependent infrastructures that can be abstracted to Diffusion and epidemic stochastic processes on NoN's. Inoperable I/O markov Models (IIM) represent useful tools to deal with disturbance propagation on networks and interdependent networks. Here The are extended here to general Markov processes on networks of networks. Among the different approach to deal with stochastic processes on network of networks, present contribution focuses on topological issues and especially on the spectral properties (of the laplacian and the adjacency matrix) that determine the epidemic threshold and the sinchronizability of the network of networks.

The concept of "Model Network of Networks" is introduced as a natural extention of "model networks" to the case of interdependent systems. The proper definition of a Model Network of Networks requires three basic ingredients: the model networks of each component network; the macroscopic topology of the system (where networks are treated as single entities) and a linkage strategy that tells which nodes of which networks are allowed to be connected. We have constructed several model Pairs of Network of different component network linked by different linkage strategies. The Spectral analyses of the resulting graphs provide evidence of emergent behaviors in large NoN's.

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## Equilibrium and Dynamic Properties of Strongly Confined Liquids

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Equilibrium and dynamic properties of strongly confined binary mixtures of simple liquids are studied using molecular dynamics and Monte Carlo simulations. We find that confinement of binary mixtures by a non-preferential repulsive wall causes, in addition to the well-known spatial modulation of the total density, a spatial modulation of the concentrations of the two components near the wall. When a repulsive wall is present, the local density of the liquid is depleted near the wall and an interface similar to that between the liquid and its vapor is created. The Guggenheim adsorption relation quantifies the concentrations of the components of a binary mixture near a liquid-vapor interface and qualitatively states that the majority or minority component enriches the interface for negative or positive mixing energy, respectively, if the surface tensions of the components are not very different from each other. We find that the Guggenheim relation remains valid for wall-induced interfaces at all temperatures and densities for systems with negative mixing energy. For systems with positive mixing energy, the Guggenheim relation remains valid at low densities, but it is violated at state points where correlations in the liquid are strong. This observation is validated from a calculation of the density profiles of the components of the mixture using classical density functional theory. Our results are relevant to recent

experiments on vapor-deposited glasses [1] in which variations in the concentrations of different components of the glass near the substrate and the interface with the vapor significantly affect the physical properties of the glass.

The spatially resolved dynamics of the confined liquid is found to oscillate in phase with the density modulation near the wall and to approach that of the bulk liquid far away from the wall. Unstructured walls make the dynamics near the wall faster than that in the bulk, whereas structured walls, such as those with a crystalline arrangement of particles, slow down the dynamics. We show that the local dynamics in systems with density modulation induced by an external potential can be substantially different from that in systems in which a similar density modulation is induced by the presence of confining walls. These results indicate that mode-coupling theories [2,3] in which the effects of confinement on the dynamics are expressed entirely in terms of the confinement-induced density modulation do not provide a complete description of the physics of the system.

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## Emergence of Periodic and Aperiodic Two-Dimensional Patterns through De-mixing

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A two dimensional (2D) binary film of thiol capped Au nanoparticles (AuNPs) and amphiphilic stearic acid (StA) is self assembled on the water surface of a Langmuir Blodgett trough. The morphology of this film is studied with a Brewster Angle Microscope (BAM) which resolves the film at the scale of few micrometers. The StA molecules form a monomolecular thin film while the AuNPs are found to self organize as a two dimensional network/ pattern of enclosed and connected micro-spaces of AuNP clusters in this StA template. The pattern is studied while varying the surface pressure  $\pi = \gamma_0 - \gamma$ , where  $\gamma_0(\gamma)$  is the surface tension of pure (monolayer-covered) water, which can in turn be readily controlled by compressing/expanding the film between the trough barriers. The network of AuNP clusters so formed through de-mixing was found to have extensive coverage in the template for concentration of AuNPs by weight,  $\rho \geq 10\%$  and  $\pi \geq 10mNm^{-1}$ . The 2D dynamics of this network which emerges from competing lipophilic interactions between AuNPs on the one hand and AuNP-StA tails on the other has been studied over a long time scale at constant  $\pi$ . The pattern lowers the number of nodes/connections ( $n$ ), evolving gradually ( $> 2$  hrs) to a state characterized by larger coherent structures and with greater in plane thickness of the AuNP threads. The pattern changes over to this lower  $n$  state, where the pattern consists of almost perfect circles minimizing the line tension of the network. In the higher  $n$  state (at short times) the mean square (ms) fluctuation of BAM intensity varies as  $F^2(l) = l^\alpha$  with  $\alpha \simeq 1.2$  over the length scales  $l < 6\mu m$  and wavy undulations about a flat plateau for length scales between  $6\mu m - 400\mu m$ . For the lower  $n$  state (at

long times), the ms fluctuation shows a single slope behavior wherein  $\alpha \simeq 0.5$  at all length scales. The ms fluctuation at small times is similar to that for a periodic wave with some randomness. The scale free behavior at large times indicates emergence of aperiodic self affine behavior from the initial quasi-periodicity.

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## Finite size effects on computer calculations of supercooled liquids

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Super-cooled liquids are out-of-equilibrium systems in which a material remains in the liquid phase at temperatures lower than its melting point. At even lower temperatures, these materials stop flowing, passing through the *glass transition*, solidifying to an *amorphous glassy phase*. Computer simulations of these systems have been done, and a problem that remains opened is how the system size influences the outcome. In this work, we present a numerical study concerning the effects of changing the system size for two different potentials in 2- and 3-dimensions: the Kob-Andersen model (Kob and Andersen, 1995) for a binary mixture

$$V_{\alpha\beta} = 4\epsilon_{\alpha\beta} \left[ \left( \frac{\sigma_{\alpha\beta}}{r} \right)^{12} - \left( \frac{\sigma_{\alpha\beta}}{r} \right)^6 \right] \quad (1)$$

with  $\alpha, \beta \in \{A, B\}$  and  $\epsilon_{AA} = 1.0$ ,  $\epsilon_{AB} = 1.5$ ,  $\epsilon_{BB} = 0.5$ ,  $\sigma_{AA} = 1.0$ ,  $\sigma_{AB} = 0.80$ ,  $\sigma_{BB} = 0.88$  and cutoff distance  $2.5\sigma_{\alpha\beta}$ ; and the power law repulsive interaction

$$\phi \left( \frac{r_{\alpha\beta}}{\lambda_{\alpha\beta}} \right) = \epsilon \left[ \left( \frac{r_{\alpha\beta}}{\lambda_{\alpha\beta}} \right)^k + \sum_{\ell=0}^q c_{2\ell} \left( \frac{r_{\alpha\beta}}{\lambda_{\alpha\beta}} \right)^{2\ell} \right], \quad (2)$$

if  $r_{\alpha\beta}/\lambda_{\alpha\beta} \leq x_c$ ,  $\phi(r_{\alpha\beta}/\lambda_{\alpha\beta}) = 0$  otherwise, used by (Karmakar and Procaccia, 2012), where  $\epsilon$  is the energy scale,  $x_c$  is the length for which the potential vanishes with  $q$  derivatives and, again,  $\alpha, \beta \in \{A, B\}$  and  $\lambda_{AA} = 1.0$ ,  $\lambda_{AB} = 1.18$  and  $\lambda_{BB} = 1.4$ . The coefficients  $c_{2\ell}$  are

$$c_{2\ell} = \frac{(-1)^{\ell+1}}{(2q-2\ell)!!(2\ell)!!} \frac{(k+2q)!!}{(k-2)!!(k+2\ell)} x_c^{-(k+2\ell)}. \quad (3)$$

The parameters chosen were  $x_c = 1.3854$ ,  $k = 10$  and  $q = 2$ , following the reference. As a first step we have reproduced the previous results in the literature, and we are implementing the numerical routine to use CUDA programs for the GPU computing of these potentials with an even larger number of particles.

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### Order parameter profiles in presence of topological defect lines

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Field theory is the natural tool to describe universal properties of statistical systems. Away from criticality, large distance correlations are determined by the low energy behavior of the underlying field theory, and a main role is played by the analytic structure of the matrix elements of local fields in the complex energy plane. Singularities on this plane have a physical meaning, simple poles being associated to bound states and branch cuts to the opening of scattering channels. In contrast with these well understood 'dynamical' singularities, additional ('kinematical') singularities have traditionally been considered as unwanted features consistent theories should be free of.

We point out that kinematical singularities at zero energy are instead essential to account for the non-locality of the order parameter field with respect to topological excitations, and that they determine the order parameter profiles in statistical systems with boundary conditions leading to the presence of topological defect lines.

The simplest example is provided by phase separation in two dimensions, for which the interface separating different phases corresponds to the trajectory of a kink. While the only available exact derivation of the order parameter variation across the interface concerned the lattice Ising model, we show how the kinematical singularity of the order parameter field accounts for two-dimensional phase separation in a general, exact and simple way. In more detail, the kinematical singularity determines the jump of the order parameter across the interface, while regular terms contribute to the interface internal structure, in particular to its branching properties.

Field theory also provides the fundamental and exact description of wetting in two dimensions. In particular, for the case of phase separation in a wedge, which provides the basic example of how the geometry of the substrate modifies adsorption properties, we determine the passage probability for the interface and exhibit the fundamental origin of the contact angle as well as of the wetting transition condition.

The role of kinematical singularities in higher dimensions is illustrated by the  $(n+1)$ -dimensional  $n$ -vector model. With suitable boundary conditions this model exhibits topological defect lines corresponding to the propagation in imaginary time of topological excitations: Ising kinks for  $n=1$ , XY vortices for  $n=2$ , and so on. We argue the form of the kinematical singularities needed to account for the variation of the order parameter. The result we obtain in this way reproduces that known for  $n=1$ , but should be exact also for  $n > 1$ .

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### Quorum Sensing: correlation in the bacterial world

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Bacteria are known to coordinate at a colony level a diverse array of physiological activities that are presumably productive only when groups of cells act in concert. This fascinating process, that eventually leads bacteria colonies to act as multicellular organisms, is known as *Quorum Sensing* (QS)[1]. From a biochemical point of view, QS is mediated by means of some small molecules called *autoinducers*, secreted by bacteria as a function of the colony growth. By sensing the concentration of such autoinducers, bacteria are able to estimate their density and initiate costly processes, that would be useless if started only by isolated individuals.

Evidently, this mechanism implicitly results in a biological example of correlation. In effect, bacteria use the autoinducers to influence the behavior of the colony, inducing gene expression or repression as a function of their density. In particular, *bioluminescence*[2], i.e. the property of some living organisms to transform chemical energy into visible light, is a phenomenon mediated by QS, widely diffused among bacteria. By means of Petri dishes spotted with the bioluminescent *Vibrio sp. PS1*, a *Vibrio harveyi* related strain, we show that in conditions of high temperature ( $30 \pm 1$  °C) and dry environment, bioluminescence is affected by QS-induced correlation. In particular, the distribution of the activation times of the bioluminescent activity in bacteria, proportional to their radiant flux, shows a fat tail for large times[3], which is well approximated by the *Gumbel distribution* of extreme value statistics. This happens despite no extremal process is known to take place. Furthermore, we found that this behavior is in accordance with the rule suggested by Bramwell for the occurrence of the Gumbel distribution in spatially averaged physical observables[4]. It is possible, indeed, to relate the total energy radiated by the bioluminescent bacteria, which is proportional to Gumbel's cumulative distribution function, to the logarithm of the bacterial growth curve, described by the *Gompertz model*[5].

This circumstance suggests an interesting relation between bacterial growth and bioluminescent emission. In effect, it is possible to relate the position and scale parameters of the Gumbel distribution ( $m$  and  $s$ ) with the biological parameters characterizing the growth of the colony under exam, i.e. the *maximum specific growth rate*  $\mu_{max}$  and the *lag time*  $\lambda$ . We present experimental evidence on this relation, trying to elucidate the complex inter-relationships between bacterial bioluminescence, QS and growth rates.

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## Penner Matrix Models Inspired by Interacting RNA

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The nonlinear Penner type external interaction  $e^{U(\phi)} = e^{\frac{N}{2} \sum_{i=1}^L (W^{-1})_i \text{Tr}[\log \phi_i^2]}$  is introduced and studied in the random matrix model of homo Ribo Nucleic Acid (RNA). The Penner interaction originally appeared in the studies of moduli space of punctured surfaces and has been applied here (for the first time) in addressing the problem of interacting RNA folding. An exact analytic formula for the generating function is derived using the orthogonal polynomial method. The partition function derived from the generating function for a given length enumerates all possible interacting RNA structure of possible topologies as well as the pairing. A numerical technique is developed to study the partition function and a general formula is obtained for all lengths. The asymptotic large length distribution functions are found and show a change in the critical exponent of the secondary structure contribution from  $L^{-3/2}$  for large  $N$  (size of matrix,  $N > L$ , where  $L$  is the length of the RNA chain) to  $L^{-1/2}$  for small  $N$  and double peak. This observation in the nonlinear model is similar to that observed in the unfolding experiments on RNA with osmolytes and monovalent cations. Preliminary results on biological networks for an enzyme will be briefly discussed.

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## Distribution of the positions of the scouts in a population of invaders with contact interactions

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Invasion is a common phenomenon in many biological or ecological processes. In particular, our motivation is the invasion of healthy tissues by tumour cells, which happens in some brain tumours called diffuse gliomas, but also when a tumour has started its metastatic transformation. Our aim is to get quantitative predictions about the invaded area (or volume), e.g. the statistical distribution of the position of the furthest individual (a cell in the context of tumours) from the source of invaders, or from the center of the expanding population if there is no source.

In the context of diffuse glioma, this information is crucial for the success of surgery and finally the survival of the patient. Indeed, it is commonly believed that isolated tumour cells left by the surgeon because they are undetectable with standard imaging techniques (e.g. MRI) are responsible for tumour regrowth, then death of the patient. In the context of ecology, this information may guide the fight against a newly introduced invasive species.

We address this question in the framework of cellular automata (or individual-based models) where agents perform

random walks with excluded volume and contact interactions, and proliferate. In particular, we study some kinds of exclusion processes with contact interactions that have been proven relevant for migrating tumour cells[1,2].

First, we establish, in two space dimensions, approximate analytic equations for the density of invaders which take nearest-neighbour correlations due to contact interactions into account. These equations take the form of coupled partial differential equations for the density and the intensity of correlations. We show, by careful comparison to stochastic simulation results, that including the correlations in this continuous space limit significantly improves the quality of the analytic approximations. They may also help to measure the intensity of interactions in experiments[3].

Then, we give (approximate) analytic predictions regarding the distribution of the furthest invader and the geometrical parameters of the convex hull of all invaders, as a function of the time allowed for invasion and of the nature and intensity contact interactions. They are successfully compared to stochastic simulations.

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## Connectivity of confined dense networks

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We consider random geometric graph models relevant to wireless mesh networks. Nodes (wireless devices) are placed randomly in a domain with a uniform distribution, since in many cases it is not desirable to specify the locations in advance. Uses include vehicular networks, sensor networks for environmental monitoring, and also ad-hoc networks for disaster recovery and military applications.

Pairwise connections are made independently with probability a specified function of the distance between the pair of nodes, either the commonly studied unit disk model (a step function from one to zero at a fixed range) or soft connection functions such as  $\exp(-r^\eta)$ , as derived in more practical Rayleigh fading models. Here the path loss exponent  $\eta$  ranges from about 2 in uncluttered to 6 in cluttered environments. In a more general anisotropic model, the connection function also depends on the orientation of the nodes.

The probability that the network is (k-)connected is estimated as a function of density using a cluster expansion approach. This leads to a double integral over the domain which can then be estimated in the high density limit to obtain an expression involving a sum of boundary components: bulk, faces, edges and corners. At the very highest densities the connectivity is dominated by the sharpest corners, but in general several boundary components are required for an accurate estimate of the connection probability.

For connectivity, we find that the disconnection probability is at a first approximation given by the probability of a single

isolated node. Similarly, network resilience is quantified by  $k$ -connectivity, the property of connection when any  $k - 1$  nodes are removed. We assume that this is given by the more tractable probability of minimum degree  $k$ . Numerical tests show that this approximation holds very well, but only for the more practical soft connection functions. Thus, incorrect choice of connection function may lead to qualitatively incorrect results.

Anisotropic connection (beamforming) enhances connectivity, but only when the path loss exponent  $\eta$  is less than the spatial dimension. In the presence of boundaries, a multi-lobed radiation pattern is required, for example with 14 lobes to mitigate the effects of a cubical corner.

Random geometric graphs and the effects of boundaries, soft connection functions and anisotropy are thus of major practical importance in the design of effective low-cost systems.

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### On the spatial distribution of the Italian primary school-size

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An increasing interest in social phenomena has been devoted in physics in the last decades. Growing literature has shown complexity features of cities, firms, or social groups. As well as other social complex systems, schools feature power law distributions, fat tails and properties incomprehensible using orthodox theories. An Italian official dataset with  $N = 17,871$  primary schools allows us to look at the Italian schooling system from a novel complex perspective, conveying information on the number of classes and students enrolled, and locations. We introduce a definition of school-size  $x_i$  as the number of students currently enrolled in the school  $i$  and we show that the PDF can be well approximated by a lognormal distribution. However, we find that the lognormal prediction does not fit appropriately the tail parts of PDF. Similarly to firms, data suggest that there are too many small schools on the left side and lesser in the upper tail which exponentially decays. In particular, despite the fat lower tail has been particularly targeted in the past years by political interest and lawful attempts, no rising of the size of the schools in the lower quantiles has been obtained.

The biggest puzzle of the school-size PDF is the two-peak shape. We test bimodality finding that the Italian primary school-size distribution is very likely to be a mixture of two laws governing small schools in the countryside and bigger ones in very dense regions, respectively. The bimodality source is addressed in the paper by introducing a new spatial interaction analysis. Motivated by the absence of any territorial constraint in school choice, according to the law n. 275/99, we introduce a measure of the average spatial interaction intensity between a school and the surrounding

ones in different Italian regions. We find that interactions are very weak, on average, for small schools, especially for countryside-based regions. This pattern involves small villages, with only one school, whose closeness coincides with the proximity of schools.

Despite the free choice, primary students generally do not move across cities to attend a school. As a result, school density and school-size are prevalently driven by the population density and, in turn, by the geographical features of the territory. Fat lower tail and bimodality are then natural characteristics of the Italian school organization that from the aftermath of the World War II has been aimed to spread education within the country.

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### Dependency structure and scaling properties of financial time series are related

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There are two main elements that define the complexity of financial time series: the first is multifractality [1], which is associated to the behavior of each single variable and the way it scales in time; the second is the structure of dependency between time series, associated with the collective behavior of the whole set of variables [2,3]. So far, these two manifestations of complexity have been investigated separately. In this talk I will point out that -in fact- they are related [4]. I will first introduce a graph-theoretic approach to extract clusters and hierarchies in an unsupervised and deterministic manner, without the use of any prior information [2] showing that applications to financial data-sets can meaningfully identify industrial activities and structural market changes [5]. I will then show new empirical observations of a deep interplay between cross-correlations hierarchical properties and multifractality [4]. In particular the degree of multifractality displayed by different stocks is found to be positively correlated to their depth in the hierarchy of cross-correlations.

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## Biomimicri, Imitation and Learning As A Method For Developing Cognitive Agents

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There is experimental research showing that Newborn infants ranging in age from 0.7 to 71 hours old were tested for their ability to imitate 2 adult facial gestures: mouth opening and tongue protrusion. At least one of the explanations of this early imitation for babies less than 3 days old is that it is instrumental, geared to better interact with the care providers. Later on, when babies begin to grow they use imitation to evaluate the consequences of actions, e.g. if a behavior or action and its consequences are rewarding, the child is very likely to imitate the same behavior or action, otherwise will stay away from it. It is conceivable that both animal and human knowledge and behavior may include a concatenation of: observation, evaluation, imitation, evaluation, and learning. Once the results of certain behavior have been shown to be good or bad, this information becomes part of what has been learned. Once a sufficient number of lessons have been learned, all these lessons become part of the animal or human toolbox to navigate through life. This is only a simplified view of the results of the studies of child development theorists, but it is a simplified model that may explain in layman language what happens. This view could help to develop man-made entities able to evolve independently in a way akin to what happens in nature. These man-made entities may take the form of software programs, of small hardware devices (e.g. robots) or both. To avoid costly mistakes we must model our man-made entities before we build them. For the purpose of modeling and simulation, structurally and architecturally simple entities can be identified with autonomous cognitive agent. We followed the philosophy of biomimicri because we believe that at various points in time natural evolution happened because of the actions of entities unable to deal with crisp values and unable to express computationally complex mathematical formulas. We show how structurally simple agents can observe, evaluate, imitate, and iteratively evaluate and imitate and, in so doing, act to their own survival and evolutionary advantage. Our talk will describe this stream of our research, based on biomimicri, and in particular on the ability to imitate and learn. Our goal is to develop simple cognitive agents able to learn, imitate, and evaluate the results of this imitation. We have already achieved good results with the simple task of crossing a highway avoiding the incoming vehicle traffic.

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## Critical Casimir Forces

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Long-ranged correlations in a fluid near its critical point lead to clearly identifiable effective forces acting on confining walls. The corresponding universal scaling functions are discussed for different boundary conditions and geometries. The theoretical predictions are compared with high precision experimental data for He<sup>4</sup> and He<sup>3</sup>/He<sup>4</sup> wetting films near the superfluid phase transition as well as with synchrotron scattering data from classical binary liquid mixtures. Direct measurements and applications for colloidal suspensions are discussed.

## Understanding the nature of climate transitions from paleoclimatic data

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It is taken for granted that the limited predictability in the initial value problem, the weather prediction, and the predictability of the statistics are two distinct problems. Lorenz (1975) dubbed this predictability of the first and the second kind respectively. Predictability of the first kind in a chaotic dynamical system is limited due to the well-known critical dependence on initial conditions. Predictability of the second kind is possible in an ergodic system, where either the dynamics is known and the phase space attractor can be characterized by simulation or the system can be observed for such long times that the statistics can be obtained from temporal averaging, assuming that the attractor does not change in time. For the climate system the distinction between predictability of the first and the second kind is fuzzy. On the one hand, the predictability horizon for a weather forecast is not related to the inverse of the Lyapunov exponent of the system. On the other hand, turning to climate change predictions, the time scales on which the system is considered quasi-stationary, such that the statistics, say mean surface temperature, can be predicted as a function of an external parameter, say atmospheric greenhouse gas concentration, is still short in comparison to slow dynamics such as the oceanic overturning. On these time scales the state of these slow variables still depends on the initial conditions. The non-linear nature of the problem furthermore opens the possibility of multiple attractors, or multiple quasi-steady states. As the paleoclimatic record shows, the climate has been jumping between different quasi-stationary climates. Such a jump happens very fast when a critical tipping point has been reached. The question is: Can a tipping point be predicted? This is a new kind of predictability (the third kind). If the tipping point is reached through a bifurcation, where the stability of the system is governed by some control parameter, changing in a predictable way to a critical value, the tipping is predictable. If the sudden jump occurs because internal chaotic fluctuations, noise, push the system across a barrier, the tipping is as unpredictable as the triggering noise.

In order to hint at an answer to this question, an analysis of the Dansgaard-Oeschger climate events observed in ice core records is presented. The result of the analysis points to a fundamental limitation in predictability of the third kind.

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### The entanglement temperature for the N-dimensional quantum walk

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Our studies suggest that if a quantum dynamics develops in a composite Hilbert space (i.e., the tensor product of several subspaces), then the behavior of an operator that belongs only to one of the subspaces may camouflage the unitary character of the global evolution. The entanglement between the position and coin state of a N-dimensional quantum walker is shown to lead to a thermodynamic theory. The entropy, in this thermodynamics, is associated to the reduced density operator for the evolution of chirality, taking a partial trace over positions. From the asymptotic reduced density matrix it is possible to define thermodynamic quantities, such as the asymptotic entanglement entropy, temperature, Helmholtz free energy, etc. We study in detail the case of a 2-dimensional quantum walk, in the case of two different initial conditions: a non-separable coin-position initial state, and a separable one. The resulting entanglement temperature is presented as function of the parameters of the system and those of the initial conditions.

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### Angular correlations of x-rays scattered from nano particles and biomolecules in solution

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In 1977 Z. Kam [1] showed theoretically that angular correlations of x-rays scattered from internally ordered objects such as proteins in solution could yield detailed information about the internal structure of the scattering particles. We will show that advances in x-ray sources and detectors have now made this possible at atomic resolution [2].

For an ensemble of  $\mathcal{O}\{10^9\}$  simple metallic nano parcels exposed to a micro-focussed synchrotron radiation x-ray beam we have been able to extract angular correlations of the data as a function of azimuthal angles around the the powder rings for wide-angle scattering  $C(q_1, q_2, \theta_{q_1, q_2}) = \langle n_{q_1} n_{q_2} \rangle - \langle n_{q_1} \rangle \langle n_{q_2} \rangle$ . For a simplified model of the internal structure for a 20nm silver nano particle as an fcc lattice confined to a sphere, the correlations may be

thought of as resulting from double bragg scattering resulting from self-interference of pairs of x-rays scattered from the same nano-particle. These may be estimated both for auto-correlations within a give ring subtending scattering vector  $q_1$ , and for inter-ring correlators between rings subtending vectors  $q_1$  and  $q_2$ .

We demonstrate that the intrinsic background noise resulting from independent scattering events from different particles can be averaged to effective convergence by averaging over  $\sim 10^3$  independent x-ray shots through different parts of the sample [3]. The resulting correlator angles  $\theta_{q_1, q_2}$  are in accord with the fcc lattice of metallic silver.

We will discuss how these measurements can provide atomic resolution constraints on atomic models of biological molecules such as DNA and proteins. Application of this Correlated X-ray Scattering (CXS) approach to time resolved measurements of chemical reactions of biomolecules using x-ray free electron lasers (xFELs) will allow atomic resolution snapshots of changing molecular conformations taken in the timescale of the  $\sim 10$ 's of femtoseconds duration of an xFEL pulse. These time scales are too short for appreciable atomic motions to take place, so that the capture of structure before the molecule is destroyed by the x-ray pulse becomes possible [4]. We will discuss how a pump-probe time series of such measurements can place constraints on the spectrum of intermediate molecular conformations obtained from Markov State Modeling of the kinetics of biomolecules[5].

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### Pattern Formation on Networks: a Continuous Time Random Walk approach

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Motivated by the recent explosion of network science, we consider pattern formation on networks including Turing Patterns, both stationary and oscillatory, as well as chemotaxis. We characterise pattern formation on networks as a build up of concentration on specific vertices. We highlight similarities and differences between pattern formation mechanisms in the continuum and on networks. A generalised master equation (GME) governing these mechanisms may be derived using a Continuous Time Random Walk (CTRW) approach. Such an approach is particularly useful when modeling non-Markovian processes such as anomalous diffusion. We also consider the application of modeling the development of prion-like diseases on the white matter transport network in the brain.

The network model we consider allows for reactions of particles on vertices and diffusion of particles between vertices. The CTRW incorporates two separate processes; first a particle waits on a vertex for some time taken from a waiting time probability density before jumping to a neighbouring node. The jump process is governed by a jump probability density. The precise incorporation of reactions

into the CTRW model for networks is nontrivial, as, even in the spatial continuum case, reaction and diffusion processes become entwined. The resultant GME provides a fundamental description of reactions, diffusion and forcing on networks. Time- and vertex-dependent forces and edge weights may be incorporated by including a bias in the jump density.

Anomalous diffusion is an ubiquitous phenomena observed in physics, biology, chemistry and sociology where the mean squared displacement of the system scales sublinearly with time. Such phenomena can be trivially incorporated using the CTRW approach by using a heavy tailed waiting time density, instead of the commonly used exponential waiting time density. We will show how this affects the dynamics of the system in a range of cases including Turing patterns and chemotaxis.

We conclude by investigating a range of applications including the spread of prion-like diseases in the brain. Such diseases are caused by misfolded proteins spreading through the brain's white matter transport network in a subdiffusive manner and locally catalysing other healthy proteins to misfold. These misfolded proteins build up in stationary plaques on neurons, causing them to malfunction. We will show how anomalous diffusion affects the observed buildup of plaques in the brain.

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### Random field Ising model with finite connectivity

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The random field Ising model (RFIM) has been attracting intense interest of researchers since its introduction by Larkin [1] more than four decades ago. From theoretical point of view, the reason comes from that this disordered model presents an impressive richness of static and dynamic behavior behind its apparently simplicity [2]. From the experimental one the interest comes from that random field (RF) type phenomenology can be found in a variety of distinct scenarios as vortices in superconductors, disordered Cerium systems and ferro-electric systems [3]. However, much of the interest in the random field Ising model for both theoretical and experimentalists can also be directly traced from the remarkable proposition by Fishman and Aharony that this model could be generated in a real diluted Ising anti ferro-magnetic field [4]. Moreover, variations of the applied magnetic field would adjust the strength of the random field. As consequence, it also allows to confirm theoretical predictions as, for instance, that the ferromagnetic (FM) phase is suppressed by the random field for a dimension  $d_c t$  with the results of the simulation.

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### The Square-Well Model within the Mean Spherical Approximation as a Reference System in Variational Calculations for Liquid Metals

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The variational method of the thermodynamic perturbation theory is widely used in structure and thermodynamics calculations for liquid and amorphous metals. Recently, the square-well model (SW) was suggested as a reference system in the variational calculations for simple liquid metals for the first time [1]. It was found that the SW-variational procedure gives a lower upper bound of the Helmholtz free energy (HFE) than the hard-sphere-variational procedure. In [1] the SW system was taken within the random phase approximation (RPA). It is a shortcoming of the work [1] since the SW-RPA solution leads to unphysical behavior of the structure factor at some sets of the SW parameters. Progress in this area can be achieved by using more accurate theories of liquids, such as the mean spherical approximation (MSA). Here, we use the SW-MSA reference system for which the semi-analytical approach developed in [2,3] is applied. Thermodynamics and structure of pure liquid alkali metals are studied. It is found that the Helmholtz free energy has not a global minimum as a function of all three SW parameters. For this reason, we suggest a minimization with respect to the hard-core diameter only at fixed SW width and SW depth. Its values are determined from the characteristics of the effective pair potentials calculated in the framework of the Animalu-Heine pseudopotential approach which used here for describing the metals under consideration. Results obtained in local HFE minimums for each metal under consideration agree better with experiment than ones obtained earlier [1] with the SW-RPA reference system.

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### Superdiffusion as a result of mechanical work

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Superdiffusion (fast diffusion) is usually considered as a result of Lévy statistics. The process, in a sense, is more random than normal diffusion. It sounds paradoxical, but such deterministic external influence as mechanical work can lead to superdiffusion. It is possible only for systems having sources of the randomness where the mechanical energy can be quickly transformed into the energy of chaotic motion. Dispersing billiards satisfy the following strong chaotic

properties: a) the central limit theorem holds; b) correlations decay exponentially; c) the particle flow is ergodic with mixing. As a result, one can define the canonical ensemble and such parameters as temperature, pressure, diffusion coefficient, mechanical work, etc. for this type of billiards and apply the theory of two-dimensional ideal gas. A quite natural generalization of the classical billiards is a system with oscillating boundaries. The unlimited linear growth of particle velocity in these systems is called the Fermi acceleration phenomenon. As is well known, Fermi acceleration leads to anomalous transport in periodical Lorenz gas with stochastically oscillating scatterers. A periodic Lorentz gas is a system containing a set of heavy discs (scatterers) embedded at sites of an infinite lattice. Particles move freely among these discs.

In our paper, we consider diffusion in Lorenz gas with random distribution in space of periodically moving scatterers. These periodic oscillations of scatterers can be interpreted as the mechanical work. This work expands on the increase of the internal energy. As is shown, the consequence of periodic oscillations of the scatterers is that the diffusion coefficient is in three times higher than for stochastic scatterers motion of the same intensity. It should be emphasized that this type of superdiffusion can be described by a standard approach, analogously to the normal diffusion in the ideal gas. In this case the diffusion coefficient will increase linearly with time as a result of heating provided by the mechanical work. The method proposed does not require complicated argumentation, and the final results are in a good agreement with the results of direct numerical simulations. This approach can still be useful for investigation of particles motion in complex landscapes, like the anomalously fast motion of metal clusters on graphite surfaces.

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### Isomorphs, hidden scale invariance, and quasiuniversality

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This presentation first establishes an approximate scaling property of the potential-energy function of a classical liquid with good isomorphs (a Roskilde-simple liquid) [1]. This pseudo-homogeneous property makes explicit that and in which sensesuch a system has a hidden scale invariance. The second part of the paper gives a potential-energy formulation of the quasiuniversality of monatomic Roskilde-simple liquids, which was recently rationalized in terms of the existence of a quasiuniversal single-parameter family of reduced-coordinate constant potential-energy [2]. The new formulation involves a quasiuniversal reduced-coordinate potential-energy function. Consequences of this are discussed.

Background: Simple liquids are traditionally defined as systems of spherically symmetric particles interacting via pairwise additive forces. It is now known, however, that a number of such systems like the Gaussian core model, the Lennard-Jones Gaussian model, the Jagla and related discontinuous-force

models, and other models exhibit quite complex behavior. On the other hand, van der Waals molecular liquids are generally regular and simple in their properties. In view of these facts we recently with Ingebrigtsen and Schrder suggested defining instead liquid simplicity from the property of strong correlations between equilibrium virial and potential energy fluctuations in the NVT ensemble [3]. In practice there is considerable overlap between the two definitions, for instance the Lennard-Jones liquid and related systems are simple in both senses. One notable difference is that realistic systems are only simple in the present meaning of the term in part of their phase diagramsimplicity does not apply near the critical point or at gas states (where different kinds of simplicity apply, of course). With regard to real liquids, it appears that most or all van der Waals bonded and metallic liquids are simple, whereas covalently bonded, hydrogen-bonded, and strongly ionic liquids are generally not simple because directional and competing interactions tend to weaken the virial potential-energy correlations. Roskilde-simple liquids are characterized by having isomorphs in their thermodynamic phase diagram. An isomorph is an equivalence class of the following equivalence relation: two state points are isomorphic if all pairs of physically relevant microconfigurations of the state points, which trivially scale into one another, have the same configuration-space canonical probability. Among pair-potential liquids only inverse-power law (IPL) systems have exact isomorphs, but all Roskilde-simple liquids have isomorphs to a good approximation. Simple liquids simple properties derive from the fact that the existence of isomorphs implies that their thermodynamic phase diagram is effectively one-dimensional instead of two-dimensional for all properties that are isomorph invariant. Examples of such properties are: Newtonian and Brownian reduced-unit dynamics, reduced-unit static structure factors of any order, the excess entropy, the isochoric heat capacity. For any simple liquid solidification defines an isomorph in the phase diagram; this implies invariance along the melting curve of, e.g., excess entropy, reduced viscosity, reduced heat conductivity, reduced diffusion constant, etc., as well as invariance of the Lindemann melting criterion.

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### IRIS observations of O IV and Si IV Lines in the Solar Transition Region and diagnostics of the $\kappa$ -distribution

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The observations of the solar transition region O IV and Si IV lines by the Interface Region Imaging Spectrograph (IRIS, Pontieu et al., 2014) show unusual line intensities ratios. The formation of these lines were investigated for both Maxwellian and non-Maxwellian conditions characterized by a  $\kappa$ -distribution with a high-energy tail. We have investigated

the formation of the O IV, Si IV and S IV lines. To do that, we utilized the state-of-art atomic data that will be part of the upcoming CHIANTI version 8. The O IV 1401.16Å line is formed at higher temperatures than the neighboring Si IV1402.77Å line. This is true for all values of  $\kappa$  considered. Taking into account the slope of the differential emission measure (DEM) in the transition region, the Si IV line can be formed predominantly at very low temperatures of  $\log(T/K) \rightarrow 4$  even for weakly non-Maxwellian situations. In contrast, the O IV 1401.16Å line will be formed at such low  $T$  only under extremely non-Maxwellian situations of  $\kappa \approx 2-3$  or in regions characterized with very steep DEM slopes. If the values of  $\kappa$  diagnosed by Dzifcakova and Kulinova (2011) are correct, the Si IV lines will be formed in different parts of the transition region than the O IV line, a situation possibly discernible by the IRIS instrument. The synthetic spectra are predicted to have lower O IV intensities compared to the purely Maxwellian transition region. No direct diagnostics of  $\kappa$  was found using O IV line ratios, due to close wavelengths of these lines, which are formed from upper levels having similar energies.

The usefulness of the O IV lines for density diagnostics depends on the ratio considered. The O IV 1404.78Å/1399.77Å ratio is particularly useful, as it is only weakly dependent on temperature and  $\kappa$ , except for extremely non-Maxwellian situation of  $\kappa \approx 2$  and very low  $\log(T/K) \rightarrow 4$ , where the density sensitivity is lost. However, the O IV 1404.78Å line is blended with the S IV1404.81Å transition. The contribution of this blend can be estimated using the S IV1406.02Å line.

It is also found that photoexcitation of the Si IV lines by photospheric and/or chromospheric radiation cannot be neglected. We recommend using realistic spectra to calculate the photoexcitation contribution to these lines. The O IV lines are not found to be sensitive to photoexcitation.

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## Phylogenetics as Quantum Computing: Concepts, Tools, Simulations

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This work puts forward a novel application of the discipline of quantum computation-information to the field of evolutionary phylogenetics. Phylogenetics' main task is to construct ancestral relationships (phylogenies), inferred by analyzing statistical data, collected for various (morphological or genotype) kinds of characters or traits, possessed by selected groups of biological organisms (taxa). This amounts to construction of phylogenetic trees with appropriate branching patterns and evolutionary lengths, that successfully reproduce statistical trends of alignments of sequences of certain characters. Various evolutionary models that compete by adjusting their tree vertex transition probabilities, to accomplish this computationally NP-hard task, are then assessed by some statistical estimation such as maximum likelihood measure. In

this work a quantum simulation of phylogenetic evolution and inference, is introduced in terms of trace preserving positive maps (a.k.a. quantum channels) operating on quantum density matrices defined for quantum systems with Hilbert space  $CN$ , which encoding states of biological taxa with  $N$  characters. Simulation of elementary operations such as speciation (branching of trees) and phyletic evolution along tree branches, are put forward utilizing conditional control-not unitary gates and quantum channels with unitary or complex matrix Kraus generators. Basic evolutionary models (Jukes-Cantor JC, Kimura two-parameter, three-parameter K2, K3, binary symmetric model B, and the Felsenstein model), are simulated and associated to quantum circuits. Specifically, the so called group-based phylogenetic models (JC, K2, K3, B), are associated to quantum walks (QW), with unitary Kraus generators (random unitary channels), and the Felsenstein model is related to post-measurement states. Simulation of iterative cherry growing and cherry pruning basic processes in a phylogenetic trees is provided and its dual character is shown in terms of the state-observable duality in the quantum setting. The central problem of phylogenetics i.e. the statistical estimation of free parameters of stochastic matrices implementing the stochastic evolution of characters along tree branches is addressed by formulating an analogous quantum maximum likelihood estimation problem for the free parameters of quantum channels operating along branches. Final remarks concern the role of quantum computation simulation of phylogenetics in issues of computational complexity.

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## Ornstein-Uhlenbeck limit for the velocity process of an $N$ -particle system interacting stochastically

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We consider a 3-dimensional  $N$ -particle system with mass  $m$  and no potential energy. The interaction is modeled as random momentum exchange between particles, obeying conservation of energy. The dynamics of the system is given by a single stochastic differential equation for a  $D = 3N$  dimensional velocity vector ( $\mathbf{V}$ ) driven by a  $D$ -dimensional Brownian noise term. A quick look at this evolution equation shows us that a single component of  $\mathbf{V}$  evolves independently of the remaining directions according to an one dimensional Ornstein-Uhlenbeck process driven by a single noise term along the same direction (corresponding to a particle moving in "white noise" with friction), when this component is small enough. Our interest, however, is to study the limiting process for the components of  $\mathbf{V}$  when these are of order one. Let  $A$  be the noise amplitude and  $k_B T/2$  be the total energy per degree of freedom. We thus consider the component  $V_1$  of  $\mathbf{V}$  and the component  $AB_1$  of the noise term driving  $\mathbf{V}$ . Call  $U_1$  the Ornstein-Uhlenbeck process driven by the noise  $AB_1$  in a viscous bath with friction rate  $mA^2/(k_B T)$ . We prove

that  $V_1$  converges in probability to this  $U_1$  as  $N \rightarrow \infty$ . The proof easily extends to any finite number  $n$

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### Sudden changes and freezing of the correlations between two qubits in cavity QED network under thermal dissipations

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Quantum correlations play a fundamental role in quantum computation and quantum information processing, where entanglement is usually considered a popular measure of such correlations. However, more general measures such as entropic quantum discord and geometric quantum discord have shown to capture non-classical correlations including the completely separable systems, e.g. DQC1 model. Recently, we proposed a scheme [1] for generation and protection of long-stable entanglement and quantum discord (QD) by the effects of the thermal environments considering the model [2] of two atoms (qubits) interacting with the fields of cavities and fibre with dissipations to thermal reservoirs. In the present study we discuss the generalization of the mentioned model, considering a versatile study of the classical and quantum correlations quantified by entropic and geometric measures for different conditions of the system under evolution for large number of total excitations. As results of our investigation, we discover various interesting phenomena in this system, such as sudden transitions in the dynamics of the classical and quantum correlations, effects of freezing of the quantum correlations, entanglement sudden-death (ESD) and revivals, etc [3]. In particular, we find that the thermal reservoirs act detrimentally on the freezing of correlations, but fortunately it is possible to restore the effect of freezing by an efficient coupling of the fibre connecting the two cavities with the remote qubits. Furthermore, for certain initial conditions, we find double sudden transitions in the dynamics of the Bures geometrical quantum discord (GQD) [4], while the classical correlations and the entropic quantum discord evidence only one sudden transition. The second transition of GQD tends to disappear at a critical temperature, so freezing these correlations. As well we discuss some ideas of the experimental realization of the proposal. We conclude that by controlling the dissipation mechanisms one may engineer the quantum correlations with multiples sudden changes and freezing periods in the temporal evolution, effects which can find practical applications. A kind of thermal critical effects in this model are expected like in other systems [5]. Hence, in the proposed research we demonstrate the existence of some interesting phenomena for quantum information applications, which also may bright more light on the fundamental understanding of the mentioned effects in the case of quantum open systems with thermal channels of dissipation.

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### Nonlinear Electron-Phonon Interaction Mechanism Explains the Exotic Superconducting Phase in a Graphene Bilayer

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In this communication we propose a different physical approach to obtain the "exotic" effects in superconductivity demonstrated recently in [1]. We show that the results and conclusions presented in this paper can be interpreted, with excellent agreement, by the existing microscopic model, published by one of us in [2] and proposed as a more general theory of unconventional superconductivity in materials (different from BCS-like) based on the concept of electron-multiphonon interaction for the electron pairing. We will present briefly the main results of this model and applying it for the case of the graphene bilayer one reproduces perfectly - qualitatively and quantitatively the phenomena demonstrated in [1], so proving a very good congruence between two different models which in fact leads to the same physical effect - unconventional superconducting phase. Our model is mainly based on the possibility of creation of the Cooper pairs in some materials via the processes of many phonons exchange between the electrons. The approach is limited to the two-phonon processes - scattering, absorption and emission, which are qualitatively sufficient to evidence interesting superconducting effects different from the conventional superconductivity in BSC theory. These processes can occur in some particular condensed systems, e.g. in multiband materials, doped compounds, etc. The role of the multiphonon processes or anharmonicity in the superconductivity of different materials (semiconductors, MgB2) was studied periodically in literature [3-5]. To the best of our knowledge, such studies followed in particular one objective - to explain the high-temperature superconductivity alternatively to the BSC theory, without evidencing any "exotic" superconducting phase, like increasing of the pairing gap in some range of temperatures or occurrence of gapless superconductivity for some specific conditions. The curious result obtained in the model of the two-phonon superconductivity [2] has been evidenced by us as an unconventional effect of the enhancement of the pairing gap with the increasing of the temperature from the absolute zero, as appear in Fig.3 of [2] which is very similar to the one shown in Fig.2a of [1]. The main conclusion of our research, is that the effects obtained in [1] intrinsically by some phenomenological assumptions, considering as a priori justification the attractive potential, Eq.(2), which is a principal ingredient for the superconductivity pairing, could be obtained by an alternative approach built on the self-consistent microscopic theory of nonlinear electron-phonon interaction. We acknowledge the Fondecyt projects no. 114099 and no.1141146.

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### Statistical Mechanics Approach for the Detection of Multiple Wireless Sources via a Sensor Network

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Anderson localization [1] is a complex quantum interference phenomenon which occurs in a disordered medium (usually disordered lattices) and turns the extended states of a metal (ballistic or diffusive) into the exponentially localized states of an insulator. The recent activity, which includes the related metal-insulator transition, is mainly due to the various choices of disorder which depend on the symmetry, e.g. the 10 symmetry classes of localization, [2] the 3 basic, orthogonal, unitary with magnetic field  $\vec{B}$  and symplectic with spin-orbit coupling, the 3 chiral, and the 4 Bogolyubov-de Gennes (BdG) for superconductivity [3]. The topological insulators in 2D (e.g. the quantum Hall effect in the presence of  $\vec{B}$  and the quantum spin Hall effect with spin-orbit coupling) are manifested via the appearance of several spectral gaps and topological invariants for the quantized conductivity. There is also current intense activity for topological p-wave superconductors which utilize the BdG Hamiltonians.

I shall review appropriate methods to study Anderson localization, especially in 1D and quasi-1D disordered systems, by finding the level-statistics from numerical diagonalization and obtaining the Lyapunov exponents from transfer matrix techniques. These methods can be combined with finite-size scaling theory in order to obtain the asymptotic behavior. I will mostly discuss Hamiltonians with off-diagonal disorder (random hoppings) for  $N$  coupled chains which belong to one of the 3 chiral classes, where at zero energy an even-odd effect is observed with a diverging density of states  $\rho(E)$  which depends on the parity of  $N$ . I shall also focus on the density of states and the localization length and discuss the observed spectral singularities approaching  $E = 0$ .

Finally I will introduce ladder Hamiltonians in the Majorana fermion basis related to the Su-Schrieffer-Heeger model [4]. I will present preliminary results concerning various models of topological insulators and disordered p-wave superconductors and check immunity of Majorana fermions to Anderson localization due to topology. I shall study the disordered Kitaev toy model [5] and its quasi-1D extensions presenting results for the density of states and the localization length.

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### Immunity of Anderson localization due to Topology

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Anderson localization [1], is a complex quantum interference phenomenon which occurs in a disordered medium (usually disordered lattices) and turns the extended states of a metal (ballistic or diffusive) into the exponentially localized states of an insulator. The activity, which includes the related metal-insulator transition, is mainly due to the various choices of disorder which depend on the symmetry, e.g. the 10 symmetry classes of localization [2] the 3 basic, orthogonal, unitary with magnetic field  $\vec{B}$  and symplectic with spin-orbit coupling, the 3 chiral, and the 4 Bogolyubov-de Gennes (BdG) for superconductivity. The topological insulators in 2D (e.g. the quantum Hall effect in the presence of  $\vec{B}$  and the quantum spin Hall effect with spin-orbit coupling) are manifested via the appearance of several spectral gaps and topological invariants for the quantized conductivity. There is also current intense activity for topological p-wave superconductors which utilize the BdG Hamiltonians [3].

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### Quantum Gibbs ensemble Monte Carlo

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Notwithstanding the quantum-classical isomorphism made

manifest by the path integral formulation of quantum statistical physics and the recognition that the path integral is a tremendously useful mathematical tool to extract exact numerical statistical properties of quantum fluids when combined with the Monte Carlo (MC) method, the development of MC methods for classical fluids is still more complete than for quantum fluids, today. Putting aside the well known sign problem for fermions another important aspect is the development of new path integral MC methods able to investigate a given fluid in different statistical ensembles.

Recently a new approach to continuous space path integral MC simulation was devised by Massimo Boninsegni [3] which makes use of the "worm algorithm" (WA) previously employed to study lattice models. This new approach is formulated in the grand canonical ensemble, unlike the conventional one of Ceperley and Pollock [2] formulated in the canonical ensemble, and does not suffer from the very unfavorable scaling of required computer time with system size, since the winding number does not become "topologically locked" in the large system size due to the allowance of an open world-line, the worm. This makes it particularly useful for the studies of critical phenomena.

MC simulations in the Gibbs ensemble (GEMC) of Athanasios Panagiotopoulos [1] have now been successfully used for several years to study first order phase transitions in classical fluids. For temperatures far below the critical point, satisfactory results for the phase coexistence densities can be obtained. Near the critical point, however, finite size effects become significant, and finite size scaling has proved to be important. According to the GEMC method, the simulation is performed in two boxes each of which contains any two coexisting phases. Equilibration in each phase is guaranteed by moving particles in the respective box. Equality of pressures is satisfied in a statistical sense by expanding the volume of one of the boxes and contracting the volume of the other. Chemical potentials are equalized by transferring particles from one box to the other.

We present a new path integral MC method, based on the WA, to study the gas-liquid coexistence line of a quantum fluid of bosons, which is the full quantum analogue of the classical GEMC method of Panagiotopoulos. As an illustrative application we apply our new algorithm to the  $^4\text{He}$  in two dimensions.

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### Statistical early-warning indicators based on Auto-Regressive Moving-Average processes

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We address the problem of defining early warning indicators of critical transition. To this purpose, we fit the relevant time series through a class of linear models, known as Auto-Regressive Moving-Average (ARMA(p,q)) models. We define two indicators representing the total order and the total persistence of the process, linked, respectively, to the shape and to the characteristic decay time of the autocorrelation function of the process. We successfully test the method

to detect transitions in a Langevin model and a 2D Ising model with nearest-neighbour interaction. We then apply the method to complex systems, namely for dynamo thresholds and financial crisis detection.

Many experimental or natural systems undergo critical transitions - sudden shifts from one to another dynamical regime. In some instances, e.g. global changes in climate science, species extinction in ecology, spin glasses, it is of crucial importance to build early warning indicators, i.e. estimates of the transition threshold based on finite time-series corresponding to situations where the bifurcation did not happen yet. The statistical approach to this issue traditionally involves so-called indicators of criticality. They are based on specific properties of ideal statistical systems (such as the Langevin or Ising model) near the transition: critical slowing down, modifications of the auto-correlation function or of the fluctuations, increase of variance and skewness, diverging susceptibility, diverging correlation length. However, it is known that, in some complex systems, these indicators fail to detect the transition: in spin glasses, no diverging correlation length has been found so far, and one has to resort to finer statistical tools (such as four point dynamical susceptibility) to detect transitions. In addition, traditional early warning indicators may be inapplicable in datasets containing a small number of observations, which is usually the case in several applications where the experiment is unique (as in financial or climate time series), difficult to repeat or to sample for a long time (as for atmospheric measurements, laboratory turbulence, etc). This suggests that indicators based on single statistical properties may not be sufficient for detecting transitions in complex systems, so that one should rather consider all the information contained in the finite-time series.

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### Application of the Confined Quantum Field Theory in the Statistical Physics and reverse entropy

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Confined Quantum Field Theory solves some fundamental problems in physics like the conflict between the theory of the relativity and the quantum field theory. Concerning statistical physics is its simple description of superconductivity and super fluidity, which is a transition from disorder to order movement of the conducting electrons.

Reverse entropy; Entropy in the information theory defined basically in this way. If it comes to us a sequence of numbers how by the knowledge on the numbers we received are we able to predict the next coming number. The entropy is a measure for this prediction. In another word if there exist a pattern in the sequence. Of course the concept of entropy comes originally from physics, more closely from the thermodynamics. The elements in the thermodynamics that originally were used to defined entropy was not of course numbers but was quantities as temperature and different type of energies. At

the beginning these quantities were defined macroscopically. In another word information about the individual atoms or electron was not essential and could be ignored. To get a better understanding and control of a physical process we must always go to the atomic level. Historically atomic and subatomic knowledge came under the name of quantum. Under the twenties centuries quantum as it was formulate gave a diffuse picture of the dynamics behavior of the elements of physics. Many experiments were miss-interpreted, for example single photon experiment by Feynman. These confusions were reflected deeply in the bases of quantum theory which literally admits that an exact knowledge of the dynamics of the elements of the physics is impossible and establish this statment as a physical law and not a weakness of quantum theory. (These questions being discussed in articles with titles Confined Quantum field Theory, Application of Confined Quantum Field Theory in the statistical physics, Application of the Confined Quantum Field Theory in high energy physics, Feynman vs. Confined Quantum Field Theory, Complex Science and in an article concerning learning physics with the title why the light changes in direction entering the water). Reverse entropy in physics means that we give a pattern to some numbers of atoms that are distributed in a random way. To do this we need a stronger theoretical base and standard quantum theory is not able to help us. The theory that can help us in this way is exactly Confined Quantum Field Theory.

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### Intrinsic Fluid Interfaces and Non-Locality

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A detailed understanding of the interface between coexisting fluid phases requires us to model physics at different length scales. First, there is a microscopic range spanning the molecular size to the bulk correlation length, which determines the surface tension  $\gamma$ . This range is often associated with an *intrinsic* interfacial density profile, which describes how the density changes across an instantaneous interfacial configuration. Second, there is a mesoscopic range from the bulk correlation length to the system size. At this scale, the interface resembles a taut drumskin described by a height variable, and one often considers the intrinsic density profile to be a sharp kink from liquid to gas. The capillary wave fluctuations of the interface are characterized by a parallel correlation length and a roughness, which are much larger than the bulk correlation length, and indeed diverge in the absence of pinning fields (gravity, walls or finite size). Almost all the controversies surrounding the physics of fluid interfaces arise directly from the difficulty of joining these two descriptions. This problem is often forced by supposing that

the intrinsic profile rigidly shifts with the undulations of the interfacial fluctuations.

In this work, we show that this concept of a rigidly shifted intrinsic interfacial profile is inconsistent with two very different recent developments in the theory of interfacial phenomena. The first one, called the Intrinsic Sampling Method (ISM) [1,2] allows one to obtain a precise intrinsic profile computed by Molecular Dynamics. The second is the derivation of a non-local interfacial Hamiltonian from a continuum microscopic model *à la* van der Waals, which accurately models the energy of long wavelength interfacial configurations [3,4]. Here we present results of an extensive Molecular Dynamics simulation of the structure and fluctuations of a liquid-gas interface, close to its triple point, in a system with cut-off Lennard-Jones interactions. The equilibrium density profile, averaged and (shape dependent) constrained intrinsic density profiles together with the fluctuations of the interfacial shape are extracted using the ISM. The correlation between fluctuations in the interfacial shape and in the intrinsic density show that the latter is not due to rigid translations of some underlying profile, as is most commonly assumed. Instead, over the whole range of wavelengths from the system size down to the molecular diameter, we see wave-vector dependent behaviour in good agreement with a non-local interfacial Hamiltonian theory specifying the shape dependence of the intrinsic profiles.

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### Dynamical theory of spin noise and relaxation: prospects for real time NMR measurements

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The dynamics of a spin system is usually calculated using the density matrix [1]. However, the usual formulation in terms of the density matrix predicts that the signal will decay to zero, and does not address the stochastic dynamics of individual spins. Spin fluctuations are to be viewed as an intrinsic quantum mechanical property of such systems immersed in random magnetic environments, and are observed as "spin noise" in the absence of any radio frequency (RF) excitation [2]. Using stochastic calculus we develop a dynamical theory of spin noise and relaxation whose origins lie in the component spin fluctuations [3]. This entails consideration of random pure states for individual protons, and how these pure states are correctly combined when the density matrix is formulated. Both the lattice and the spins are treated quantum mechanically. Such treatment incorporates both the processes of spin-spin and (finite temperature) spin-lattice relaxation. Our results reveal the intimate connections between spin noise and conventional spin relaxation, in terms of a modified spin density (MSD), distinct from the density matrix, which is necessary to describe non-ensemble averaged properties of spin systems. With the prospect of ultra-fast digitization, the role of spin noise in real time parameter

extraction for (NMR) spin systems, and the advantage over standard techniques, is of essential importance, especially for systems containing a small number of spins. In this presentation we outline prospects for harnessing the recent dynamical theory in terms of spin noise measurement, with attention to real time properties. Indeed, we argue that such a dynamical theory is the only possibility for tracking NMR parameters in real time - any ensemble average technique necessarily fails. With regard to experimental relevance, for a population of size  $N$ , (transverse) spin noise scales as its square root  $\sqrt{N}$ , as compared to the FID which scales as  $\epsilon N$ , where  $\epsilon$  is the (dimensionless) Boltzmann parameter and is typically small (at lab field strengths). Thus, there exists a critical value of the population below which spin noise becomes the dominant effect. For low field / high temperature this critical value is large, and here a spin noise dominated regime acquires extra significance, experimentally. While our results are shown for simulated data, it should be appreciated that corresponding results have been obtained successfully in the context of scattering [4]. The issues at stake in NMR experiments are the filtering of additive noise and ability to digitize fast enough to apply the stochastic volatility technique.

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## Growing Layer Capacity in the Multilayer Particle Deposition Process

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Suppose we have a strip  $L(n)$  of width  $2n + 1$  and infinite height consisting of sites  $(x, r)$  with  $x \in \{-n, \dots, -2, -1, 0, 1, 2, \dots, n\}$  and  $r \in N^+$ . During the deposition process particles arrive randomly at each site  $x$  according to independent Poisson processes  $N_t(x)$ . Particles are deposited in the lowest possible layer but the horizontal distance between any two particles must be at least 2. When a particle cannot be deposited in a certain layer it will try to deposit in the layer above. Once a lattice site is occupied, the particle stays there during the entire process. The model can be formulated more precisely in the following way:

1. The state-space is  $F := (L, N^+)^{\{0,1\}}$ .
2. The process  $\kappa_t(x, r) = 1$  if there is a particle at  $(x, r)$  at time  $t$  and 0 otherwise.
3. When a particle arrives at site  $x$  at time  $t$ , it will be deposited at the height of site  $x$  which is defined as  $h_t(x) := \min\{r : \kappa_t(y, r) = 0, \forall y \in N_x\}$ , where neighborhood set  $N_x$  consists of site  $x$  and the sites with distance 1 from it. At the start of the process the heights of all sites are 0, or  $h_0(x) = 0, \forall x$ . But after some time there will be a difference between the height at the border sites ( $-n$  and  $n$ ) and all the other sites because the neighborhood  $N_x$  for border sites is smaller than at sites in the middle. For example,  $N_0 = \{-1, 0, 1\}$  whereas  $N_n = \{n - 1, n\}$ . The growth rate of the height at the border sites is thus  $2/3$  of that of other sites.

This results in a lower expected value for  $h_t(n)$  than  $h_t(0)$ .

When border site heights tend to lag behind with respect to the heights of other sites it will in turn affect the process at the next-border sites ( $-(n - 1)$  and  $(n - 1)$ ) because in effect particles who are dropped on these sites now only see a neighborhood of two sites instead of three. Indeed, after a while the particles of the border sites are deposited too low to be able to interfere with them.

So, after a while the next-border sites will start lagging behind too and will in turn affect their neighbors in the same way. At a certain point in time only the last 3 center sites  $\{-1, 0, 1\}$  are not yet affected by the above described boundary effect. The process continues at the center essentially as if it were a three-site system which has already been analyzed in a previous paper by Fleurke and Van Enter.

In this talk it is shown that for each system with  $n \geq 1$  the above described boundary effect results in a higher average deposition rate in higher layers than in lower layers.

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## Large deviations, fluctuation theorems and model systems

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There is a current interest in fluctuating small systems in contact with heat reservoirs and driven by external forces. This focus is driven by the recent possibilities of direct manipulation of nano systems and bio molecules. These techniques also permit direct experimental access to the probability distributions for the work or heat exchanged with the environment. Moreover, these single molecule techniques have also yielded access to the so called fluctuation theorems, which relate the probability of observing entropy-generated trajectories, with that of observing entropy-consuming trajectories. Altogether, there is a general renewed theoretical interest in small non equilibrium systems.

Reviewing and supplementing the work of Saito and Dhar, we derive the large deviation function for the harmonic chain driven at the end points by heat reservoirs at different temperatures. We formulate the problem in terms of Langevin equations and carry out a standard path integral calculation in combination with a transfer matrix method. The large deviation function is given in terms of the transmission Green's function or end-to-end propagator for the chain and is consistent with the Gallavotti-Cohen fluctuation theorem. We also give a proof of the fluctuation theorem on the basis of the Fokker-Planck equation for the heat. We make contact with a single particle model suggested by Derrida and Brunet and also discuss the case of two particles some detail. We support our findings with numerical simulations.

We extend the work of Kannan et al. and derive the cumulant generating function for the alternating mass harmonic chain consisting of  $N$  particles and driven by heat reservoirs. The transmission Greens function exhibits a two band structure arising from the acoustical and optical branches of the phonon dispersion law. We discuss the cumulant generating function and present a closed expression independent of  $N$  in the large  $N$  limit, in accordance with the absence of local

thermodynamic equilibrium.

We consider a harmonically bound Brownian particle coupled to two distinct heat reservoirs at different temperatures. We show that the presence of a harmonic trap does not change the large deviation function from the case of a free Brownian particle discussed by Derrida and Brunet and Visco. Likewise, the Gallavotti-Cohen fluctuation theorem related to the entropy production at the heat sources remains in force.

We derive the integral fluctuation theorem proposed by Seifert within a Fokker-Planck framework and show that it is a consequence of the Gallavotti-Cohen symmetry.

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## Two dimensional gas of hard needles: a simulation study

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Molecular dynamics (MD) simulation of anisotropic hard objects such as ellipsoids and spherocylinders has been the subject of exploration in past decades. It has been shown the alignment of non-spherical molecules can lead to a diversity of phases, mainly orientational in nature, in liquid crystals. Despite the profound insight obtained via tremendous Monte Carlo (MC) simulations of the static phases, many dynamical, transportational and structural properties namely kinetic arrest and glassy behaviours have only been poorly understood. In spite of employment of other simulation techniques like Brownian dynamics and theoretical approaches such as kinetic theory, density functional and hydrodynamics equations approach, event-oriented MD remains as an efficient tool for probing the dynamical aspects of hard gases of non-spherical objects. Among elongated and anisotropic hard bodies, infinitely thin needles have received quite notable attention. The first MD simulation attempt were carried out for a three dimensional gas of hard needles by Frenkel and Maguire. In this talk the dynamics of a two dimensional gas of hard needles is simulated by event-oriented molecular dynamics method. Excluded volume makes the problem nontrivial. We have computed various quantities such as translational and rotational diffusion constants and intermediate self scattering function. Their dependence on density have been extensively explored. Despite there is no positional ordering in the system, nontrivial behaviour in the rotational degree of freedom is observed. Moreover, we have seen slowing down in the angular part of the motion. It is shown that above a certain number density of the needles, the rotational mean squared displacement exhibits a three stage regime including a plateau. Regarding the temporal behaviour of the system, our results show that many of the temporal autocorrelation functions both translational and angular exhibit a sort of slow dynamics and multi step relaxation. The most interesting feature of our simulation is the existence

of three regimes in the angular mean squared displacement. This can be attributed to slow dynamics. Our findings show relaxing the translational degrees of freedom does not smear out angular slow dynamics. Density dependence of translational and rotational diffusion coefficients has been obtained and compared to three dimensional results. In 2D dependence of the translational diffusion coefficient on density qualitatively resembles to 3D. Rotational diffusion constant exhibits an algebraic decay but with a larger exponent than in 3D. Comparison to the existing results in the literature is done.

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## Description of conflicting traffic flows by totally asymmetric simple exclusion processes

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Modelling a vast variety of non equilibrium phenomena has constituted the subject of intensive research by statistical physicists. In particular, vehicular dynamics has been one of these fascinating issues. While the existing results in highway traffic needs further manipulations in order to find direct applications, researches on city traffic seem to have more feasibility in practical applications. Recently, notable attention have paid to controlling traffic flow at intersections and other designations such as roundabouts. In this respect, we intend to study another aspect of traffic flow at intersections. In principle, the vehicular flow at an intersection can be controlled via two schemes. In the first scheme the traffic is controlled without traffic lights. In the second scheme, signalized traffic lights control the flow. In the former scheme, approaching car to the intersection yields to the traffic in its perpendicular direction by adjusting its velocity to a safe value to avoid collision. The basic question is that under what circumstances the intersection should be controlled by traffic lights? In order to capture the basic features of this problem, we construct a simple stochastic model. The vehicular dynamics is represented by asymmetric simple exclusion process (ASEP). The intersection point is the place where two chains representing the streets interact with each other. It is a well-established fact that a single static impurity can strongly affect the characteristics of ASEP both in closed and open boundary condition. In addition, the characteristics of ASEP in the presence of moving impurities has been studied and shown to exhibit disorder-induced phase transitions. Besides relevance to traffic flow, the investigation of ASEP in the presence of small amount disorder has recently revealed the existence of novel aspects of the interplay of disorder and drive. In our model, the effect of the perpendicular chain can be interpreted as a single dynamic site-wise disorder which to our knowledge has not been investigated. Two perpendicular chains interact with each other via the intersection point. Using Monte Carlo simulations and numerics, we have obtained the dependence of each chain current on its own and on its perpendicular chain density. It is verified that the chains can maintain large currents up to rather a high density. Interaction of two chains can effectively be considered as a dynamic impurity. For some values of global densities in the chains, the interaction of chains leads to formation of high density region behind

the intersection point which is segregated from a low density region afterwards. By scanning of the phase space, we have obtained the model's phase diagram. Two phases of jamming (density segregated) and regulated (uniform density) flows are identified.

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### Degree-corrected blockmodel: Benchmark graphs for testing community detection algorithms

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Of particular interest in recent years has been the community structure in networks [1,2]. As a community (module or block), one understands a group of nodes that is densely connected internally but sparsely connected externally. To name a few, communities may be groups of related individuals in social networks, sets of web-pages on the same topic, biochemical pathways in metabolic networks, and groups of countries in the world trade network that signed regional trade agreements. The above examples show that the group membership is related to the function of a node in the network. For this reason, aside from the small-world effect and scale-free degree distributions, the community structure is considered one of the most important topological properties of complex networks, yet this structure is not fully understood and not well captured by network models.

We develop an exponential random graph approach [3,4] to networks with community structure [5]. To this end we mainly built upon the idea of blockmodels. We consider both, the classical blockmodel and its degree corrected counterpart, and study many of their properties analytically. We show that in the degree-corrected blockmodel, node degrees display an interesting scaling property, which is reminiscent of what is observed in real-world fractal networks [6,7]. In Ref. [7], the authors argued that the self-similarity of real-world complex networks with power-law degree distributions results from the scaling property of the node degrees, which arises when the networks undergo a renormalization procedure that coarse-grains their nodes into boxes (blocks). It is remarkable, that the scaling property reported by Song et al. is similar to the scale-transformation observed in the degree-corrected blockmodel. The scaling feature comes as a surprise, especially that in this study, contrary to what is suggested in the literature, the scaling property is not attributed to any specific network construction procedure. It is an intrinsic feature of the degree-corrected blockmodel. A short description of Monte Carlo simulations of the models is also given in the hope of being useful to others working in the field.

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### International trade network: fractal properties and globalization puzzle

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The gravity model of trade is one of the most recognizable empirical models in economics. Drawing from Newton's law of gravity, the gravity model relates trade volume,  $T_{ij}$ , between two countries,  $i$  and  $j$ , positively to the product of their GDP's, i.e.  $Q_i Q_j$ , and negatively to the geographic distance,  $r_{ij}$ , between them. The simplest form of the gravity equation for the bilateral trade volume is  $T_{ij} = G \frac{Q_i Q_j}{r_{ij}^\alpha}$ , where  $\alpha$  is the distance coefficient, which is calculated from the real data analysis and  $G$  is a constant. The model successfully explains trading patterns, but the growing in time distance coefficient,  $\alpha$ , seems to indicate that the role of distance increases over time. It stands in sharp contrast to the common perception of the globalization process, i.e. that due to declining communication and transport costs, distance becomes less and less important. This, in essence, captures the notion of the so-called *distance puzzle*. Many explanations for this puzzle have been proposed in the literature. However, since the price of the solutions is the significantly increased complexity of the model, which contrasts with the simplicity of the original model, none of these explanations has been wholly satisfactory. We argue that the  $\alpha$  coefficient is strictly related to the fractal dimension of the international trade network (ITN) and the changes of both quantities are strongly correlated. It is commonly accepted that the uneven spatial distribution of socio-economic activity can have a fractal dimension, that is, it can repeat itself at different levels of spatial aggregation. In most cases, the conjecture about the fractal character of studied objects (e.g. transport systems, or wealth and population distribution) is usually based on scaling laws observed therein. However, it must be stressed that the existence of a dependence following a power law does not necessarily imply the existence of a fractal structure. Taking above into account, a number of observed spatial fractal socio-economic systems drops drastically. Here, we propose the two distinct methods, both exploiting spatial properties of ITN, which allow to estimate its fractal dimension. The first one is a box counting method - a classical tool for the analysis of the dimension of fractal objects. The second one exploits the simple decision-based model which is somehow related to the recently introduced radiation model for mobility and migration patterns. Having found evidence of ITN fractality, we show that it can shed light on the origin of the globalization puzzle.

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## Stability condition for diffusion of artificial language with linguistic neutrality

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Since the dynamics of language death was shown as a simple model by Abrams and Strogatz [1], language dynamics has attracted the interest of many statistical physicists in the context of complex adaptive systems. While several researchers found that different languages can stably coexist in bilingual systems [2] and multi-language systems [3], the question of the mechanisms of the coexistence is still open.

The ways to communicate between native speakers of different languages are to learn and use the language of the other or for both of them to learn and use a third language. The former causes an inequality between native speakers and second language speakers. Thus, to pursue the latter, some tried to introduce artificial languages. For example, Esperanto was created as an international auxiliary language with the aim of equal second language for all people in the world. The Esperanto movement holds onto the ideal of linguistic neutrality, i.e. Esperanto should not be anyone's native language. However, native speakers appeared as Esperanto spread among people. It is feared that this emergence of native speakers will destroy the linguistic neutrality. The question we have to consider here is whether Esperanto can spread while maintaining linguistic neutrality, i.e. whether the society with no native Esperantists but many second language speakers of Esperanto is able to exist stably.

The aim of this study is to validate the possibility of the diffusion of an artificial language with linguistic neutrality based on the framework of the evolutionary game theory. I modeled the three-language (A, B, and E) system including bilinguals by the replicator-mutator equation [4]. Each speaker is assumed to have his/her own native language and second language, and to prefer to use the native language. I also assume that languages A, B, and E are linguistically very distant each other, so that successful communication is only possible when the interaction takes place in one language. When the communication successes between native speakers (or between second language speakers), both get payoffs equal to 1. On the other hand, when it successes between a native speaker and a second language speaker, they get payoffs  $1 + v$  for the native speaker and  $1 - v$  for the second language speaker;  $v$  denotes the inequality parameter, i.e. the difference of gains between native and second language speakers. Bilingual speakers always pay the learning cost  $c_X (X \in \{A, B, E\})$  for their second language.

From the results of the linear stability analysis and numerical simulations, I obtained the phase diagram of the inequality parameter  $v$  and the learning cost for E,  $c_E$ . According to this phase diagram, the condition for the spread of an artificial language E with maintenance of linguistic neutrality was revealed. In addition, I showed that bilinguals and monolinguals could coexist in two-language system if one imposes a large cost on monolinguals.

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## Statistical physics approach to quantifying differences in myelinated nerve fibers: application to aging detection

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The nervous system is a complex network allowing the transmission of signals between interconnected neurons across distances which vary from fractions of millimeters to meters. Axons following similar paths are often bundled together, forming nerves in the peripheral nervous system and tracts in the central nervous system. The proper functioning of such tracts depends on axon characteristics such as size, density and spatial organization. The axons populating different tracts or bundles change during development and aging as well as a consequence of pathology and environmental influences. It is therefore important to have a means for to discern which features best characterize the observed differences. Typically, studies of differences observed in nerve fibers are limited to one or just a few geometrical properties, chosen to measure an evident and already observed difference. This methodology presents several problems, as some of the differences may be subtle and not easily identified by visual inspection and hence not chosen for quantification, restricting the identification of potential differences. We present here [1] a new method to uncover which features of axons are most affected by an underlying biological process. We consider a large set of candidate features representative of diverse types of possible differences in axons (e.g. density, shape of axons, spatial order, etc.) and then use the feature selection technique to identify which combinations of such features yield the best discrimination between axons of two distinct groups. This approach enables the identification from the list of candidate features, of the set of features that, when taken together, best discriminates between groups in a general dataset. We apply this methodology to characterize changes observed with normal aging in the myelinated nerve fibers of the fornix [2] of young and old rhesus monkeys. Previous studies [3] have shown that the axon density declines with age. Our methodology shows that density related features, in particular the fraction of occupied axon area and the effective local density, which measures how closely axons are packed, are the best discriminating features between the young and old age groups. These features are shown to be better than simple density alone, which provides an accuracy of only 75% when assigning samples to their age groups. Furthermore, we show that using the combination of fraction of occupied axon area and effective local density is enough to characterize the age group separation in the samples observed, with an accuracy of 90%, and that the addition of other features adds little to the accuracy.

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## Reconstruction real economic and financial networks from partial information: WTW and e-Mid cases

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In this talk we present a novel method to reconstruct global topological properties of a complex network starting from limited information on the graph connectivity features. The method is applied to both binary and weighted networks without assuming any *a priori* global structure of the topology of the unknown node-node connections. In the binary case we assume to know for all the nodes a non-topological quantity that we interpret as a sort of *fitness* to create connections to other nodes. Moreover, we assume to know the degree, i.e. the number of connections, only for a subset of the nodes of the network. In addition to that, in the case of weighted network case we assume to know also the strength of each node but to ignore the single weights of links. We then use appropriate fitness models of networks [1], calibrated on the subset of nodes for which degrees are known, in order to generate, through a maximal likelihood argument, ensembles of networks which describe as better as possible the topological properties of the real network studied. Here, we focus on the reconstruction of the topological properties that are strictly relevant for processes of contagion and distress propagation in networks [2] and risk management, i.e. network density, nearest neighbor binary and weighted connectivity and *k*-core structure, and we study how well these properties can be estimated as a function of the fraction of the nodes of the network for which we assume to know the degree and which are used for the calibration of the ensembles of artificial networks we use to mimic the real one [3]. Finally, we also study how well the resilience to distress propagation in the network can be estimated using our method. We perform a first test on ensembles of synthetic networks generated with the Exponential Random Graph model and weighted generalizations, which allows to apply common tools from statistical mechanics. This permits to evaluate the level of approximation in the reconstruction due only to the partial knowledge of the node degrees. We then perform a second test on empirical networks taken from economic and financial contexts: the International Trade Network (ITN) or binary World Trade Web and the e-Mid electronic market for Interbank Deposits. This further step permits to evaluate also the part of the error in the reconstruction due to the use of fitness models to mimic this kind of real networks. In both cases, we find that a subset as small as 10% of nodes can be enough to estimate the properties of the network along with its resilience with an error of 5–10%.

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## Critical behavior of the system with a small-world property

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We study the critical behavior of the systems with small world properties. For the construction of the free energy functional for the system with small world properties, we define the symmetry of regular subgraph and construct integer basis for a given irreducible representation. As the symmetry of a regular graph, we take one of the discrete subgroups of the Galilean group.

Then the structure of systems disorder we classify at the level of the corresponding generalized entropy. Classification of generalized entropies allows using various distribution functions for describing distribution of weak disorder in the system [1]. This allows us to classify the different systems with small world property.

With a nonlocal interaction taken into account a free energy functional of the system can be presented in the form

$F[\eta] = F_0[\eta] + F_I[\eta]$ , where

$$F_0[\eta] = \int_R dr dt \int_R dr' dt' \left\{ \frac{1}{2} \frac{\partial \eta(r, t)}{\partial t} g_0(r, t, r', t') \right. \\ \left. \times \frac{\partial \eta(r', t')}{\partial t'} + \frac{1}{2} \frac{\partial \eta(r, t)}{\partial r} g_1(r, t, r', t') \frac{\partial \eta(r', t')}{\partial r'} \right\}$$

and

$$F_I[\eta] = - \int_R dr dt \int_R dr' dt' V(\eta(r, t), u(r', t')). \quad (4)$$

The equation of motion for the order parameter is derived from the stationary principle Gateaux derivative  $\delta F[\eta, h] = 0$  for any *h*.

We show that if a structure has a small-world effect the equation of motion for the major parameter is an integro-differential equation [2]. When the structures are strongly inhomogeneous the space and time dependencies of the order parameters are described by a fractional differential equation. Linearization of the equation of motion for the order parameter made it possible to define a non-linear dispersion law. Within the framework of the renormalization group method this property taken into account makes it possible to obtain a critical indexes dependence on the parameter of the system complexity. We show that a small world property of systems taken into account leads to a new class of universality. Based on fMRI data, we construct the brain functional network [3]. We determined the degree distribution and other characteristic functions of network. By using the maximum likelihood method, we have shown that degree distribution of the network is described *q*-exponential distribution. We try to describe some properties of these networks using our results.

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## Fundamental energy limits in the physics of small-scale computing systems

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Is it possible to operate a computing device with zero energy expenditure? This question has been addressed during the second half of last century by a number of scientists and has led to a version of the second principle of thermodynamics that, assuming the Shannon information as a special form of the Gibbs-Boltzmann entropy, establishes that necessary condition to operate a computing device with zero energy dissipated is that the computing process does not decrease information[1,2]. This result, often invoked as "Landauer principle", has been recently put under experimental test[3] with the aim of exploring the limits in low power computation. It is a well-known fact that in the last forty years the semiconductor industry has been driven by its ability to scale down the size of the CMOSFET devices, and to increase computing capability density up to a point where the power dissipated in heat during computation has become a serious limitation[4]. Thus the fundamental limits in computing are rapidly becoming a strategic issue for the development of future ICT.

In modern computers information is processed through binary switches, usually realized with transistors, i.e. microelectronic devices. Thus binary switches represent a paradigmatic example of "small scale physical systems" employed in the process of information.

In order to take into account a realistic representation of the switch dynamics we assume a single-dof dynamical model for the switch that is coupled to a thermal bath at temperature  $T$ . Although the switch is isolated, exchanges of heat  $Q$  between the switch and the thermal bath are possible. Moreover, due to the coupling with the thermal bath a fluctuating force ( $t$ ) appears. At thermal equilibrium the Fluctuation-Dissipation theorem links ( $t$ ) and the dissipative force. According to this description the switch dynamics can be described in terms of a Langevin equation, where the fluctuating force appears together with a dissipative term.

In this talk we discuss the role of switching procedure with reference to the fundamental limits in minimum energy dissipation. We show that the minimum energy depends on the switching procedure and test this result with micromagnetic simulations of a nanoscale switch realized with single cylindrical element of permalloy (NiFe). Finally we establish a relation between minimum energy and switching error probability[5].

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## Reciprocity and optimal scales of weighted networks

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In directed networks, reciprocal links have dramatic effects on dynamical processes, network growth, and higher-order structures such as motifs and communities. While the reciprocity of binary networks has been extensively studied, that of weighted networks is still poorly understood, implying an ever-increasing gap between the availability of weighted network data and our understanding of their dyadic properties. Here we introduce a general approach to the reciprocity of weighted networks, and define quantities and null models that consistently capture empirical reciprocity patterns at different structural levels. We show that, counter-intuitively, previous reciprocity measures based on the similarity of mutual weights are uninformative. By contrast, our measures allow to consistently classify different weighted networks according to their reciprocity, track the evolution of a network's reciprocity over time, identify patterns at the level of dyads and vertices, and distinguish the effects of flux (im)balances or other (a)symmetries from a true tendency towards (anti-)reciprocation. We also address the more general problem that the analysis of weighted networks suffers from two long-standing problems of arbitrariness. On one hand, the definitions of topological properties introduced for binary graphs can be generalized in non-unique ways to weighted networks. On the other hand, even when a definition is given, there is no natural choice of the (optimal) scale of link intensities (e.g. the money unit in economic networks). Here we show that these two seemingly independent problems can be regarded as intimately related, and propose a common solution to both. Using a formalism that we recently proposed in order to map a weighted network to an ensemble of binary graphs, we introduce an information-theoretic approach leading to the least biased generalization of binary properties to weighted networks, and at the same time fixing the optimal scale of link intensities. We illustrate our method on various social and economic networks. We show that our approach provides a straightforward generalization of any network measure defined on unweighted networks. All these measures are well established for unweighted networks but have hitherto proven difficult to define for weighted networks.

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### The generalized Bose-Fermi distribution: general formalism and applications to the enhanced reconstruction of weighted networks

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We derive a class of generalized distributions, unifying the Bose and Fermi ones, that describe any system where the first-occupation energies or probabilities are different from subsequent ones, as in the presence of thresholds, saturation, or aging. The distributions adequately characterize a number of physical systems and have important applications to the analysis of weighted networks. First, they completely describe the structural correlations of weighted networks, which turn out to be stronger than expected and to determine significant topological biases. This shows that the null behavior of weighted networks is different from what was previously believed, and that a systematic redefinition of weighted properties is necessary. Second, they provide an enhanced method to reconstruct real-world weighted networks from purely local information. Whenever the whole structure of a network is unknown, one must resort to reconstruction methods that identify the least biased ensemble of networks consistent with the partial information available. A challenging case, frequently encountered due to privacy issues in the analysis of interbank flows and Big Data, is when there is only local (node-specific) aggregate information available. For binary networks, the relevant ensemble is one where the degree (number of links) of each node is constrained to its observed value. However, for weighted networks the problem is much more complicated. While the naive approach prescribes to constrain the strengths (total link weights) of all nodes, recent counter-intuitive results suggest that in weighted networks the degrees are often more informative than the strengths. This implies that the reconstruction of weighted networks would be significantly enhanced by the specification of both strengths and degrees, a computationally hard and bias-prone procedure. Here we solve this problem by introducing an analytical and unbiased maximum-entropy method that works in the shortest possible time and does not require the explicit generation of reconstructed samples. We consider several real-world examples and show that, while the strengths alone give poor results, the additional knowledge of the degrees yields accurately reconstructed networks. Information-theoretic criteria rigorously confirm that the degree sequence, as soon as it is non-trivial, is irreducible to the strength sequence. Our results have strong implications for the analysis of motifs and communities and whenever the reconstructed ensemble is required as a null model to detect higher-order patterns.

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### A complex network approach for rating of socio-economic indicators. The UN EGDI case study.

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In recent years two struggling positions have risen regarding the use of composite indicators as reliable tools for policy makers. Simplicity is probably the mainstay for their use, on the contrary several studies recently have demonstrated possible misleading conclusions deriving from the underlying uncertainty. In this paper we present an alternative point of view, eventually mediating between the previously mentioned opposite schools of thinking. The presented analysis was performed on the UN data used for the e-Government survey 2012, thus the focus is on the e-Government Development Index (EGDI). In the current work we propose a comparison between standard approaches for data mining with a complex network framework. The underlying motivation of this study is the possibility of capturing data characteristics and relevant features on the basis of topological considerations which standard approaches cannot unveil. According to this we used the EGDI data to explore the similarities among the different member states. Correlations between nations were used to build a weighted network whose properties and communities were then investigated. The network communities were adopted as a reference for the development of a new weighted indicator. In fact, we propose to assign a continuous score as a performance index, intended as a membership ranking to be assigned to all member states beside the EGDI index studied in the present case. The performance index of each nation belonging to a determined community was calculated firstly detecting the remaining most similar community members and then averaging the related EGDIs. The average was weighted over the relative correlations in order to give consider as major contributions those yielded by most correlated nations. The proposed methodology offers two different possibilities to modify the ranking concept. The first proposes the use of the identified communities as rating classes; within them the nations are marked with a plus or minus if the relative EGDI is higher or lower than the proposed weighted indicator. The second suggests instead the adoption of a plus and the minus signs along with the original rating. The main result of the proposed methodology is the definition of a new concept of ranking. Moreover, the complex network framework naturally involves the possibility to incorporate temporal evolution through network dynamics and as a consequence it could be favorably used with data sets whose size is ineluctably increasing over time.

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## Instabilities in granular fluids at moderate densities

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Although hydrodynamics is frequently used to describe rapid granular flows, there are still some open questions about the domain of validity of this description. One of them lies in the transition from kinetic theory to hydrodynamics beyond the quasielastic limit. The reason of this concern resides in the fact that the inverse of the cooling rate (which measures the rate of energy loss due to collisional dissipation) introduces a new time scale not present for elastic collisions (ordinary fluids). The variation of the (granular) temperature over this new time scale is faster than over the usual hydrodynamic time scale. However, as the inelasticity increases, it is possible that the system could lack a separation of time scales between the hydrodynamic and the pure kinetic excitations such that there is no *aging* to hydrodynamics or, in the language of kinetic theory, there is no normal solution at finite dissipation. Needless to say, to definitively address the validity of hydrodynamics for dissipative systems, the complete spectrum of the (linearized) Enskog-Boltzmann collision operator must be known. More specifically, knowing this spectrum allows one to see if the hydrodynamic modes (density, velocity, and temperature) decay more slowly than the remaining kinetic excitations at large times. On the other hand, the complex mathematical structure of the above collision operator makes an exact solution to the (inelastic) Enskog kinetic equation intractable, even for studying the relaxation of small spatial perturbations of the homogeneous cooling state. An alternative route is to compare the (approximate) theoretical solutions to the Enskog equations (which are obtained by assuming the validity of a normal solution) with molecular dynamics (MD) simulations.

In this talk we give some insight into the above problem by determining the critical length scale  $L_c$  for the onset of instabilities in freely cooling flows of granular fluids at moderate densities. To get  $L_c$ , we will perform a linear stability analysis of the Navier-Stokes (NS) granular hydrodynamic equations [1] where the explicit form of the transport coefficients is obtained from a solution of the Enskog equation [2]. The theoretical predictions for  $L_c$  are tested against MD simulations in flows of strong dissipation and moderate volume fractions. We find excellent agreement between MD and kinetic theory for the onset of velocity vortices, indicating the applicability of NS hydrodynamics to monodisperse [3,4] and polydisperse [5] granular flows even for strong inelasticity and finite density.

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## Approaches to Derivation of the Boltzmann Kinetic Equation

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In the talk the possible approaches to the rigorous derivation of the Boltzmann kinetic equation with hard sphere collisions from underlying dynamics are considered. In particular, a formalism for the description of the evolution of infinitely many hard spheres within the framework of marginal observables governed by the dual BBGKY hierarchy in the Boltzmann-Grad scaling limit is developed. The limit dynamics is described by the set of recurrence evolution equations, namely by the dual Boltzmann hierarchy. The links of the dual Boltzmann hierarchy for the limit marginal observables and the Boltzmann kinetic equation with hard spheres collisions are established. We prove that the mean value functional of the constructed limit additive-type marginal observables is equivalent to the mean value functional determined by the one-particle marginal distribution function governed by the Boltzmann equation. In the general case of the limit  $k$ -ary-type marginal observables the corresponding mean value functionals are equivalent to the mean value functional determined by the  $k$  times product of a one-particle marginal distribution function, i.e. the property of the propagation of initial chaos takes place.

Also we give consideration to one more approach of the description of the kinetic evolution of hard spheres in terms of a one-particle distribution function governed by the non-Markovian generalization of the Enskog kinetic equation and the Boltzmann-Grad asymptotic behavior of its non-perturbative solution is established. We note that within the framework of the perturbation theory a particular case of this approach reduces to the Bogolyubov's method of the derivation of the Boltzmann equation.

Moreover, we consider the problem of the rigorous description of the kinetic evolution in the presence of initial correlations of a system of particles with hard spheres collisions. The developed approaches give an opportunity to derive the kinetic equations with initial correlations that may characterize the condensed states of large particle systems.

The obtained results are applied to the description of the kinetic evolution of interacting stochastic Markovian processes, modeling the microscopic evolution of soft active matter. We note that the developed formalism to the description of the collective behavior within the framework of the marginal observables is in fact the best mathematically fully consistent formulation for systems of mathematical biology.

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## Spitzer identities and Wiener-Hopf factorisations through fast Hilbert transform - Application to the pricing of exotic derivatives

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Spitzer identities are closed expressions for the  $z$ -transform (or moment generating function) of the characteristic function of a stochastic process  $X(t)$  with independent and identically distributed increments. For instance, if  $p_M(x, n)$  is the probability density function of the maximum  $M = \max_n X(n\Delta t)$  of  $X(t)$  monitored on a discrete grid of points in time, then

$$\begin{aligned}\tilde{p}_{X,M}(\xi, q) &= \sum_{n=0}^{\infty} q^n \hat{p}_{X,M}(\xi, n) = \frac{1}{\Phi(\xi, q)} - e^{iu\xi} \frac{Q_+(\xi, q)}{\Phi_-(\xi, q)} \\ &= e^{iu\xi} \frac{Q_-(\xi, q)}{\Phi_-(\xi, q)}\end{aligned}$$

where  $\Phi(\xi, q) = 1 - q\Psi(\xi, \Delta t)$  and  $\Psi(\xi, \Delta t)$  is the characteristic function of  $X(t)$ .

The application of Spitzer identities has been difficult because it requires the factorisation of a complex function into its + and - parts. The same operation is required for the solution of linear integral equations of convolution type with constant limits of integration,

$$\lambda f(x) - \int_a^b k(x-x')f(x')dx' = g(x), \quad x \in (a, b);$$

this is known as a Wiener-Hopf equation if  $a = -\infty$  or  $b = +\infty$ , and as a Fredholm equation if both integration limits are finite.

We express the factorisation through the Plemelj-Sokhotski formulas

$$\begin{aligned}\hat{f}_+(\xi) &= \frac{1}{2}[\hat{f}(\xi) + i\mathcal{H}\hat{f}(\xi)] \\ \hat{f}_-(\xi) &= \frac{1}{2}[\hat{f}(\xi) - i\mathcal{H}\hat{f}(\xi)]\end{aligned}$$

in terms of the Hilbert transform

$$\mathcal{H}_x f(x) = \text{p.v.} \frac{1}{\pi x} * f(x) = \text{p.v.} \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{f(x')}{x-x'} dx'$$

and compute the latter by two fast Fourier transforms combined with a sinc function expansion, which provides exponential convergence of the error with the number of grid points. We bind from above the computational cost of the inverse  $z$ -transform using the Euler summation, a convergence-acceleration technique for alternating series.

The resulting methods can be used to solve many disparate problems in engineering mathematics, mathematical physics, etc., from diffusion in the presence of barriers to the scattering of electromagnetic waves. We present an application in finance, the pricing of exotic (i.e. path-dependent) derivatives, such as barrier, lookback, Bermudan, occupation time, quantile or step options, when the underlying is modelled as an exponential Lévy process, showing the two equivalent solutions based on a Spitzer identity or on a Wiener-Hopf-type equation.

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## Fokker-Planck equation and thermodynamic system analysis

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Stochastic differential equations are important tools in modelling complex systems in physics, chemistry, biology and even finance and economics. The non-linear Fokker-Planck equation or forward Kolmogorov equation is currently successfully applied in order to describe in a phenomenological way anomalous correlated diffusion. The resulting distribution-dependent stochastic equation is then used for a deep analysis of irreversibility and it gives an excellent approximation near the free energy minimum, just as Boltzmann's definition of entropy follows from finding the maximum entropy state. Under certain conditions on the drift and diffusion coefficients, the stationary solution of a Fokker-Planck equation of the type that we consider here satisfies a variational principle: it minimizes a certain convex free energy functional over an appropriate admissible class of probability densities. This free energy functional decreases in time for any solution of the Fokker-Planck equation. It is then possible to describe in suitable mathematical terms the evolution and the probability density for stochastic processes associated with their stochastic differential equations, stressing their time-dependence in which randomness plays a crucial role. The solution of the stochastic differential equation leads to information on the probability transition function of the stochastic processes but it also allows to find out peculiar stationary distribution conditions and time evolution function of the irreversible complex system under investigation. A link between Fokker-Planck dynamics and the free energy functional is presented and discussed: the approach has been particularly successful to describe metastability and hysteresis phenomena. The fundamental role of dissipation analysis in metastable systems is investigated and discussed. A link between Fokker-Planck equation applied to dissipation and entropy generation is developed and a statistical approach for irreversible phenomena application coming from the preliminary results of this research is suggested. The major novelty of our approach is that this formulation enables us to reveal an appealing, and previously unexplored, relationship between the Fokker-Planck equation and the associated free energy functional. Namely, we point out that the dynamics may be regarded as a gradient flux, or a steepest descent, for the free energy.

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## Thermodynamic instabilities in high energy heavy-ion collisions

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One of the very interesting aspects of high energy heavy-ion collisions experiments is a detailed study of the thermody-

namical properties of strongly interacting nuclear matter away from the nuclear ground state. In this direction, many efforts were focused on searching for possible phase transitions in such collisions. In this contribution, we investigate the possible thermodynamic instability in a warm and dense nuclear medium where a phase transition from nucleonic matter to resonance-dominated  $\Delta$ -matter can take place. Such a phase transition is characterized by both mechanical instability (fluctuations on the baryon density) and by chemical-diffusive instability (fluctuations on the isospin concentration) in asymmetric nuclear matter. Similarly to the liquid-gas phase transition, the nucleonic and the  $\Delta$ -matter phase have a different isospin density in the mixed phase. In the liquid-gas phase transition, the process of producing a larger neutron excess in the gas phase is referred to as isospin fractionation. A similar effects can occur in the nucleon- $\Delta$  matter phase transition due essentially to a  $\Delta^-$  excess in the  $\Delta$ -matter phase in asymmetric nuclear matter. In this context, we study the hadronic equation of state by means of an effective relativistic mean field model with the inclusion of the full octet of baryons, the  $\Delta$ -isobar degrees of freedom, and the lightest pseudoscalar and vector mesons. These last particles are considered in the so-called one-body contribution, taking into account their effective chemical potentials depending on the self-consistent interaction between baryons. The main goal is to investigate how the constraints on the global conservation of the baryon number, electric charge fraction, and strangeness neutrality, in the presence of  $\Delta$ -isobar degrees of freedom, hyperons, and strange mesons, influence the behavior of the EOS in a regime of finite values of baryon density and temperature. Moreover, we show the relevance of  $\Delta$ -isobars for different coupling constants and how their presence influences several particle ratios and strangeness production for different parameters sets, compatible with experimental constraints. Finally, we will investigate the presence of thermodynamic instabilities in a hot and dense nuclear medium where phases with different values of antibaryon-baryon ratios and strangeness content may coexist. Such a physical regime could be in principle investigated in the future high-energy compressed nuclear matter experiments where will make it possible to create compressed baryonic matter with a high net baryon density.

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## Time Operator, Age and Mixing of Markov chains

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Extending the Time Operator, originally introduced for highly unstable Dynamical Systems, to regular Markov Chains allows to relate the Mixing Time with Internal Age. We construct an exact mixing time formula for two-state regular Markov chains as a function of the initial distribution and the chosen accuracy  $\epsilon$ . The evolution of the distance of initial distributions from equilibrium is compared to the evolution of Shannon entropy and more general entropy functions, namely the Rényi entropies and the Tsallis entropies. These results are illustrated with several examples.

The self-adjoint operator with spectral projections the con-

ditional expectations  $E_t$  on the Hilbert space of fluctuations is called *the Time Operator of the stochastic process*  $X_t, t = 1, 2, \dots$ :

$$T = \sum_{t=1}^{\infty} t(E_t \ominus E_{t-1}) \quad (1)$$

The Time Operator  $T$  of a two-state regular Markov Chain  $X_t, t = 1, 2, \dots$  has been constructed explicitly. The internal Age of a two-state regular Markov Chain  $X_t, t = 1, 2, \dots$  is the Rayleigh quotient (expectation) of the Time Operator  $T$  of  $X_t$ -:

$$Age(X_t) = \frac{1}{\|X_t - \|^2} \quad (2)$$

We define the mixing time  $t_{mix}(\epsilon)$  as the minimal time  $t$  such that the differences  $t - Age(X_t)$  become constant. For times  $t$  related to the differences  $t - Age(X_t)$ .

The evolution of probabilities of any initial distribution  $\rho(0) = \rho = \begin{pmatrix} \rho_0 \\ \rho_1 \end{pmatrix}$ , with  $\rho_0 + \rho_1 = 1$  is given by the formula:

$$\rho(t) = \begin{pmatrix} \rho_0(t) \\ \rho_1(t) \end{pmatrix} = \begin{pmatrix} \rho_{eq,0} \\ \rho_{eq,1} \end{pmatrix} + \begin{pmatrix} \rho_0 - \rho_{eq,0} \\ \rho_1 - \rho_{eq,1} \end{pmatrix} \gamma^t \quad (3)$$

where  $\gamma$  is the second eigenvalue of the stochastic transition probability matrix  $W = \{w_{\kappa,\lambda}\}, \kappa, \lambda = 0, 1$  and  $\rho_{eq} = \begin{pmatrix} \rho_{eq,0} \\ \rho_{eq,1} \end{pmatrix}$  is the equilibrium probability distribution.

The mixing time of any initial distribution  $\rho$  is defined in terms of the family of  $p$ -norms  $\|\cdot\|_p$ , the family of Rényi Entropies and the family of Tsallis Entropies which include the Shannon Entropy and the Boltzmann-Gibbs Entropy correspondingly.

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## Stochastic oscillator with a random mass

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The simplest, the most general and the most used model of the harmonic oscillator. This model is used for the description of different phenomena in physics, chemistry, biology and social science. However, this model requires an improvement. In order to describe the real phenomena, one has to add some random forces to the appropriate equation. These noises may have an internal origin or an external one, and they enter the equations additively or multiplicatively, respectively. Additive noise arises from non-zero temperatures or from very rapid dynamics of certain degrees of freedom. Multiplicative noise arises from the stochastic nature of external fields and boundary conditions. Oscillator equations may contain random frequency, random damping or random mass. The first two cases have been intensively studied. The main aim of this talk is the detailed analysis of the oscillator with random mass. As a simple physical example one can mention Brownian motion with adhesion. Usual Brownian motion describes a particle subject to a dynamic systematic force, proportional to velocity, and an additive random force exerted on the Brownian particle by the molecules of the surrounding medium. Brownian motion with adhesion means that the molecules of the surrounding medium not only randomly collide with the Brownian particle, which produce

its well-known zigzag motion, but they also stick to the Brownian particle for some (random) time, thereby changing its mass, which is described by an additional multiplicative random force in the corresponding dynamic equation. The fluctuations of mass are modeled as a dichotomous noise, and the first two moments of coordinates show non-monotonic dependence on the parameters of oscillator and noise. In the presence of an additional periodic force an oscillator with random mass is characterized by the stochastic resonance phenomenon, where the appearance of noise increases the input signal. In the absence of additive white noise

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### Chaos-geometric attractor and quantum neural networks approach to simulation of chaotic evolutionary dynamics of complex systems

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Most important, the fundamental issue in the description of the dynamics of the statistically complex system is its ability to forecast its future evolution, i.e. predictability of behavior. Now for the analysis of time series of fundamental dynamic parameters there are developed and implemented a variety of methods, in particular, the nonlinear spectral and trend analysis, the study of Markov chains, wavelet and multifractal analysis, the formalism of the matrix memory and the method of evolution propagators etc. Most of the cited approaches are defined as the methods of a chaos theory. Nonlinear simulation and forecasting chaotic evolutionary dynamics of the large statistical systems can be effectively performed using the concept of compact geometric attractors. In our paper we present a new approach to analyze and predict the nonlinear dynamics of these systems based on the concept of geometric attractors, chaos theory methods and algorithms for quantum neural network simulation. In terms of the modern theory of neural systems, and neuro-informatics, the process of modelling the evolution of the system can be generalized to describe some evolutionary dynamic neuro-equations (miemo-dynamic equations). Imitating the further evolution of a complex system as the evolution of a neural network with the corresponding elements of the self-study, self-adaptation, etc., it becomes possible to significantly improve the prediction of evolutionary dynamics of a chaotic system. We have developed a software package for numerical modeling of the dynamics of the photon echo neural network. It has the following key features: multi-layering, possibility of introducing training, feedback and controlled noise. The results of the PC simulation of dynamics of the quantum multilayer neural networks with the input rectangular, sinusoidal pure soliton and noisy soliton-like pulses sequence allow to make conclusion about sufficiently high-quality processing the input signals of very different shapes and complexity. A stochastic resonance effect is discovered in our PC experiment for noisy soliton-like pulse input. Using phase space information on the evolution of the processes in time and results of the neural network modelling techniques can be

considered as one of the fundamentally new approaches in the construction of global nonlinear models of the most effective and accurate description of the structure of the corresponding attractor and in further optimal simulation of the statistically complex evolutionary processes.

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### Global mechanisms in the Earth atmosphere models, energy and angle momentum balance and teleconnection: Statistical and dynamical modelling

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The satellite data and data of observing the radio-waveguide parameters (especially in the low troposphere layers) by means of radio-technical devices (in the ultra short-wave diapason) is the informative basis of the modern atmosphere long-term forecasts. As any water quantities in atmosphere are formed on the basis of the cycle- and front-genesis (or in the convective non-stability lines) one can introduce the corresponding model on the basis of thermodynamics and hydromechanics of the corresponding processes. For example, physics of these processes can coincide with the a soliton mechanics, which has the long-periodical basis of the energy support. The action mechanics of such a soliton defines the key thermo-hydro-dynamical parameters of the atmosphere ultra-short-wave radio-waveguide. We present principally new non-linear statistical and dynamical methods of monitoring and modelling the Earth system low-frequency scale processes on the basis of observing some summated contributions of low frequency oscillations for geophysical factors. They base on the energy and angle moment balance relations with stochastic elements and new scheme for calculation of the macro-turbulence regime in typical atmospheric processes (Glushkov et al, 2006), which are known as atmospheric circulation forms. The balance analysis allows to predict the large-scaled atmospheric transformations and teleconnection phenomena and to give their quantitative description. We carried out a series of PC experiments at the Pacific ocean region in order to study global mechanisms in the atmospheric models and check the seasonal sequences of the conservation (or disbalance) of the Earth atmosphere angle momentum and to provide new predictors for the long-termed and super long-termed forecasts of the low frequency atmospheric processes. The current function (complex velocity) fields are calculated for typical atmospheric circulations forms. The experiments allowed quantitatively defining a direct link between an atmospheric turnover and atmospheric circulation forms through the front divider position and typical low frequency process of conservation of the angle moment balance. Besides, we have adapted the modified theory of the macro-turbulence for possible using the atmosphere radio-waveguides as a special effective predictors in the long-termed

plan. The results of analysis for the temporal variation of the North-Atlantic and Southern oscillations (NAO and SO) and their effect on the spatiotemporal distribution of precipitation in the Europe during 1960-2010 years on the basis of non-decimated wavelet decomposition method are listed too.

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### Effective temperature in an active matter system

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Active matter refers to systems driven out of equilibrium by internal or external energy sources. They are characterized by many peculiar properties not present in their passive counterparts, like clustering, anomalous diffusion, giant fluctuations [1,2]. For these systems an open question is the coherent definition of an effective temperature [3,4].

In our work we consider a two-dimensional system of active dumbbells [?], each made by two beads kept together by a spring, and interacting through a Weeks-Chandler-Anderson (WCA) potential. They are immersed in an implicit solvent modeled by the Langevin equation. The activity is modeled by a constant force acting on the principal direction of the dumbbell. This model wants to be a coarse-grained characterization of the behavior of simple bacteria and tries to study them from a statistical point of view.

The aim of our study is to use tracers, mechanically coupled with the dumbbells through the WCA potential and not coupled to the thermal bath, to test the notion of effective temperature and the use of different methods to measure it through the presence of the tracer.

We first analyzed the diffusive properties of our model in the absence of tracers, always in a regime without clustering. For a single active dumbbell we analytically calculated the diffusion coefficient for its center of mass together with the displacement induced by a pulling force, arriving to a definition of an effective temperature through the fluctuation dissipation relation. Then we calculated numerically the above quantities for systems with different densities, temperature and activities. We found non trivial non monotonic behavior of diffusion and mobility when the activity is varied, at fixed density. The Einstein relation results verified in terms of an effective temperature not depending on the density.

On the other hand, we calculated the effective temperature of the active system by introducing spherical tracers and studying their velocity distribution. The tracer velocity distributions result to be gaussian and from that a temperature can be evaluated. We also considered a version of the fluctuation dissipation relation for the tracers, giving another estimate of the effective temperature. We found that these

effective temperatures, for high values of the masses of the tracer, converge to the value of the effective temperature previously calculated in the system without tracer. This suggests that the study of the dynamics of tracers, more accessible experimentally than that of the non-equilibrium system itself, is a convenient way to analyze non-equilibrium systems.

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### Financial herding of three agent groups under the impact of exogenous noise

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Socio-economic sciences face complexity right from the start as they are intrinsically complex at many different levels. Financial markets are among the most interesting examples of such complexity. There is no direct way to gain insights into the nature of microscopic interactions in financial markets. Agent based modeling framework, which may provide qualitative and quantitative understanding of the financial markets, is very ambiguous with scarce empirical evidence. Looking for the ideal agent based approach we would consider as a primary necessity to build bridges between microscopic, agent based, and macroscopic, phenomenological, modeling [1]. Following this trace of thought it is rational to combine two origins of the noise: exogenous one, related to the information flow, and endogenous one, arising from the complex stochastic dynamics of agents.

From the agent, market trader, point of view there are two fundamental dilemmas: a) to follow rational, based on long term fundamental values, thinking or to speculate counting on sophisticated strategies tracking much more rapid price movement tendencies and b) dilemma to buy or sell stocks if agent already has chosen to acts as speculating chartist. If dilemma (a) considers choice between common sense rationality and heterogenous rationality, then dilemma (b) considers heterogenous rationality as irrelevant for the macroscopic outcome of the whole macroscopic system and simplifies it to the alternatives buy or sell stocks. Such simplification of possible agent's choices can be considered as herding process in three groups of agents [2]. Herding is one of the behavior peculiarities considered in behavioral finance [3].

In this contribution we consider a three state agent based herding model of the financial markets. The derived set of stochastic differential equations describes underlying macroscopic dynamics of agent population and log price of the asset in financial market. The obtained stochastic process is then subjected to the exogenous noise, which shapes instantaneous return fluctuations. We test both Gaussian and q-Gaussian noise as a source of the short term fluctuations. The resulting model of the return in the financial markets with the same set of parameters reproduces empirical probability and spectral

densities of absolute return observed in New York, Warsaw and NASDAQ OMX Vilnius Stock Exchanges. Our result confirms the prevalent idea in behavioral finance that herding interactions may be dominant over agent rationality and contribute towards bubble formation.

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### Modeling the interplay between protein and lipid aggregation in supported membrane

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We present a theoretical model that deals with the complex interplay between lipid species segregation in a bilayer membrane and the self aggregation of protein attached to one of the lipid species. This model is an extension of a previous one [1,2] used to study the filaments and bundles formed by the self-aggregating cytoskeletal protein FtsZ, a protein found in prokaryotes and which is one of the key characters in bacterial reproduction. Recent experiments with FtsZ mutants covalently attached to maleimide lipids found a new range of structures that required new ideas to understand them, like the interplay between torsion and an specific anchoring of the protein filaments.

However, this model was not able to reproduce the unexpected polymorphism observed under conditions in which the composition of the lipid membrane produces the segregation of two phases with different composition; at low temperatures or high concentrations of linker lipid. Under these circumstances the lipid bilayer has to be treated like an active component of the system, rather than as a passive substrate. Biological membranes are very complex, and lateral segregation of lipids and proteins into domains of different sizes and life times play an important role in signaling pathways, endocytosis, cell polarity and migration, neuronal growth and a variety of disease settings.

In order to look into this new phenomena, we expanded our model by including a lipid bilayer with two lipid components. In contrast to previous models that consider the membrane only as a passive element for the planar confinement of the proteins, ours describes the system including dynamic interactions between protein monomers, the interactions between lipid components, and also a mixed term considering both protein-lipid interactions.

Our preliminary results helped us to understand the experimental results, and suggest that lipid segregation may affect the length and curvature of protein filaments and that the dynamic behavior of the lipids and proteins might have different time scales, giving rise to “memory effects”.

This simple model that considers a dynamic protein assembly on a fluid and active lipid surface is a starting point which can be further refined with additional elements. In this way it could be easily applied to other biologically relevant situations

in which the interplay between protein and lipid aggregation are both needed to fully describe the system [3].

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### Liquid-crystal patterns of rectangular particles in a square nanocavity

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Hard rectangular particles in two dimensions have attracted a lot of interest in recent years, due to their ability to form exotic nematic phases (1) and their peculiar nature of the crystal phase near close-packing (2). In Using density-functional theory we analyze liquid-crystal patterns and phase behaviour in a fluid of hard rectangular particles of length  $L$  and width  $D$  confined in a two-dimensional square nanocavity (3). Patterning in the cavity is governed by surface-induced order as well as capillary and frustration effects and depends on the relative values of the particle aspect ratio and the cavity size  $H$ . Ordering may be very different from bulk behaviour when the cavity size is a few times the particle length. Bulk and confinement properties are obtained for different particle aspect ratios,  $\kappa = L/D = 1, 3$  and 6.

In bulk the isotropic phase is always stable at low packing fractions and nematic, smectic, columnar, and crystal phases can be stabilized at higher densities depending on the particle size. In the confined fluid surface-induced frustration leads to fourfold symmetry breaking in all phases (which become twofold symmetric). The restricted-orientation approximation used in our theoretical approximation prevents director distortion by construction, so that frustration in the director orientation is relaxed by the creation of domain walls (where the director changes by  $90^\circ$ ); this configuration is necessary to stabilize periodic phases at high density. For square particles,  $\kappa = 1$ , the crystal becomes stable, with commensurate transitions taking place as the cavity size is varied. These transitions involve structures with different number of peaks in the local density. For more anisotropic particles the commensurate transitions involve columnar phases (with different number of columns) for  $\kappa = 3$  or smectic structures (with different number of layers) for  $\kappa = 6$ . In the latter case, at lower densities there is a symmetry-breaking isotropic to nematic transition exhibiting nonmonotonic behavior with cavity size.

Apart from the present application in a confinement setup, our model can be used to explore the bulk region near close packing in order to elucidate the possible existence of disordered phases at close packing. Finally, results involving relaxation dynamics (using dynamic density-functional theory) from different structures and phase transitions are also presented, and an application involving the growth of a bacteria along its longitudinal direction is shown.

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## Molecular motors operating in viscoelastic cytosol: anomalous vs. normal transport and its efficiency

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Discovery of anomalously slow passive subdiffusion of vesicles, RNA messengers and other submicron particles in living cells provokes a number of intriguing questions. In particular, how do molecular motors like kinesins actively transport such freely subdiffusing particles? Is motor-mediated transport normal or it also becomes anomalously slow? Can the same motors in the same cells realize both normal and anomalous transport and under which conditions? We answer these intriguing questions [1] within a non-Markovian generalization of standard Markovian continuous diffusion model of molecular motors with two conformational states by taking into account slowly decaying memory effects caused by the viscoelasticity of cytosol considered as a highly crowded complex polymeric fluid. Influence of such complex medium is characterized by a memory friction with fractional friction coefficient and by thermal fractional Gaussian noise which are related by fluctuation-dissipation relation. Long-time memory cutoff corresponding to maximal temporal range of subdiffusion makes the integral friction coefficient finite. Large enhancement of such efficient friction coefficient reflects largely enhanced macroscopic viscosity of cytosol for submicron particles with respect to one of its water constituent. In this study, we rely upon and develop further general approach to anomalous Brownian motors [2-5] which is based on Generalized Langevin Equation with power-law decaying memory kernel, thermal  $1/f$  noise, and approximate multi-dimensional Markovian embedding of such profoundly non-Markovian dynamics within a generalized Maxwell-Langevin model of viscoelasticity [3,4]. The approach has a firm statistic-mechanical foundation with vivid physical interpretation making it especially attractive in biophysical applications [3]. For realistic parameters, we show that the transport can be both normal and anomalously slow, depending on the amplitude of binding potential of the motor protein to microtubule and external loading force acting on the motor, turnover frequency of motor enzyme and the cargo size (determining relevant fractional friction coefficient). Strikingly enough, our anomalous Brownian motor can operate as perfect ratchets with the motor steps completely synchronized with the enzymatic turnovers, if the cargo is not too large and the motor operates sufficiently slow. Thereby we show how a power stroke operating mechanism can overcome subdiffusional restrictions caused by macromolecular crowding in living cells. Such active molecular transport can be highly efficient despite subdiffusion.

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## laPENSOcosì: unveiling political opinion structure with a web-experiment

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In the United States hundreds of millions of dollars are yearly spent for presidential campaigns, but the very mechanisms through which public opinion can be successfully shifted are largely understood. Available data have been studied seeking for universals and experiments have been proposed aiming at highlighting different mechanisms influencing voters opinions on candidates. However, comprehensive and focused databases of political opinions are lacking. While in fact political elections outcomes constitute precious databases to investigate voters behavior, the very voters opinion could be hidden by many different factors, as for instance tactical voting and limitations imposed by the election rules. Moreover, the electoral system usually allows the expression of a single positive vote per elective assembly, thus individuals are called to compress all their interpretation of the political landscape in a single preference. Most importantly, voters are bound to express positive votes, so that only approval and not disappointment can be directly expressed.

Here we propose an experiment aiming at overcoming these limitations, providing a more complete picture of the citizens perception and allowing in this way a quantitative analysis of the complex structure of individuals opinions. In particular, following a general trend emerged in the last few years, where web-games are adopted as interesting laboratory to run experiments in the social sciences, we propose a web game/experiment in which people are directly called to express their opinions on political subjects. The data gathered with the *laPENSOcosì* experiment reflect the general negative sentiment the Italian population has towards its political class. More interestingly, we found that opinions follow a power law distribution, with a negative exponent whose absolute value decreases with the level of optimism, but that remains almost constant across different political orientations. This result strongly recall a well known law linking stimuli and perception, i.e., the Weber-Fechner law. This law has been observed in several situation involving physical perceptions, e.g. for light brightness or for noise intensity, but also in other context, e.g. for numerical cognition and recently in subjective perception on the quality of a service. The presence of a Weber-Fechner-like law in political perception is a novel aspect. This finding could foster other experiments in this direction and can have a profound impact on the way models of opinion dynamics are constructed.

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## Rigorous results on first-passage times for surface-mediated diffusion

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We present an exact calculation of the mean first-passage time (FPT) to a target on the surface of a 2D or 3D spherical domain by surface-mediated diffusion. In this model, a molecule alternates phases of surface diffusion on the domain boundary and phases of bulk diffusion, with diffusion coefficients  $D_1$  and  $D_2$  respectively. Random durations of each surface (adsorbed) phase are exponentially distributed, with a given desorption rate. The process is stopped when the molecule arrives on the target on the surface (either from the bulk or from the surface). Two mean FPTs are introduced, for molecules started on the surface, and for the molecules started in the bulk. These mean FPTs satisfy two coupled diffusion PDEs with mixed boundary conditions. The spherical symmetry allows one to benefit from specific relations between the eigenbases of the Laplace operators in the bulk and on the surface in order to reduce these coupled PDEs to an integral equation. We then provide two analytical solutions of this integral equation: an exact solution that requires an inversion of an explicitly given matrix, and a very accurate approximate solution which is fully explicit. These solutions are provided for normal and biased diffusion (in the presence of a radial force) for a variety of simple domains such as disk, annulus, sector, sphere, and even rectangles. In particular, the mean FPT is obtained in a general annulus with an arbitrary number of regularly spaced targets on a partially reflecting surface. Our method is applicable to extended targets of arbitrary size that goes beyond the narrow escape limit. In the framework of this minimal model of surface-mediated reactions, we show analytically that the mean reaction time can be minimized as a function of the desorption rate from the surface. As a consequence, an intermittent exploration may enhance search and reaction, as compared to pure surface diffusion or pure bulk diffusion. In fact, we determine conditions on the  $D_2/D_1$  ratio under which surface diffusion, bulk diffusion, or intermittent diffusion is optimal. This analytical approach opens a way to understand intermittent search processes (such as e.g. facilitated diffusion) in natural systems and puts forward a general mechanism of enhancement and regulation of chemical and biological reactivity. Higher-order moments and the probability distribution of the first-passage time can also be derived.

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## Recent developments in multifractal analysis of data with application to finance

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Possible reasons for the appearance of false multifractal signals in real records of data will be discussed. Particular emphasis will be given to the influence of random fluctuations, finite size effects (FSE) as well as the influence of broad data distributions on multifractal records. Detailed semi analytical formula linking the amount of such false multifractal bias with persistency level in data, its length and maximal fluctuation moment order  $q$  used to amplify the ratio of respective small and large fluctuations in time series will be provided. An implementation of proposed corrections for calculations of the actual threshold of multiscaling phenomena in complex systems will be shown in the examples taken from real multifractal financial data. A study is performed within MF DFA. It is found this effect is quite important for shorter or persistent series. It is linear with respect to autocorrelation exponent  $\gamma$ . Its strength decays according to power law with respect to the length of time series  $L$ . The obtained formulas are significant in any interdisciplinary application of multifractality, including physics, financial data analysis or physiology, because they allow to separate the true multifractal phenomena from the apparent (artificial) multifractal effects. They should be a helpful tool of the first choice to decide whether we do in particular case with the signal with real multiscaling properties or not. It will be shown how to draw proper conclusions regarding multiscaling properties of complex systems from the multifractal analysis based on interpretation of the observed  $\Delta h$  (generalized Hurst exponent spread) or  $\Delta \alpha$  (Holder exponent spread) in multifractal techniques used so far.

In particular, we will analyze also quantitatively the corrections to the spread of the generalized Hurst exponent profile  $h$ . These will allow to extend the previously found formulas describing the level of artificial multiscaling in finite signals for large  $q$  to an arbitrary narrower range of  $q$  moments used in MF DFA technique in distinct applications by a number of authors. Examples of the implementation of proposed corrections to real multifractal financial data will be provided at the end. Finally I shall discuss the naked influence of broad data distributions on multifractal findings. It will be indicated that the latter effect is crucial for extraction of "real" multiscaling phenomena within MF DFA and for some financial markets it entirely suppresses multifractality caused by different autocorrelation nature of data at distinct time scales.

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## Fokker-Planck approach to propagation of light in one-dimensional disordered photonic crystals

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Influence of disorder on propagation of light in one-dimensional nominally periodic structures is studied analytically using statistical approach based on a Fokker-Planck type equation. A variety of the models with short-range disorder has been considered, allowing for both independent and correlated fluctuations of the refractive index and layer widths within a period. In order to describe evolution of the electromagnetic wave along a structure under the conditions of small dielectric contrast  $\eta$  and disorder amplitudes statistically we introduce slowly varying angle variable  $\Psi$  and derive the Fokker-Planck equation for the distribution function of this variable [1], which can be rewritten in the stationary case (i.e. in the limit of a long sample) as

$$(1 + \gamma^2 \sin^2 \Psi) \frac{dF}{d\Psi} - \left[ \alpha(G + \sin \Psi) - \beta \cos \Psi - \frac{\gamma^2}{2} \sin 2\Psi \right] F = -J, \quad (1)$$

where  $J$  is a stationary flow, and relations between the Greek letters  $\alpha$ ,  $\beta$ ,  $\gamma$  and amplitudes of the fluctuations are given in Ref. [1]. Practically, we need only solution of the homogeneous equation,  $F_0$  (which can be found again in Ref. [1]), and, using it, general answers for the light localization length and density of the optical modes [2] are readily obtained, which can be made explicit in the particular cases of purely geometric disorder, disorder with equal dielectric fluctuations, uncorrelated dielectric component, and for the limiting case of small disorder amplitudes as compared to dielectric contrast of a photonic crystal  $\eta$ . Our derivation shows that initial region of dependence of light localization length  $\xi$  on a disorder parameter  $\delta$  is universal, provided that the latter is defined in a proper way, see Ref. [1]. However, this universality disappears at  $\delta \sim \sqrt{\eta}$ , and subsequent behaviour is specific to a model. Interestingly, in most of the models, except for a particular case of the purely geometric disorder,  $\xi(\delta)$  is a non-monotonic function, and indeed this fact can be explained by crossover between weak disorder regime describable by graceful suppression of the reflecting properties of a photonic crystal and strong disorder regime, when periodic component of the refractive index can be almost fully neglected. In addition, we have checked practical consistency of the theory by comparison with the numerical computations carried out for non-small values of the dielectric contrast. Also, good agreement with the experimental data of Ref. [3] managing small contrast three-dimensional photonic crystals has been achieved. This work was partially supported by NSH-5062.2014.2.

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## Modelling the integer quantum Hall effect at finite temperature in the quantum wells, quantum dots and graphene

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The quantum Hall effect (QHE) is nice example of a phenomenon where disorder produces deterministic result with the precision limited by our tools of measurement, the feature honoured by the two Nobel Prizes. Another Nobel Prize, awarded in 2010, was shared by the QHE with other exciting electronic phenomena in graphene, where it was used to prove monoatomic structure and, amazingly, is observable at 300K [1]. However, theoretical background for description of the QHE at finite temperatures is not very rich, to put it mildly. The trivial part of our knowledge is that the QHE is being smeared when temperature is increased due to dephasing introduced by the electron-phonon and electron-electron interactions, and less trivial fact originating from the concept of scaling is a power dependence of the transition widths between adjacent plateaus of the QHE on the temperature with some exponent  $\kappa$ , which is believed to be fixed for a certain class of universality [2]. Unfortunately, scaling theory of the QHE gives no quantitative information, even on the value of  $\kappa$ , and, surprisingly, no numerical calculations of conductivity in the QHE regime taking into account real inelastic processes were available until recently [3].

In this talk we present a theory suitable for description of the integer QHE at finite temperatures, which takes into account the electron-phonon interaction, and numerical results for the temperature and filling factor dependencies of the conductivity tensor components for three material systems: semiconductor quantum wells (QWs), quantum dots (QDs) and graphene. We start up with the exact eigenfunctions of the two-dimensional electron gas in the short-range or long-range disorder potential and high magnetic field, found implicitly using high-efficient method of kernel polynomials [4], and use them to calculate transition rates between the one-particle electron states, which determine conductivity via Kubo formula. We have obtained interesting results for all three kinds of the studied systems. In the case of QWs with the short-range disorder, our calculations give the value of  $\kappa$  close to 0.4, in agreement with most of the experiments, while for the long-range disorder it is significantly smaller. For the QDs we have shown possibility to increase the maximum temperature for observation of the QHE using strong confinement potential of the QDs, up to  $T \sim 50 - 100K$ . The most exciting results refer to graphene, for which we were able to reproduce the room-temperature integer QHE taking into account strictly two-dimensional (atomic) confinement for both electrons and phonons. Our study was supported by MK-718.2014.2.

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## Emergence of Complexity from Cooperative Interaction

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Here we illustrate one of the possible perspectives that we guess will expand and generate a unifying vision of complexity in the next few years. (1) The fall of reductionism. According to Schweber [Schweber, 1993] the triumph of renormalization group theory makes reductionism superseded by the study of emergent phenomena. On the other hand renormalization group theory affords the most advanced theoretical understanding of phase transitions, which, in turn, led the investigators to go beyond the single unit pictures while emphasizing the collective and cooperative behavior. (2) Emergence of intelligence and cognition. The results of the research work on sets of cooperating animals are changing the concept itself of organism. The communality of interest and minimal conflict leads to a definition of organism, called organismality [Queller, 2009] extending from the ordinary concept to human society. The cooperative action of many units, at the same time, is thought to be equivalent to generating the intelligence of a Turing machine [Rosenfeld, 2013]. (3) Game theory. The essential role of cooperation raises the important issue of explaining its origin, in spite of the advantages that a single individual may derive from defection, this being a compelling motivation for game theory in finite populations [No. wak, 2004]. The study of collective behavior and evolutionary games is becoming increasingly popular and the authors of this field are currently addressing the challenge of establishing a connection with neurophysiology and sociology [Perc, 2013]. (4) Criticality in the brain. On the other hand, the study of brain dynamics remains of central interest [Werner, 2013] with the key hypothesis that the brain is a complex system operating at criticality. This brings us back to the phase transition issue under the important condition of a finite number of interacting units. (5) Interdisciplinary research. Advances in game theory must take into account that behavioral psychology yields experimental evidence [Kearns, 2012] on the social nature of human brain thereby substantiating the conjecture of the existence of the group mind [McDougall, 1928], which is an explanation of the emergence of cooperation resting on psychology. The growing networks of links between disciplines seem to obey the same cooperative processes as those hypothesized to be at the basis of emergence of intelligence [Villa, 2013]. (6) Progress in Statistical Physics. Science of Complexity will not rest on traditional physics, and we argue that physics has to be extended and generalized so as to properly cope with the non ergodic nature of complex systems [Burov, 2010; Aquino, 2010].

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## Sine-Gordon Breathers generation in driven long Josephson junctions

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The exclusive generation of breathers in Josephson junctions (JJ) is computationally investigated. The analyzed system is a long JJ (LJJ) stimulated by an external driving signal. The electrostatics of a LJJ is ruled by the *sine-Gordon* (SG) equation. This is a partial differential equation for the JJ *order parameter*  $\varphi$ , that is the phase difference between the macroscopic wave functions describing the superconducting condensate in the two electrodes. The SG model used is perturbed including a damping term, setted to work in underdamped regime, and an applied bias current. Different SG travelling wave solutions are well-known: plasma waves, one-soliton solutions, i.e. *kink* and *antikink*, N-soliton solutions, stationary or moving *breathers*. In particular, a breather is a bound pair of a kink and an antikink oscillating with an internal frequency  $\omega$ . Considerable amount of theoretical and computational studies, about breathers in LJJ, exists, despite of the absence of experimental works devoted to their detection. Many impediments affect experimental observation of breathers in LJJ. First, an efficient generation and entrapment is necessary, and second, a breather doesn't manifest itself as clear mean voltage difference or magnetic flux through the junction. This work is devoted to establish an efficient experimental setting to generate a SG breather in a LJJ, exploiting a well-known phenomenon, the *nonlinear supratransmission* (NLS). Due to the nonlinearity of the medium, that is the junction, energy transmission is possible also irradiating the system with a signal of frequency in ranges usually forbidden. In our model, one end of the junction is driven by a sinusoidal pulse of amplitude  $A$  and frequency  $\omega$  lower than the plasma frequency of the JJ. To well reproduce a meaningful experimental signal, that is no abrupt variations during the switching on/off regimes, the driving amplitude is smoothly increased/reduced according to Gaussian profiles. In a 2D parameters space  $(A, \omega)$ , a region with absence of NLS, that is without energy flowing into the system, is evident. Otherwise every combinations of SG solutions are generated and propagate along the junction. When this occurs, in correspondence of specific  $(A, \omega)$  values, creation of only breathers is also detectable. The analysis is improved varying the damping parameter, the duration of the driving pulse and the applied bias current. To check the robustness of the generated breathers, a Gaussian noise source is inserted into the perturbed SG model, and the percentage of surviving breathers is calculated.

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## A two-order parameter model for the analysis of phase transitions

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The main propose of this report is to present the analysis of phase transitions in the presence of an intermediate state by using the model with two order parameters. The parametric modelling of phase transitions and analysis of the role of an intermediate liquid state in irreversible relaxation processes at low temperatures were performed. As an example, one can consider the systems with two stable states  $L1$  and  $C$  that means liquid and crystalline, respectively, and the third one - intermediate fluid state, namely  $L2$  [1]. Such  $L2$  state has been experimentally discovered in supercooled liquids [2]. The model includes two order parameters and three control parameters in the Landau-type kinetic general potential of 6<sup>th</sup> degree, and has been developed to study the impact of heterogeneity on phase transitions in the presence of an intermediate fluid state [3]. We noticed that the presence of the intermediate liquid state may indeed enhance the nucleation rate, and, furthermore, an increase in the heterogeneity of system accelerates the transition dynamics. In the previous work, analytical solutions were also obtained according to the Descartes-Euler method for solving polynomial equations. Depending on the values of its control parameters, the potential has one, two or three possible minima, and the problem dealt with the construction of the equilibrium phase diagrams. It is also worth mentioning that the previously obtained results are general and suggest a complete set of different transition scenarios in the entire parameter plane with two control parameters [4].

In general, the largest and smallest values of order parameter for the bifurcational analysis correspond to minima of free energy functional  $F$ , while the intermediate value corresponds to an unstable state ( $F$  has a local maximum or saddle point), and these three extrema are identified with the crystalline and two liquid phases [5]. In case of a single - component glass which can be characterized in terms of the pressure  $P$  and volume  $V$ , the relation between  $P$  and  $V$  could be obtained using the equation  $P(V, T, x, y) = -(\partial F / \partial V)_{T, x, y}$ . Note that  $P(V, T, x, y)$  can be derived from experimental data and this equation may be further used to determine the  $V$ -dependence of  $F(V, T, x, y)$ . Then  $F$  can be applied to get the entropy  $S = -(\partial F / \partial T)_{V, x, y}$ , and in this way specific heats and other thermodynamic quantities for the system may be defined; see, for example, the well-known handbook Statistical Physics of L.D. Landau and E.M. Lifshitz.

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## Work distribution and fluctuation-dissipation relations for systems driven by external Levy noises

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Superlinear and sublinear diffusional transport and nonexponential relaxation kinetics are ubiquitous in nature and have been observed and analyzed in a number of systems ranging from hydrodynamic flows and plasma physics to transport in crowded environment inside cells and price variations in economy. Usually, a starting point of description of systems exhibiting anomalous transport properties involves use of continuous time random walk (CTRW) and fractional differential diffusion equations of the Fokker-Planck type (FFPE). Although both concepts turn to be extremely powerful in applications, there is still lack of full understanding of their links with first principles of mechanics and thermodynamics [1]. Among various problems addressed in this field in recent years is the origin of superdiffusive transport in momentum space investigated in a number of studies. In particular, based on the Langevin model with linear friction proportional to velocity and non-Gaussian noise, superdiffusive transport in velocity space for a magnetized plasma has been analyzed [2,3]. Resulting FFPE has been further shown to describe the evolution of the velocity distribution function of strongly nonequilibrium and hot plasmas obtained in tokomaks. On the other hand, anomalous transport is also relevant for analysis of signal transmission and detection in biological systems [1]. In this field Brownian-ratchet models provide mechanisms by which operation of biomolecular motors is investigated. In recent analysis it has been shown that the minimal setups of simple ratcheting potentials and additive white symmetric Lévy (non-Gaussian) noise are sufficient to produce directional transport and inversion of currents can be obtained by considering a time periodic modulation of the chirality of the Lévy noise [4,5].

Here we continue this line of research and analyze thermodynamic energetics of a linear system subject to Lévy fluctuations. We assume that the motion of a test particle is described by a stochastic differential equation (SDE). We study fluctuation-dissipation relations for a linear system coupled to thermal bath and subject to independent deterministic and random forces. At the level of a corresponding Langevin equation, inclusion of thermal bath is modeled by white Gaussian noise, whereas external random force is represented by Cauchy noise term. Additionally, the system is perturbed by a deterministic time-dependent force field. By use of generalized susceptibility which is a function of the unperturbed equilibrium system, we relate the correlation between spontaneous fluctuations with the response of the system to external drivings.

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## The transition to synchronisation on hierarchical networks

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We study the transition to synchronisation in branching hierarchical lattices using the evolution of Chate Manneville maps placed on a triangular lattice. Connections are generated between the layers of the triangular lattice assuming that each site is connected to its neighbours on the layer above with equal probability. The maps are diffusively coupled, and the map parameters increase hierarchically, depending on the map parameters at the sites they are coupled to in the previous layer. Thus the system forms clusters, with the cluster with the largest number of connected sites being called the maximal cluster. The system shows a transition to synchronisation which is second order in nature, with associated critical exponents. However, the V-lattice, which is a special realisation of this lattice, and where the maximal cluster is V-shaped and contains the largest number of sites which can be seen in the lattice realisations, shows a transition to synchronisation which appears to be discontinuous. This raises the possibility of the transition belonging to the class of explosive synchronisation with the explosive nature depending on the nature of the substrate.

We compare our results with those seen for explosive percolation transitions seen on these networks [1],[2], where the transmission of packets has been considered. Each node can accommodate a limited number of packets, depending on its capacity, and the packets hop from node to node, following the links between the nodes. The traffic flow in these models can be represented by the site percolation problem. The transition to percolation on the regular clusters seen here was seen to be continuous, with a full set of critical exponents whereas a transition to explosive percolation was seen on the V-lattice, with continuous transitions being seen for the usual clusters. The load bearing versions of these networks also show that the V-lattice shows power-law behaviour for the failure rates, with sigmoidal failure rates for the other realisations. The distribution of lengths of the avalanches for the V lattice also scales as a power law, unlike the behaviour of the usual realisations which are multimodal Gaussians. These lead to the belief that the V-lattice is the critical realisation of these networks, for these cases. The asymmetric nature of the links on the V-lattice is a major factor leading to criticality. We speculate on whether a similar structure can lead to critical cascading behaviour in the case of the synchronisation transition, as well as in real life applications.

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## Self-avoiding walks and related models under compression

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Many lattice models, such as self-avoiding walks, self-avoiding polygons, self-avoiding bridges, and simpler models like Dyck

paths, have generating functions  $A(x) = \sum a_n x^n$ , (where the coefficient  $a_n$  gives the number of such objects of size  $n$ ) with algebraic singularities. The coefficients thus behave as  $a_n \sim B\mu^n n^g$ . This remains true when such models are considered in a half-space, with their origin anchored in the surface. In this situation, each such object achieves a maximum height  $h$ . If we associate with each such object a fugacity  $y$  conjugate to the height  $h$  of the object, this can model the behaviour of such objects subject to a stretching force ( $y > 1$ ), no force ( $y = 1$ ) or a compressive force ( $y < 1$ ). We find that, for all the models considered, the critical behaviour is different in the three regimes. When there is no force ( $y = 1$ ), the usual algebraic singularity is observed. For  $y > 1$ , – the stretched regime – we find the generating function has a simple pole, but the radius of convergence  $1/\mu(y)$  decreases as  $y$  increases.

For the compressive case however, we find, in all cases, that  $a_n \sim C\mu^n \mu_1^{\sigma} n^g$ , where  $\mu(y) = \mu(1)$ , and typically  $\sigma = 1/2$  or  $1/3$ . In the case of Dyck paths this result can be proved [1] (we find  $\sigma = 1/3$ ). In the other cases mentioned, we obtain estimates of  $\mu$ ,  $\mu_1$ ,  $\sigma$  and  $g$  by careful numerical work. We discuss these results, and the newly developed numerical methods whereby they are obtained.

This behaviour is reminiscent of that found by Owczarek, Prellberg and Brak [2] for self-interacting partially directed self-avoiding walks in the collapsed phase. They conjectured a similar behaviour for interacting self-avoiding walks in general, in the collapsed phase. That is to say, they conjectured that the partition function of such walks of length  $L$  should behave as  $Q_L \sim q_0 \mu_0^L \mu_1^{L\sigma} L^g$ , and argued that  $\log \mu_1(T)$  is proportional to a temperature dependent surface free-energy, while  $\sigma$  should be close to  $1/2$ . Duplantier [3] pointed out that such behaviour was observed even earlier, in the case of dense two-dimensional polymers, and further calculated various critical parameters.

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## Sessile drop evaporation from isolating and conducting surfaces

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There has been a continual interest in understanding the evaporation of sessile drops. This apparently simple problem, with a number of important applications, has repeatedly posed new challenges and lead to surprising results. As an example, Ristenpart and coworkers (2007) found that the flow pattern (the circulation) inside an evaporating drop can be reversed through a sufficient change in the substrate conductivity. In a seminal paper, Deegan and coworkers (1997) demonstrated that such flow patterns can explain the formation of ring-shaped ‘coffee stains’, for cases where a pinned evaporating drop contains dispersed material. Several authors have emphasized the role of substrate conductivity, whereas variations in substrate temperature have been investigated by Sobac and Brutin (2012).

We have studied experimentally the evaporation of sessile drops at cold (unheated) surfaces. We obtain evaporation rates as functions of time by placing a substrate plate with the drop on a balance with readability 0.1 mg. The evaporation process depends on a continuous heat flow from the surroundings to the drop and the plate it rests on - by conduction, radiation, and convection. There is a temperature gradient in the plate towards the droplet. The temperature of the droplet remains lower than that of the surroundings throughout the evaporation process. A proper description depends on a decomposition of the needed heat flow into the various contributions. In our experimental setup, the heat flow as a function of time is measured through the evaporation rate. Furthermore, we have restricted the role of heat conduction to the drop by using a thick slab of a material with very low conductivity between the substrate plate and the balance.

We have investigated the evaporation of water droplets with three different materials used for substrate plates: Poly-Methyl-Methacrylate (PMMA) plate, Aluminum (Al) alloy plate and Al foil. Surface areas were from  $2 \times 2 \text{ cm}^2$  to  $10 \times 10 \text{ cm}^2$ , initial droplet masses from 0.05 g to 0.4 g. Photos from above were used to determine drop area, as function of time. Wetting conditions vary; typically we observe spherical cap shape for drops at PMMA and more oblate shape for droplet water at Al-surfaces. The drops at Al substrates are pinned.

The instantaneous evaporation rates of droplets from Al plates are about twice as large as for PMMA plates. This precludes a description with diffusion of water vapor a limiting mechanism. On the other hand, had conduction through the substrate plates been the limiting mechanism, the difference between the cases should have been much larger.

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## Relativistic diffusion

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Diffusion of relativistic particles is discussed. It is shown that relativistic invariance imposes strong restrictions on the form of the relativistic diffusion. A relativistic diffusion must be formulated in the phase space. Its non-relativistic version coincides with the Kramers diffusion. We discuss diffusions which equilibrate to Juttner, quantum Bose-Einstein and Fermi-Dirac distribution as well as distributions resulting from non-extensive statistical physics. The equilibrium is determined by the maximum of entropy. It is shown that the classical Boltzmann entropy leads to linear diffusions whereas quantum entropies require non-linear relativistic diffusion equations. We derive the relativistic diffusions as an approximation of an interaction with a random environment. We apply Kubo approach relating short time random dynamics with long time diffusion. In such an approach the diffusion generator is defined as an expectation value of the square of the Liouville operator. The relativistic invariance determines in the unique way the coefficients of the second order derivatives of the diffusion generator. We confirm this observation by an explicit calculation of the diffusion

generator in an environment of random electromagnetic fields (photons) and random gravitational waves. The coefficients of the first order derivatives depend on dynamics and are not universal. However, the principle of the detailed balance can determine the part of the diffusion generator which describes the approach to the equilibrium. We apply relativistic diffusion to astrophysics and cosmology. We approximate the effect of the interaction of an unknown matter (dark matter) by the relativistic diffusion using the fact that it is uniquely determined. The energy-momentum of the diffusion is not conserved. In order to preserve the energy-momentum we introduce the compensating energy-momentum of the fluid of the dark matter. The diffusion of massless particles in a homogeneous universe is exactly soluble. We treat inhomogeneities in a description of the structure formation as a perturbation of the homogeneous soluble case. We suggest that the structure formation in cosmology can be treated on the basis of non-linear relativistic diffusion in an analogous way as the well-known description of the structure formation in chemical reactions based on non-linear reaction-diffusion equations.

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## Transparently controlling chaos-assisted quantum transport and localization

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**Presentation: Oral**

The contradiction between chaos-assisted quantum tunneling and chaos-related localization has been found before some years. One hand, it is known that Anderson localization (AL) was related to disorder, and chaos can substitute for disorder in Anderson's scenario, leading to dynamical localization [1]. Such a dynamical localization was first directly observed in a system that is classically chaotic [2]. Recently, we further found that the high-chaoticity could replace higher disorder and aids AL in a driven optical lattice system. On the other hand, Lin and Ballentine have proved that the tunneling rate can be highly enhanced due to the periodic modulation associated with classical chaos [3]. Subsequently, many theoretical and experimental works demonstrated that classical chaos enhances quantum tunneling rate drastically [4], even without a barrier to tunnel through. Why can chaos promote tunneling and localization which are totally contrary phenomenon?

In this paper, we will give an answer to this interesting question. We study the chaos-related quantum transport of single particles in one-dimensional tilted optical lattices consisted of some amplitude-modulated laser standing waves and a time-dependent linear potential. We aim at applying the transparent control method based on the combined modulation and exact solutions [5] to investigate the dependence of the different chaotic characteristics on manipulating quantum transport for single particles in the optical lattices with any number of sites. Setting different combined modulations

to the lattice depth and tilt, we emphatically study the quantum transport crossing the chaotic sea of classical phase space. Combining analytical and numerical calculations, the previous perplexity on chaos-assisted tunneling or localization is cleared, namely chaos may aid or suppress quantum tunneling, depending on the selected parameter regions. Different parameter regions are shown in which chaos assists tunneling or aids localization. Several schemes for transparently controlling particle to rapidly transport and to localize at some given places in the phase space are provided, by adjusting the controllable parameters to change between the different regions. The results of single particles can be extended to some two-level many-body interacting systems, and could be applied to explore the corresponding manipulation schemes of quantum information transport and processing.

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### Decoherence in the quantum walk on the line with two entangled particles

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The discrete-time quantum walk, the quantum version of classical random walks, were first introduced in 1993 [1] and have since then been a topic of research within the context of quantum information and computation [2]. Some of the algorithms in quantum information has been shown to have a successful implementation using quantum walks. Several systems have been proposed as candidates to implement quantum walks. These proposals include atoms traps in optical lattice, cavity quantum electrodynamics and nuclear magnetic resonance in solid substrates. However, all these proposed implementations face the obstacle of decoherence due to environmental noise and imperfection. Decoherence in the quantum walks has been considered recently for example in Refs [3-5]. Numerical simulations of the effect of different kinds of measurements have shown that the quantum walk properties are highly sensitive to decoherent events. So far decoherence in quantum walk has been discussed with one particle. However the decoherence in a quantum walk with two or more entangled particles may contain rich physics more than single particle. Two particles in a system can be interacting or non-interacting. For more particles quantum dynamics will live in a composite Hilbert space. The particles can be considered separable or non-separable in Hilbert space. The quantum walk definition is very easy for separable systems but for non-separable systems may be entangled. In this case maximally entangled coins are considered to define discrete-time quantum walk process. In this study we consider a quantum walk on the line with two separable and entangled particles. Firstly we will give brief introduction for discrete and continuous time quantum walk formalism with one and multi-particles. Secondly we consider two unitary operators to

represent our quantum equivalence of a coin and then we have two walkers. There are many features that can be explored including the notion of superposition and entangled states. Entanglement can be studied in the coin degree of freedom as well as in the particle degree of freedom. In this part we will present numerical results of the decoherence in quantum walk for a several decoherence mechanisms such as absorption traps, broken links and different boundary conditions. We will discuss the decoherence for different coin operators and the results are compared.

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### A Monte Carlo Method for Two-Phase Flow in Porous Media

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The flow of a single fluid in a porous medium is well understood today. However, when this single fluid is replaced by two or more immiscible fluids, the situation becomes very complex and correspondingly less understood. For three decades, the instabilities caused by one fluid invading the pore space of another have been studied and a rich phase diagram has been revealed characterized by the wetting properties of the fluids with respect to the pore walls, their viscosities and the speed at which the invasion happens.

The situation when two immiscible fluids flow simultaneously in the porous medium has been much less studied. At very slow flow rates, typically one of the fluids will be held in place by capillary forces whereas the other one percolates and is then free to flow. At higher flow rates, both fluids will flow and the interfaces between them will change continuously. This simultaneous flow will reach steady state where the macroscopic quantities characterizing the flow will have well defined average values that remain unchanged or change slowly compared to the time scale associated with the flow at the pore level.

In Hansen and Ramstad (2009), it was proposed that a statistical mechanics describing steady-state flow of immiscible fluids in porous media could be built based on knowledge of the configurational probability, where configurations are defined by the distribution of interfaces between the two fluids in the porous medium. It was speculated that a Monte Carlo algorithm based on the configurational probability could be constructed which would be much more efficient numerically than time integration of the position of the interfaces.

Based on Sinha et al. (2013), we derive the configurational probability for two-phase flow in porous media. We find that it is inversely proportional to the inverse total flow rate in the porous medium.

We construct a Monte Carlo algorithm based on this probability and compare it to time integration of the same system. We find the Monte Carlo method to be significantly more efficient than time integration.

Lastly, we discuss the theoretical ramifications of the existence of a configurational probability for this system in terms of developing a pseudo-thermodynamics for the macroscopic variables describing the fluids, see e.g. Knudsen and Hansen (2002).

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## Digital Society and Economy 4.0 - The Ultimate Challenges for Complexity Science

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The digital revolution is changing human history. The invention of the steam engine turned agricultural society ("economy 1.0") into industrial society ("economy 2.0"), and wide-spread education turned it into service society ("economy 3.0"). Now, the invention of computers, the Internet, the World Wide Web, and Social Media are transforming service societies into digital societies ("economy 4.0"). With computers reaching the level of human brainpower in about 10 years, with intelligent service robots, and the Big Data tsunami, 50 percent of jobs in the industrial and service sectors will be lost within the next 20 years. And most of our current institutions will fundamentally change: the way we educate (MOOCs Massively Open On-line Courses and personalized education), the way we do research (Big Data analytics), the way we move (self-driving Google cars) or transport goods (drones), the way we go shopping (take Amazon and eBay), the way we produce (3D printers), but also our health system (personalized medicine), and most likely politics (participation of citizens) and the entire economy as well (with the makers community, the emerging sharing economy, and prosumers, i.e. co-producing consumers). Financial business, which used to be the domain of banks, is increasingly replaced by algorithmic trading, Paypal, Bitcoin, and Google Wallet, etc. Moreover, the biggest share of the insurance business is now in financial products such as credit default swaps. Even wars seem to change from conventional battlefield wars to cyberwars. How will this change our society? And what contributions can complexity science make to contribute to a better world?

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## How much opinion content do literary texts have?

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By studying the frequency of word usage, previous studies in different languages have reported evidence that words with a positive emotional content are more frequently used. Also, the correlation between word polarity and information content has been studied, reporting that positive words carry less information. These previous studies have used different corpora in several languages, mainly taken from Web-based content (online forums, blogs, Twitter, etc.). Our goal in the present work is to assess the opinion charge of a literary work as well as to looking for possible pattern after analyzing the statistics of the time series obtained after replacing each word in the text by its opinion polarity value. Therefore, we focus on single-author works, for which the writing dynamics clearly differs from the Web-based one. Thus, our approach supplements the previous studies based on a high number of authors in the opinion online forum, and then, this allows to compare the behavior of an ensemble of authors on shorter texts versus the longer texts produced by single authors under more scrutiny.

Our analysis is comprised of two approaches: (i) the study of the word frequency distribution (ranking of words) to compute an average value of the polarities and the objectivity; and, (ii) the substitution of every word in the text with its corresponding entries for opinion polarity in the SentiwordNet database (SWN), to compute for each series its empirical cumulative distribution functions (CDFs) to assess which polarity distribution (positive or negative) exhibits higher relative probability. However, since SWN provides polarity values for the different usage of the words and the information about the context is lacking, we develop an algorithm to compute the polarity values of a word considering the different cases provided in SWN.

The input data is comprised of 37 ebooks in English language downloaded mainly from the websites of the Gutenberg Project (<http://www.gutenberg.org/>) and the Project Gutenberg Australia (<http://gutenberg.net.au/>). There was not a particular strategy to select the titles, other than considering well-known works as well as some that have been considered polemic or which treated polemic topics; and the selected books were first published at different epochs, therefore giving diversity in time.

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## Entanglement generation is not necessary for optimal work extraction

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Energy storage and its subsequent extraction has always been a central topic of thermodynamics, due to its obvious fundamental and practical importance. On the other hand, entanglement is a key feature of quantum mechanical systems. It is one of the cornerstones of quantum information theory and it proved to be important in different physical phenomena as, to name a couple, quantum phase transitions or fractional quantum Hall effect.

In quantum thermodynamics, entanglement is also connected to work extraction from multipartite systems. Indeed, a classical result on work extraction is the no-go theorem called Thomson's formulation of the second law of thermodynamics: no work can be extracted from a thermal state via cyclic Hamiltonian processes. It is extended to finite quantum systems by introducing the concept of passive states, those not capable of giving out energy during any cyclic Hamiltonian process [1]. Thermal states are passive, while not all passive states are thermal. Indeed, several copies of states of the latter type may contain population inversions, allowing for some work extraction. On the other hand, thermal states do not share this ability of "activation" since any combination (tensor product) of thermal states is a thermal, thus passive, state. One can, however, get some work out of a set of locally thermal states by initially correlating them, the microcanonical state being a prominent example. But in all cases entangling operations are needed to extract the stored work [2].

On the other hand, that an entangling operation is needed for a process does not imply that entanglement is generated during its execution. The scope of this work is to clarify these connections.

We consider reversible work extraction from identical noninteracting quantum systems [3]. From an ensemble of individually passive states, work can be produced only via global unitary (and thus entangling) operations. However, we show here that there always exists a method to extract all possible work without creating any entanglement during the process, at the price of generically requiring more operations (i.e., additional time). We then study faster methods to extract work and provide a quantitative relation between the amount of generated multipartite entanglement and extractable work. Our results suggest a general relation between entanglement generation and the power of work extraction. Moreover, the task of population exchange is essential in other processes as diverse as dynamical polarisation of spins in NMR physics or operation of quantum thermal machines (see, e.g., [4,5]), etc., so our results have implications also beyond the problem of wo

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## The kappa-Weibull Distribution and Weakest-Link Scaling

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The times between consecutive earthquakes are referred to as earthquake recurrence intervals (ERI). The ERI probability distribution is a topic of continuing research interest, since it is a key element for earthquake risk assessment. Various probability distributions have been proposed as potential ERI models including the Gaussian, lognormal, Poisson, gamma, and the Weibull models. Some proposals are purely based on statistical evidence, others are obtained in the framework of numerical simulations, such as the epidemic type aftershock sequence (ETAS) model, whereas others are based on the principle of universality and notions borrowed from the theories of self-organized criticality and of critical phenomena. We present a different theoretical approach that develops a connection between the weakest-link scaling theory and the ERI distribution of tectonic earthquakes [1]. This is based on the fact that tectonic earthquakes result from the interplay of a driving force (plate motion) with mechanical defects (faults) on the Earths crust. The latter is composed of brittle material (rock), and the experimental evidence supports the Weibull strength distribution model. On the other hand, the Weibull distribution is typically obtained at the limit of a very large system (as measured in terms of links or representative elementary volumes RVEs).

We propose an extension of the weakest-link theory for finite-size systems [2] that leads to a generalization of the Weibull probability distribution into the so-called kappa-Weibull distribution [3]. The latter has a power-law right tail for finite-size systems, whereas it tends to the classical Weibull model for large systems. We discuss the impact of the power-law tail on the ERI hazard rate. We show that the power-law tail is able to better explain the tail of some data-based ERI distributions. We also show that the recurrence interval distribution of avalanches in simulated fiber bundle models is in good agreement with the kappa-Weibull model. We also present statistical analysis based on the Kolmogorov-Smirnov test [4] and the Akaike Information Criterion to investigate the agreement of empirical ERI distributions from Cretan and South California seismic catalogues with different theoretical models including the Weibull and kappa-Weibull models. We suggest that the kappa-Weibull distribution may also be relevant in the description of the strength statistics of quasibrittle materials.

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## Stochastic Local Interaction Model for Scattered Spatial Data

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We propose a Stochastic Local Interaction (SLI) model for incomplete (scattered) data in  $D$ -dimensional spaces. The SLI model incorporates concepts from statistical physics and machine learning. We use the idea of local interactions from statistical physics to impose correlations between neighboring points. From the field of machine learning we import kernel functions to implement local interactions between incomplete data distributed on regular grids or on irregular sampling networks. We employ kernel functions with locally adaptive bandwidth which is learned from the sampling spatial distribution [1]. The bandwidth is selected so as to allow mostly local contributions. The SLI model is sufficiently general for application in both the physical position space (i.e., in Euclidean space of dimension less than or equal to three) and higher-dimensional, abstract feature spaces provided a notion of distance is defined.

SLI is based on an explicit joint probability density function determined from the local interactions. The SLI model extends previous work on Spartan spatial random fields [2,3] which were based on a scalar field determined in continuum space to explicitly discrete sampling configurations. This is achieved by handling the irregularity of the sampling locations in terms of kernel functions with appropriately adjusted local bandwidth. The SLI precision matrix is explicitly derived from the local interactions and thus its calculation does not require the inversion of the covariance matrix, which is the main computational bottleneck for spatial models [4].

The prediction of missing data is based on maximizing the joint probability density function of the data and the prediction. The SLI model derives its computational efficiency from the local nature of the interactions. This allows for the likelihood function to be efficiently calculated and maximized. The model can be used for the interpolation and simulation of large data sets. In addition, it is possible to incorporate the SLI model in the Bayesian statistical framework. We will present applications of the model to different real and simulated data sets and we will investigate its performance using various statistical cross validation measures. We will also consider the role of the kernel function in the cross-validation performance of the model.

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## Functional relation between fluctuation and node degree in coupled stochastic dynamical systems

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Recently there have been growing interests for inferring the network structure from measuring nodal time series. The

source of these interests arises from many scientific areas. Examples are interacting proteins or genes, complex brain networks, ecological food webs, etc.

In this regard, recently Wang et al. (2009) showed that from collective dynamics of oscillator network system that in the presence of noisy, the time averaged fluctuation of each node scales with node degree in coupled stochastic dynamical systems. We extend the approaches for obtaining functional relation to the weighted network whose link weight is dependent on the node degree such as proportional to  $1/k^\beta$ , where  $k$  is degree of the node. The functional relation was measured in various network, such as Erdos Renyi random graphs, Watts-Strogatz small world networks, Barabasi-Albert scale-free networks, and configuration models, by using consensus dynamics, Rössler system, and Kuramoto oscillators. For the network with strong heterogeneity in degree distribution, we find that the theoretical result derived from the approaches in Wang et al. (2009) shows disagreement with numerical results especially in the case of when  $\beta$  is larger than 1 and configuration model irrespective of dynamical models. We found that this disagreement comes from neglecting fluctuation from first neighbor, which is marginal in previous studies. However, in our cases, the the contributed fluctuation from its neighbor becomes significant. In this respect, we propose novel approaches using the average of higher order moments and improve the accuracy of functional relation between noisy fluctuation and node degree by exploiting Fourier transformation. Our analytic results compensate previous disagreement significantly in lower degree nodes, and predict numerical results precisely. Also, we investigate the functional relation of noisy fluctuation versus node input strength. And more realistic situations where the dependence of interaction on degree are randomized, we can observe the influence coming from lower degree nodes, and confirm that this kind of fluctuation can give erratic results when one try to infer characteristics of network from nodal time series. We conclude that this novel approach should be considered when one inferring network from nodal time series, especially when the interaction between nodes are dependent on their degrees.

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## Geometry-induced superdiffusion and velocity anomaly in driven crowded systems

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Determining the dynamics of a driven tracer particle (TP) in a crowded medium constitutes a recurrent question of non equilibrium statistical physics. Examples of such situations include self-propelled particles in crowded environments, such as molecular motors or motile living cells. Another important application is given by active microrheology, which monitors the response of a medium whilst in the presence of a TP manipulated by an external force, and which is used to analyze different systems such as colloidal suspensions, glass

forming liquids, fluid interfaces or live cells.

Behavior beyond the force-velocity relation was recently addressed numerically by Winter et al., who studied via Molecular Dynamics simulations the dynamics of an externally driven TP in a glass-forming liquid.

It was recognized that whilst the TP moves ballistically, the variance of the TP position along the force direction grows surprisingly in a superdiffusive manner with respect to time. Very recently, we reproduced such superdiffusive fluctuations analytically in a simple discrete model in which the tracer performs a random walk biased by an external force, in a dense bath of particles performing symmetric random walks constrained by hard-core interactions. This description then relies on two paradigmatic models of statistical mechanics: the symmetric and asymmetric exclusion processes. In our model, the motion of the TP is mediated by successive visits of vacancies, whose density is denoted as  $\rho_0$ . It was found that in 2D and quasi-1D systems such as stripes or capillaries there exists a long-lived superdiffusion induced by the anomalous return statistics of the vacancies to the TP position, crossing-over to a diffusive behavior after a time  $t_\times$  which scales as  $1/\rho_0^2$ . These predictions were confirmed by numerical simulations, and were qualitatively extended to more complex continuous systems, such as Lennard-Jones particles or granular fluids. We also showed that the behavior of the mean itself of the TP position displays a striking anomaly in confined geometries, which apparently has been left aside up to now. Indeed, a scaled form of the mean-position is found which, very surprisingly, after a long-lived plateau drops to a lower ultimate value. The transition from the “high” velocity to the “low” velocity takes place after a time comparable to  $t_\times$ . However, it can be shown that the cross-over from superdiffusion to diffusion and the velocity anomaly are not controlled by the same criteria.

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### Thermal fluctuations and stability of a metallic particle levitated by the Casimir effect near a solid-liquid interface

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The Casimir force arising from a change in zero-point vacuum can be repulsive [1]. It enables levitation of a small metallic particle in liquid. For example, the Casimir force acting on a gold sphere immersed in bromobenzene can be repulsive near a silica plate. However, this levitation is not stable, because the Casimir force changes from repulsive to attractive as the separation decreases. In liquid, the position of a particle is constantly changed by bombardments from surrounding molecules, and the particle fluctuates about the equilibrium position. When the gold particle is very close to the silica plate, the Casimir force changes from repulsive to attractive, and the particle eventually adheres to the surface. Accordingly, the Brownian motion of a levitated particle is important in considering the stability of levitation. The distribution function of a levitated particle is highly im-

portant. However, calculating it is not easy. If the levitation height is considerably smaller than the particle radius, the proximity force approximation (PFA) is valid, and the Lifshitz formula between parallel plates, which has been used in many studies, is applicable. On the other hand, if the levitation height is larger than the particle radius, recently developed calculation methods such as finite-difference frequency-domain and geometric optics approximation for the configuration between a sphere and a plate are required. In this study, the scattering-matrix formulation is employed to determine the Casimir force [2]. The Casimir force is linearly proportional to the particle radius within the PFA, and the gravitational force is proportional to the cube of the radius. Thus, the levitation height decreases with the radius. The estimated levitation height, for example, is 82 nm for a 500-nm radius gold particle [3].

The time evolution of the particle distribution starting from an equilibrium position, where the Casimir force was balanced with gravitational force, is considered by solving the Langevin equation by using the Monte Carlo method. The levitated particle can move both upward and downward; however, because a potential barrier exists near the surface, the distribution is asymmetric, and the gold particle is more frequently found above the starting position. The escape rate from a metastable position is calculated by solving the Fokker-Plank equation. The Casimir force cannot be expressed by a simple function; therefore, we used the finite element method, in which the space is divided into a set of small segments and the distribution of particles is expressed by a linear combination of trigonometric functions. The duration of levitation increases as the particle radius increases up to approximately  $2.3 \mu\text{m}$ . As an example, we show that a  $1\text{-}\mu\text{m}$   $m$ -diameter gold particle can be levitated for a considerably long time by the repulsive Casimir force at room temperature. These numerically obtained escape rates agree well with the Kramers' escape theory.

The levitation of a nanoparticle can be realized not only near a liquid-solid interface but also near gas-liquid and liquid-liquid interfaces. Herein, we discuss the stability of quantum levitation near a liquid-liquid interface.

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### Financial regulations and bank credit to the real economy

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The modelling of interbank markets has typically focused on them in isolation. This leaves unanswered questions concerning the impact of financial regulation on real economic performance. We present a new agent-based model focusing on the linkage between the interbank market and the real economy with a stylised central bank acting as lender of last resort. Using this model we address the tradeoff between stability and economic performance for different structures of

the interbank market. We also explore the efficacy of recent regulatory changes using our richer model. The results of our model provide some support to the concerns raised by critics of the recent tightening of regulatory capital requirements under the new Basel framework. While low ceilings on leverage ratios can protect banks from idiosyncratic as well as systemic risk, they do have an anti-competitive effect which hurts borrowers in the real economy, especially in times when the demand for bank credit is high. At the same time, relaxed leverage ceilings can make banks particularly vulnerable to systemic failure in times when demand for bank credit is low, so while counter-cyclical leverage constraints, as proposed by some observers, might encourage the banking system to extend cheaper credit to the real sector it also exposes it to greater systemic risk especially when it is already vulnerable to such risk. Other results include the possibility that greater bank connectivity can have a non-monotonic effect on bank stability, first increasing the risk of contagion and then decreasing it. Bank prots (resp: defaults) tend to increase (resp: decrease) with interbank connectivity when leverage ceilings are low, to decrease (resp: increase) with connectivity when leverage ceilings are high and to be U-shaped (resp: inverse U-shaped) in connectivity under medium default ceilings. These relationships can be explained by taking into account the dual nature of the risk-sharing role of the interbank market, either as stabilising or a source of contagion, and then accounting for the effect of leverage on individual banks' riskiness. Under conditions of low leverage, individual banks are already relatively safe (especially when the real economy is not too small relative to the banking sector). Expanding interbank lending diminishes risk even further and makes them even safer. However, when banks are individually risky, then expanding interbank exposure can lead to contagion.

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### Mathematical model for hit phenomena and its application to analyze popularity of weekly TV drama

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A mathematical model for the hit phenomenon in entertainment within a society has been presented as a stochastic process of interactions of human dynamics in the sense of many-body theory in physics[1]. In the model, we write down the equation of intention of individual persons in the society as

$$\frac{dI_i(t)}{dt} = \sum_{\xi} c_{\xi} A_{\xi}(t) + \sum_j d_{ij} I_j(t) + \sum_j \sum_k p_{ijk} I_j(t) I_k(t) \quad (1)$$

where  $d_{ij}$ ,  $h_{ijk}$ , and  $f_i(t)$  are the coefficient of the direct communication, the coefficient of the indirect communication, and the random effect for person i, respectively. We consider the above equation for every persons in the society. The advertise-

ment and publicity effects are include in  $A_{\xi}(t)$  which is treated as an external force. The index  $\xi$  means the multi media exposure. The advertisement and publicity effects are mainly come from the broadcast of the drama itself and the recorded viewing are also considered using the digital TV system in Japan. In the model we use only the time distribution of advertisement budget as an input. Word-of-mouth (WOM) represented by posts on social network systems like blog or twitter is used as observed data which can be compared with the calculated results of the model. The unit of time is a day.

Using the mean field approximation, we obtain the following equation used in the calculation. The detailed derivation is shown in ref.[1].

$$\frac{dI(t)}{dt} = \sum_{\xi} c_{\xi} A_{\xi}(t) + DI(t) + PI^2(t) \quad (2)$$

Using this equation, our calculations for the Japanese motion picture market have agreed very well with the actual residue distribution in time[1]. The other example of the application of our model is the Japanese local events[2]. In this study we apply our mathematical model for hit phenomena for weekly TV drama in Japan. We calculate more than twenty weekly drama as example of analysis where we adjust the parameters for first three terms. For the rest of terms, we predict the intention of society for 4th to 10th term. The calculation agrees very well with the counts of the observed blog posting. Since the model is based on the stochastic process, it will be possible to extend the model to many other social phenomena in the real society. Therefore, the mathematical model of hit phenomena we presented will be applicable to many marketing problem where the time-dependence of the popularity is significant.

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### Network analysis of inter-firm payment flows using real monetary transaction records between firms

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To understand a firm's economic activity, it is important to investigate not only the firm itself but also the structure of interactions on an inter-firm network. We analyzed empirically network structures formed by monetary transactions between firms. The dataset we used is compiled from individual transactions in records of bank transfers in 2012, which is provided directly by three financial institutions in Japan, not via public open data supplied from the Bank of Japan. The data contains information on money flow (annual transaction volumes), and we can directly observe the money flow from a

buyer to a supplier. The network is consisted of 115,025 nodes representing firms and 2,149,120 directed edges representing transactions between firms. The directed edge stands for flow of money, and the direction of edge is from buyer to supplier. We found that distributions of indegree and outdegree follow power law distribution, showing directed scale-free networks. We also found that the amount of annual transaction volumes, which can be regarded as the weight of the edge, follows power law distribution. We calculated the average degree of the nearest neighbors and found that larger degree nodes tend to be connected with smaller degree nodes. This indicate that the network has a negative degree-degree correlation.

We can obtain the significant information of each firm by using the correlation between the PageRank and the financial evaluation of each firms. To evaluate financial condition of each firm, we can use two different measurements. One is to use published accounting and financial data, that is measuring the valuation of the firm itself. These measures are composed of accounting-based measurements (revenue, asset, debt, and so on) obtained from financial statements. Credit rating, the risk of default by non-fulfillment of an obligation of debt repayment can also be used as financial evaluation of each firm. The other is to use customer profitability data, that is a evaluation of the firm value as customer. Customer profitability is measured by using 36 months accounting data archived internally by the financial institutions, that is focal company of this study. We calculate customer profitability by summing up the each revenue excluding the expense associated with the specific customers over 36 months.

The low PageRank with high financial evaluation means that the firm should be connected to more firms in business and banks should support it. The high PageRank with low financial evaluation means the possibility of the chain bankruptcies around the firm.

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### Exact finite-size corrections and corner free energies for the $c=-2$ universality class

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We consider (a) the partition functions of the anisotropic dimer model on the rectangular  $(2M - 1) \times (2N - 1)$  lattice with free and cylindrical boundary conditions with a single monomer residing on the boundary and (b) the partition function of the anisotropic spanning tree on an  $M \times N$  rectangular lattice with free boundary conditions. We express (a) and (b) in terms of a principal partition function with twisted boundary conditions. Based on these expressions, we derive the exact asymptotic expansions of the free energy for both cases (a) and (b). We show that the exact asymptotic expansion for the free energy for these models can be written as

$$F = S f_{\text{bulk}} + 2N f_{1s} + 2M f_{2s} + f_0 + \sum_{p=1}^{\infty} \frac{f_p}{S^p}, \quad (1)$$

where  $S = M \times N$  represents the area of the lattice,  $f_{\text{bulk}}$  is the free energy per unit,  $f_{1s}$  and  $f_{2s}$  are the free energies per unit edge length in the horizontal and vertical directions respectively. All coefficients in the expansion (1) are expressed through analytical functions. In general, the coefficients  $f_{\text{bulk}}$ ,  $f_{1s}$  and  $f_{2s}$  are non-universal, but the coefficient  $f_0$  is supposed to be universal, depending only on the shape of the system and, possibly, the nature of the boundary conditions.

In some two-dimensional geometries, the value of  $f_0$  is known to be simply related to the conformal anomaly number  $c$  of the theory. Cardy and Peschel have shown that corners on the boundary induce a trace anomaly in the stress tensor. This gives rise to a term in  $f_0$  proportional to  $\ln S$ , where  $S$  is the area of the domain. Later, Kleban and Vassileva have shown that in rectangular geometry in addition to corner contribution proportional to  $\ln S$  the term  $f_0$  contains a term depending on the aspect ratio, namely, the term  $f_0$  contains the universal part  $f_{\text{univ}}$  given by

$$f_{\text{univ}} = -\frac{c}{8} \ln S + \frac{c}{4} \ln [\eta(q)\eta(q')]. \quad (2)$$

Here  $c$  is the central charge,  $q = \exp(-2\pi\xi)$ ,  $q' = \exp(-2\pi/\xi)$ ,  $\xi$  is the aspect ratio and  $\eta$  the Dedekind eta function.

We confirm the conformal field theory prediction for the corner free energy of these models (2), and find the central charge is  $c = -2$ .

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### The effect of censorship and community on emergence of monopoly

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Social networks can be described as networks with complex topology, in a sense that they are expected to exhibit a power-law behavior, if their development is according to the preferential attachment model of Barabasi-Albert. BA model assumes availability of nodes degree information for each new node introduced to the system. However, in real social networks, the nodes do not have complete information about the degree of other nodes. This is due to the fact that the origin of the information is not very clear. The information may root in either cultural reasons or government's obstruction of information flow (censorship). Another important player is the media or advertising systems. They can make someone look more powerful than they actually are. In this work, we are controlling these factors in our model with the rate of information diffusion ( $r$ ), where  $r = 0$  represents a random graph, and  $r = 1$  represents the Barabasi-Albert model. However in the general case, the diffusion factor can be smaller or larger than unity. We state that when information flow is obstructed ( $r < 1$ ) the topology of the network begins to change. In the extreme case where  $r \gg 1$ , scale-free networks tend to random networks. Hence, one could easily get the impression on how the media shapes the structure of social networks.

The results show a random graph  $r = 0$ , and a BA network  $r = 1$ , have the same clustering coefficient. The networks with

a low information rate  $r \sim 0.05$  have the highest clustering coefficient. This means that the censorship would make the people of the community become closer to each other. When in a community censorship is widespread, that community would not be controllable as expected. This means that the number of driver nodes that need to be governed in order to control the community becomes too much. This makes the process of control costly and expensive. By destroying the nodes and links, our results show that transparent networks would be in a more stable state. In the end, I focus on the collective effect of other nodes on the growth dynamics of the BA model. This is achieved by solving the BA equation in a fractional space, resulting in a characteristic time scale ( $\tau^*$ ). At times before  $\tau^*$ , the node degrees grow more rapidly, where at times after this threshold the collective effects slow down the node degree growth. To state clearer, at first the community helps elevate the people, but later after passing the threshold time, the community plays a negative role in the growth process of the community members. This is readily observed in reality.

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### Feynman checkerboard picture and neutrino oscillations

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In the first part of this talk I will explain some essentials of Feynman's checkerboard (also chessboard) picture (FCP). In its essence the FCP represents a relativistic quantum random walk in 1+1 dimensions. The latter was devised by R.P. Feynman in 1950's in order to describe an evolution of a massive Dirac particle and provide a novel explanation of the geometrical origin of spin. In mathematical terms the underlying (Euclidean-space) stochastic process for the FCP can be viewed as a specific form of a Poisson process in which the direction of motion and helicity are simultaneously changed at random Poisson distributed points. I will also point out a non-trivial (time-scale dependent) scaling of the underlying random walk.

After this prologue I will report on our recent works [1,2] where we elucidated how the checkerboard picture can be equivalently rephrased in the framework of superstatistics paradigm known from complex dynamical systems. I will namely argue that a doubly stochastic process with two vastly separated time scales (much longer and much shorter than particle's Compton wave length) can give rise to the genuine FCP when properly marginalized according to superstatistics prescription. In addition, the superstatistics framework allows for a straightforward generalization to 1+2 and 1+3 dimensions.

In the second part of my talk the Feynman checkerboard paradigm will be applied to two-(Dirac)fermion mixing [3]. From this, the phenomenologically observed neutrino oscillations will emerge as an important byproduct. The Euclidean formulation of the checkerboard picture will prove instrumen-

tal in guiding one's intuition about the underlying inner workings of neutrino mixing. In particular, the mixing can be envisaged as resulting from a probability bias of the transitions in the turning points between states with opposite helicities in the guiding Poisson stochastic process.

In the final part, I will present modifications needed in the FCP to include mixing of massive scalar (Klein-Gordon) particles and their oscillations. This will be also briefly discussed with the help of the Feshbach-Villars representation of the Klein-Gordon equation. Results obtained are pertinent for scalar or pseudoscalar meson oscillations, e.g.,  $\eta - \eta'$  oscillations.

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### Label Uncertainty, Irreversibility and Thermal Efficiency

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There are many physical situations where one has to infer under incomplete information. Statistical inference has emerged as a powerful tool in recent years with applications in many diverse areas of research, such as particle physics, cosmology and astrophysics, machine learning and so on. Inference or inductive reasoning is intimately tied with the epistemic or subjective interpretation of probability. This was also the earliest meaning that was ascribed to the notion of probability which was called by Bernoulli as the "art of conjecturing" or later interpreted as the degree of rational belief. In this presentation, we apply the problem of inference to the process of work extraction from two finite, constant heat capacity reservoirs, when the final thermodynamic coordinates of the process are not fully specified. First we discuss the case when the values of final temperatures have been specified but it is not known as to which reservoir a specific value refers to. This scenario is termed as label uncertainty. Given a pair of possible values, we find the estimate by invoking Laplace's principle of insufficient reason which assigns equal probabilities for the occurrence of different values, in the absence of any evidence to the contrary. The estimates for thermal efficiency indicate that the uncertainty about exact labels, reduces the maximal efficiency below the Carnot value, its minimum value being the well known Curzon-Ahlborn value. Thus we obtain the latter efficiency from a novel perspective. We map the inferred properties of the incomplete model to a model with complete information but with an additional source of thermodynamic irreversibility. In the second part, we make an average estimate of the efficiency, assuming this time that even the values of the temperatures are not available. This may be achieved by averaging over a uniform prior distribution. It is found that if the labels are known with certainty but just the values of temperature are unknown, then in the near-equilibrium limit the efficiency scales as 1/2 of Carnot value. On the other hand, if there is maximal uncertainty in the labels alongwith the uncertainty in value, then the average estimate for efficiency drops to 1/3 of Carnot value. The connection of these inferences with certain optimal results

from the finite-time models of heat engines will be pointed out.

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### Evolution equations for modelling the medico-demographic state in population

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This presentation surveys a collection of demographic models, that can be used for the statistics of the medico-demographic state of population. The models describing the time evolution of the medico-demographic indicators of population: population size, population age distributions, fertility, mortality, etc. The evolution of population is considered to be driven by birth-death processes, migration and deceases. For each models a respective set of evolution equations is formulated. These equations include kinetic parameters that have the meaning of the transition rates and whose values are linked to the characteristics of current medico-demographic state. We consider a number of evolution demographic models of different levels the lowest of which includes uni-gender population, where, in addition, newborns appear irrespective of the population distribution over ages. Then we introduce more sophisticated models that distinguish the ages of the members of community and their gender. Next we study the model of migration, which takes into account the exchange of population between several localities. We also consider the situations, where infections and deceases affect the medico-demographic state and thus the evolution of the population. Of special interest is the evolution model of the marriage-divorce process. In this case we show how to formulate the model in terms of the probability to find exactly a given number of families in a finite-sized population group. The steady-state solution to this equation is shown to be expressed in terms of special functions.

Then we describe a linear model of population dynamics. In this model the Malthus process of population growth is reformulated in terms of the probability  $w(n,t)$  to find exactly  $n$  individuals at time  $t$  assuming that both the birth and the death rates are linear functions of the population size. The master equation for  $w(n,t)$  is solved exactly. It is shown that  $w(n,t)$  strongly deviates from the Poisson distribution and is expressed in terms of Laguerre's polynomials. The asymptotic analysis of the distribution is presented.

After that we show how the parameters of the evolution models are linked to the standard medico-demographic indicators. The basic idea is to replace the standard medico-demographic set of characteristics by the set of constants entering the birth-death equation of the evolution models. We also demonstrate that the set of the models considered above is enough for modeling any medico-demographic situation. The results of our study are illustrated by some numerical examples.

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### Non-trivial discontinuous percolation model in cluster aggregation processes

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Recently interest of discontinuous percolation transitions (DPT) has been boosted by the cascading-failure model in multi-layer networks and the explosive percolation model with competitive dynamics. For the former model, the PT arises through fragmentation processes. The control parameter of this PT is taken as the number of intentionally removed links alone, which is different from the standard control parameter, the number of actually removed links including not only the number of links intentionally removed but also the number of links successively removed by cascading dynamics. Moreover, the order parameter is the fraction of nodes belonging to the mutually connected component in multi-layer networks. Due to this difference, the DPT can be induced. If the standard control and order parameters are used, then a continuous PT would be induced. For the latter model, the PT arises through cluster aggregation processes under the suppressive effect. Starting from a fixed number of isolated nodes, at each time step, a link is added between two unconnected nodes following a given rule so-called the Achlioptas process, which brings out the suppression effect against the growth of the largest cluster. This explosive percolation model shows a drastic PT in finite-sized system and thus was regarded as a DPT model, but after extensive researches, it turns out to be continuous in the thermodynamic limit. However, when the model is modified as the spanning cluster-avoiding model in Euclidean space, a DPT can take place, but it is trivial as the percolation threshold locates at unity and the order parameter is increased in all the way up to unity. Hence, it is timely demanding to introduce a non-trivial DPT that takes place at a finite threshold through the cluster aggregation process. Here, we introduce a stochastic model and solve analytically the increase of the order parameter using the Smoluchowski equation. Moreover, we provide the necessary condition under which a DPT takes place at a finite threshold, and by which we classify existing percolation models into several types, exhibiting continuous PT, trivial DPT, or nontrivial DPT. We anticipate this theoretical framework for the DPT to serve as a platform for further research on abrupt phase transitions in non-equilibrium systems.

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## Market bubbles and crashes from the point of view of statistical physics

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In the last few decades, the asset markets have been frequently visited by bubbles and the subsequent crashes. The increasingly frequent market crashes have attracted the attention of the general public. Although many academics, practitioners and policy makers have studied questions related to collapsing asset price bubbles, the questions, how asset bubbles come about, why it persists, and what causes a crash, have been the greatest myths. What is the origin of bubbles? Why are asset prices deviated away from fundamental value? In this talk we propose a new model to explore a mechanism of a bubble and its subsequent collapse. We consider a stock market that the two contrary types of investors coexist. One is arbitragers who invest based on their fundamental value they predict, whereas another is noise-traders whose investment is driven by expectations about what other investors predict, rather than expectations on their fundamentals. Our model shows that coexistence of the two contrary types of investors in the asset market is the key to understand a mechanism of stock market bubble and its subsequent crash. To model the interaction among noise-traders, Ising model is applied. The market price of the risky asset is determined by the noise-traders sentiment. Firstly, we demonstrate that as the conformity effect among noise traders (the extent that each noise-trader is influenced by the decisions of other noise-traders) is strengthened, noise traders begin to follow the herd, and the noise-traders herding behavior destabilize the risky-asset price, and the deviation of the risky asset price from the fundamental price has been enlarging in a long run. Enhancing the noise-traders bullish sentiment gives cause to a bubble, and their positive feedback trading prolongs bubble. Generation of bubble is described as second-order phase transition in Ising model. In the second half of bubble, run up of risky asset price come to an end as the noise-traders sentiment approaches to a limit of the bullish sentiment, that is, almost all the noise-trader s demand for the risky asset are buyers of the risky asset. For the noise-traders excess demand for the risky asset price is little or nothing. Thirdly, we demonstrate that decreasing the expected price momentum leads necessarily to market crash. The market crash is equivalent to the first-order phase transition. Finally, we demonstrate that after a crash, the noise-traders sentiment approaches to a limit of the bearish sentiment, the process of generating a bubble begins again.

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## Geometric aspects of a non-extensive entropy

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Relying on our previous work, we are searching for a local invariant in the configuration / phase space of a system of many degrees of freedom described by the Tsallis entropy

which can effectively describe the underlying dynamical behaviour.

For the case of the Boltzmann/Gibbs/Shannon entropy such a quantity is the Ricci tensor of the configuration / phase space. Relying on developments of the last fifteen years on optimal transportation, we suggest the use of the generalised Bakry-Emery-Ricci curvature for that goal.

In this framework, we provide an isoperimetric interpretation of the non-extensive (entropic) parameter.

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## Entropic forces between polymers and repulsive surfaces

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In a single molecule stretching experiment one end of a polymer is grafted to a flat stage while the other end is attached either to the tip of an atomic force microscope, or to a microneedle, or to a bead held by a magnetic field or by optical tweezers. When a complex biological molecule is strongly deformed its force-extension curve is dominated by the details of its energy landscape. However, for small deformations, and especially for homogeneous polymers, the entropy plays an important role. Recently, the accuracy of the experiments reached a point where deformations smaller than the equilibrium size of the polymer  $R_0$  and the corresponding forces can be measured. The number of configurations of a polymer is reduced in the presence of an obstacle, and the resulting loss of entropy produces repulsive forces. In the small deformation regime the polymer partially envelops the probe to which it is attached, and therefore the shape of the probe inevitably influences the outcome of the experiment. We examine the purely entropic contribution to force-displacement relations in such situations. The elastic response of a flexible polymer attached to the rounded tip of a probe with radius of curvature  $R$ , differs qualitatively for large and small values of the ratio  $s = R_0/R$ . The scaled compliance (inverse force constant)  $S/R_0^2$ , is anisotropic, and quite large in the direction parallel to the surface when  $s \sim 1$  [1]. When a polymer attached to a sharp (scale-free) tip approaches a plate, then for separation  $h \ll R_0$  the polymer-mediated force between the tip and the plate is  $F = Ak_B T/h$ . The amplitude  $A$  is universal and can be related to exponents governing the anomalous scaling of polymer correlations in the presence of obstacles [2, 3]. Its numerical value can be found from the solution of diffusion equation for ideal polymers, and by the means of  $\epsilon$ -expansion for polymers in good solvents. If one end of an infinite ideal polymer is held near a repulsive surface, then the monomer density is related to the electrostatic potential near a conducting surface of a charge placed at the point where the polymer end is held [4]. Pressure of the polymer on the surface is then related to the surface charge density distribution in the electrostatic problem. Pressure distribution near sharp tips resembles (but is not identical with) the behavior of an electric field near a conductor.

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### Hedging against Default Risk in a Heterogeneous Leveraged Market

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The existence of power-laws in finance and economics has widely been discussed since the seminal studies of Mandelbrot and Fama about the non-Gaussianity and the extremely anomalous “variability” of financial prices. A considerable amount of research in Financial Economics over the last twenty years has strengthened and expanded the links between (Financial) Economics and Statistical Mechanics. These links have taken the form of both theoretical and computational tools, such as heterogeneous agent-based models. The investigation of the latter class of models has allowed the study of emergent phenomena, which are impossible to understand within the rational representative agent framework. The heterogeneity of the interacting agents is a key feature in these models and in many cases is the source of the observed complexity [1-3].

Our work discusses the use of interest rate as a hedging tool against the default risk of heterogeneous hedge funds. To accomplish this, we use the framework of Thurner, Farmer and Geanakoplos, which describes a market with two assets (one risky and one riskless), HFs, noise traders, investors and a commercial bank. Here the heterogeneity stems from the different precision of the signal received from the market by each HF. Our model extends the one discussed in Ref. [4] by focusing on the survival statistics (failure function) of the HFs which are used by the commercial bank in order to charge an interest on the loans that it extends to the HFs. We show that the failure function of the HFs is qualitatively different when observed on the micro and the macro level. Specifically, the failure function of all HFs decays exponentially on the micro-level, indicating thus a constant default rate. However, on the aggregate level, we prove that the failure function tends to a power-law, with an exponent such that the variance becomes infinite (heavy-tail). This leads to the underestimation of the default risk of the best informed HFs, while the converse is true for the HFs receiving the least precise signal. Moreover, we show that there is a heterogeneous effect of hedging on the market, whereas the market as a whole becomes more unstable. Thus, hedging against default risk through charging interest rates, results into a feedback effect, increasing risk itself.

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### A universal mechanism for long-range cross-correlations

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Complex systems usually consist of several dynamical components interacting in a non-linear fashion. Cross-correlations are then used in order to explore the inter-dependence in the time evolution of these components measured in terms of specific quantities characterizing each component. Of special interest is the case of long-range (power-law) cross-correlations (LRCC), which, being scale-free, may be associated with the appearance of characteristics of criticality in the dynamics of the considered complex system. Such a behaviour has been observed in a broad range of systems, arising in disciplines as diverse as physics, biology and economics [1]. All these findings indicate that the presence of power-law cross-correlations is a quite general property of the dynamics of complex systems. Even more, very recently geometry-induced power-law cross-correlations have been also observed in a coarse-grained description of the dynamics of an ensemble of non-interacting particles propagating in a Lorentz channel [2]. This clearly poses the question of the origin and mechanisms of cross-correlations in particle systems.

Up to now the theoretical treatment of cross-correlations is based on statistical approaches and their microscopic origin is to a large extent unclear. I will present a dynamical mechanism leading to LRCC [3] and show specifically that intermittent dynamics, characterized by long intervals of regular evolution (laminar phases) interrupted by short bursts of abrupt evolution (irregular phases), obeyed by each component separately, generates LRCC between the different components, even if they do not interact with each other. It will be demonstrated that the emergence of LRCC is of geometrical origin: in a system with a large number of particles the ensemble of their intermittent trajectories forms a random fractal set in time. The two-point correlation function of this set can be identified with the cross-correlation function between intermittent trajectories of different particles, which appear in the set with probability one. In addition, I will provide strong evidence that a sufficient condition for the emergence of such scale-free LRCC is the divergence of the mean length of the laminar phase in the intermittent dynamics of each component. As a final remark, the emergence of power-law cross-correlations in the absence of inter-particle interactions is a very welcome feature in the context of critical phenomena, since it could explain universality aspects.

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## Computational Social Sciences: studies of in vivo sociality

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Over the past decade Network Theory approach has turned out to be a powerful methodology to investigate complex systems of various sorts by giving us through data analysis, modeling, and simulation a deeper insight into the structure, function, and response of these systems. In today's society human social interactions happen increasingly through ICT, the events of which leave behind digital "footprints", forming large datasets. Analysing such a dataset of mobile phone communication-logs gave us a proxy of social network that confirmed the Granovetterian picture for the network structure, i.e. being modular showing communities with strong internal ties and weaker external ties linking them. Yet the same dataset allows us to look at the nature of social interaction in more detail and from a different perspective, namely Dunbarian egocentric perspective, due to it having demographic data in the form of gender and age information of the individual service subscribers. With this we have got even deeper insight into the gender and age-related social behavior patterns and dynamics of close human relationships. Our analysis results demonstrate sex differences in the gender-bias of preferred relationships that reflect the way the reproductive investment strategies of both sexes change across the lifespan, in particular women's shifting patterns of investment in reproduction and parental care. These empirical findings inspired us to take the next step in network theory, namely developing models to catch some salient features of social networks and processes of human sociality in them. One of our first models, based on network sociology mechanisms for making friends, turned out to produce many empirically observed Granovetterian features of social networks, like meso-scale community and macro-scale topology formation. We are currently extending the modeling approach towards Dunbarian egocentric picture by including the fact that dyadic social interactions take place at different levels or contexts. To summarize we believe that the network theory approach to social systems combined with computational data analysis, modeling, and simulation, can open up a new and quantitative perspective for studying and even predicting collective social phenomena.

## Variational wavefunction for multi-species spinful fermionic systems - application to superfluids and superconductors

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We introduce a new fermionic variational wavefunction  $\Psi$ , generalizing the Bardeen-Cooper-Schrieffer (BCS) wavefunction, which is suitable for interacting multi-species spinful systems and sustaining superfluidity. Applications include quark matter, nuclei, neutron stars, superconducting grains, cold atoms, graphene, and the high temperature superconductors. E.g. in solid state physics, electrons in different bands with different dispersion relations and effective masses,

correspond to different fermion species.

A wide class of Hamiltonians, comprising interactions and hybridization of arbitrary momentum dependence between different fermion species, can be treated in a comprehensive manner. This is the case, as both the intra-species and the inter-species interactions are treated on equally rigorous footing. This is accomplished via the introduction of a *new quantum index* attached to the fermions. The index is consistent with known fermionic physics. Moreover, it allows to consider a multitude of heretofore unaccounted for fermion-fermion correlations. These correlations allow  $\Psi$  to take a *comprehensive account of the generic momentum dependence* of both the intra-species and the inter-species interactions.

We have derived the finite temperature version of the theory, thus obtaining the renormalized quasiparticle dispersion relations, and we discuss the appearance of charge and spin density wave (CDW/SDW) order. These two types of order may be present irrespectively of the existence of superconductivity in the system (opposite to what happens with more conventional treatments of CDW/SDW).

We present numerical solutions for two electron species in 2 dimensions. Based on these solutions, we show that, for equivalent spin up and down fermions, the Fermi occupation factor (per spin) equals 1/2 deep in the Fermi sea. This constitutes a unique experimental prediction of the theory, both for the normal and superfluid states. Interestingly, this result, obtained in the *thermodynamic limit*, is consistent with Fermi occupation factor inequalities for finite systems of electrons, derived (in a different context) [2,3].

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## 1/f noise from the nonlinear transformations of the variables

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The origin of the low-frequency noise with power spectrum  $1/f^\beta$  (also known as  $1/f$  fluctuations or flicker noise), observable for about eighty years in different systems, from physics to financial markets, still remains a challenge. Different models and theories have been proposed for explanation of this ubiquitous phenomenon. Recently, the stochastic model of  $1/f^\beta$  noise, based on the nonlinear stochastic differential equations

$$dx = \left( \eta - \frac{\lambda}{2} \right) x^{2\eta-1} dt + x^\eta dW_t, \quad (1)$$

where  $x$  is the signal with  $1/f^\beta$  spectrum,  $\eta \neq 1$  is the nonlinearity exponent,  $\lambda$  is the exponent of the steady-state distribution  $P_{ss} \sim x^{-\lambda}$ ,  $\beta = 1 + (\lambda - 3) / 2 (\eta - 1)$ , and  $W_t$  is a Wiener process (Brownian motion), has been proposed and analyzed [1, 2]. Eq. (1) may be derived from the point process model [1], from scaling properties of the signal [3] or from the agent-based herding model [4]. Note that special cases of Eq. (1) are: (i) an equation with the additive noise ( $\eta = 0$ ) and nonlinear drift, i.e., the Bessel process of order  $N = 1 - \lambda$ , (ii) the order  $N$

squared Bessel process when  $\eta = 1/2$  and  $N = 2(1 - \lambda)$ , (iii) special exponential restrictions of the variable  $x$  in Eq. (1) yield Constant Elasticity of Variance (CEV) process or (iv) Cox-Ingersoll-Ross (CIR) process (when  $\eta = 1/2$ ) [4]. On the other hand, we will show that processes with  $1/f^\beta$  spectrum may yield from the nonlinear transformation of the variable

$$x = 1/y^\delta \quad (2)$$

of the widespread processes  $y$ , e.g., from the Brownian motion, Bessel or similar familiar processes. Here we employ the self-similarity property of the nonlinear transformation (2) of Eq. (1) [5], relating the parameters of the equations (1) for  $x$  and for  $y$  variables:  $\eta_y = 1 - \delta(\eta_x - 1)$  and  $\lambda_y = 1 - \delta(\lambda_x - 1)$ , and  $\beta_y = \beta_x + (1 + \delta)/\delta(\eta_x - 1)$ . Therefore, from the  $N$ -dimensional Wiener process yielding the Bessel process with index  $\nu = N/2 - 1 = -1, -1/2, 0$ , and  $1/2$  as a result transformation of the variable (2) with the appropriate  $\delta = -1/2, 1/2, 1, 3/2$  and  $2$  one can obtain  $1/f^\beta$  noise processes.

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### Data-driven Seed Selection for Spread of Influence in Temporal Social Networks

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The problem of finding optimal set of humans for influencing others in the social network extracted from data about human social activities has been widely studied. Because it is NP-hard, some heuristics were proposed to find sub-optimal solutions. Still, one of the commonly used assumption is the one that seeds are chosen on the static network, not the dynamic one. This static approach is in fact far from the real-world networks, where new nodes may appear and old ones dynamically disappear in course of time. The main purpose of this research is to analyse how the results of one of the typical models for spread of influence - linear threshold - differ depending on the strategy of building the social network used later for choosing seeds. To show the impact of network creation strategy on the final number of influenced nodes - outcome of spread of influence, the results for various approaches were studied, both static and temporal with different granularities, i.e. various number of time windows. Social networks for each time window encapsulated dynamic changes in the network structure. Calculation of various node structural measures like degree or betweenness respected these changes by means of forgetting mechanism - more recent data had greater influence on node measure values. These measures were, in turn, used for node ranking and their selection for seeding. All concepts were applied to experimental verification on several real datasets. The results revealed that temporal approach is always better than static

and the granularity in the temporal social network strongly influences the number of finally influenced nodes.

Based on the analysis of datasets authors found that the behaviour of nodes is correlated with network measures. As a result, new theoretical model of preferential presence is introduced. It takes account of the correlation of network measures and the presence of a node in the dynamic network. The use of this model may simplify further work with artificial networks, since modelling the user presence still requires some preliminary assumptions and this model may make this assumptions more realistic.

Additionally, the research revealed that the stability of network measures may be used to determine the window size used for generation the temporal social network used for seeding and this window size may result with better outcomes of the spread of influence process.

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### Thermodynamics of bosonic systems in the canonical ensemble

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For many-particle quantum systems (bosons and fermions), calculating the canonical partition function is a very hard task. This is due to the difficulty to manage the indistinguishability of identical quantum particles. For this reason, the use of the canonical ensemble is avoided. As the grand canonical partition function is more tractable, thermodynamic quantities are calculated using the grand canonical ensemble. The results deduced from these two ensembles are supposed to be the same, at least in the thermodynamic limit. But during the last two decades, this equivalence has been subject to a renewed interest [1,2]. It has been shown first, that the microcanonical and the grand canonical ensembles are not equivalent for the calculation of the ground state population in the Bose Einstein condensate. Later, the equivalence between the microcanonical and the canonical ensembles has been studied deeply. In particular, it has been proved that these ensembles are equivalent only when the microcanonical entropy is a concave function of the energy. For systems of interacting spins, it has been shown that the equivalence is realized only for short range interactions. In a previous work [3], we have shown that statistical ensembles are not always equivalent, particularly for quantum systems with bounded energy spectrums: the relative discrepancy between canonical and grand canonical mean energies can reach 17% for such system, and this relative discrepancy does not vanish in the thermodynamic limit. For this kind of systems, the use of the canonical ensemble is then mandatory when the particle number is constant if we want to obtain precise results for thermodynamic quantities.

In this work, we present a study of thermodynamic properties of bosonic systems in the frame of the canonical ensembles (systems with fixed number of particles in contact with a thermal bath of fixed temperature). We consider bosonic systems with a bounded energy spectrum. We calculate the canonical partition function for a general spectrum with

energies  $\epsilon_i$  ( $i = 1, N$ ). We use this general form to obtain the partition function expression for systems with bounded spectrum of equally spaced energies. Then we determine the thermodynamic properties (mean energy and heat capacity) of these systems. We plot these quantities as functions of temperature. The heat capacity goes always to zero for both high and low temperatures. We notice that there is not equipartition energy theorem for bosonic systems when the energy spectrum is bounded.

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### Effect of potential barrier on correlation characteristics of steady-state Lévy flights in bistable potential

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The study of relaxation processes and anomalous diffusion in nonlinear dynamical systems subjected to the Lévy noise is of interest for several reasons. From the nonequilibrium statistical physics point of view such systems represent paradigmatic examples of non-Brownian processes in the systems far from equilibrium. Also, it has been demonstrated that such systems can be used as “minimal” models for the description of such diverse phenomena as anomalous transport in turbulent plasmas, abrupt climatic changes, population dynamics, etc.

Specifically, investigations of superdiffusion induced by Lévy noise in different potential profiles remain an open problem in statistical physics. Such type of anomalous diffusion is called Lévy flights and can be described by Fokker-Planck equation with the fractional space derivative. Some results in this area were obtained by analytical calculations and a lot of other only by numerical simulations. Analytical calculations of temporal characteristics of Lévy flights meet some difficulties because Markovian theory of first-passage times assumes the presence of some boundary conditions which are not so evident for discontinuous Markovian process having long jumps.

It should be emphasized that, unlike the normal diffusion, the stationary probability density functions for Lévy flights are bimodal, and, starting with the quartic potential, there is a confinement of Lévy flights, i.e., the variance of particle displacement is finite. In principle, one can find the correlation function and the power spectral density of Lévy motion in a steady state and such time characteristic as the correlation time.

Using the method previously developed in Refs. [1-2] and the form of stationary probability density function recently found in Ref. [3], from the fractional Fokker-Planck equation we obtain the exact equation to calculate the correlation time of steady-state Lévy flights in the bistable symmetric quartic potential. This equation can be solved exactly by Fourier transform only in the case of Lévy index  $\alpha = 1$  (Cauchy noise). As our analysis shown, the correlation time of stationary Lévy flights decreases with increasing both noise intensity and the steepness of a potential according to a power law. A comparison with the result for monostable quartic potential [2] is performed, and a role of potential barrier on

this temporal parameter is revealed.

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### Molecular dynamics simulation of self-diffusion of gases and liquids in conditions of phase transition

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Diffusion simulation of gases and liquids by direct numerical simulations methods has both theoretical and practical value. On the one hand it gives the chance to check up theories and hypotheses which are used in the physics of liquids and gases. On the other hand, it allows receiving new information about diffusion mechanisms in the conditions of vapor - liquid phase transition. Such information is important for studying of nanoparticles production from supersaturated vapor by vapor condensation [1, 2]. In this work, the self-diffusion coefficients in molecular systems of Lennard-Jones particles have been calculated by the molecular dynamics method. The time dependences of velocity autocorrelation functions of particles were calculated. Then the self-diffusion coefficients were calculated using the Green-Kubo formulae. The calculations were made in a wide range of density and temperature of the systems. Special attention was given to the simulations of the systems in the region of vapor - liquid phase transition. The time dependences of the velocity autocorrelation functions of molecules depended on density and temperature of the system. For low density systems exponential dependences of the autocorrelation functions were observed and the self-diffusion coefficients correlated with conclusions of the Chapman-Enskog theory. The autocorrelation function of diphasic systems consisted of two parts with strongly different relaxation times [3]. The first part was an exponential one. It was connected with the diffusion of the molecules in vapor. Another part corresponded to the molecular diffusion in liquid droplets. In homogeneous systems a universal dependence of the reduced diffusion coefficients (in relation to the Chapman-Enskog self-diffusion coefficients) on density was found out. Deviations from this dependence were observed in a phase transition region with the liquid droplets in the vapor or the vapor babbles in the liquid. Thus, the formation of the droplets in the vapor lead to decrease of the reduced diffusion coefficients, but the formation of the vapor babbles in the liquid leads to increase of the reduced diffusion coefficients in comparison with these in homogeneous systems under the same conditions. The comparison of the calculations results with experimental data on diffusion in gaseous and liquid argon has shown good agreement.

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### Studying interaction dynamics of chaotic systems within a non-linear prediction statistical method: Application to neurophysiology

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Many diseases of the nervous system, including epilepsy and Parkinson's disease associated with abnormal synchronization large groups of neurons in the brain. A sign of Parkinson's disease is the synchronization of neurons in the ranks of the thalamus and basal ganglia. Standard deep electrical deep brain stimulation has been found empirically, the mechanism of its effect has not yet been elucidated, and it has restrictions, such as those associated with side effects. Confirmation that the tremor caused synchronous neuronal activity in nuclei of the thalamus and basal ganglia, would presumably result in a softer therapies with fewer side effects. In this connection of the relevance of the problem of determining the nature of the links between different areas of the brain and the muscles of patients. Paper is devoted to an employing a variety of techniques for characterizing dynamics of the nonlinear neuro-physiological systems identifying the presence of chaotic elements. To analyze measured time histories of the neurophysiological system responses the phase space of these systems was reconstructed by delay embedding. The mutual information approach, correlation integral analysis, false nearest neighbour algorithm, Lyapunov exponent's analysis, and surrogate data method are used for comprehensive characterization. The correlation dimension method provided a low fractal-dimensional attractor thus suggesting a possibility of the existence of chaotic behavior. Statistical significance of the results was confirmed by testing for a surrogate data. We also present the concrete numerical results regarding the ensembles fluctuations of spontaneous Parkinsonian tremor of a few patients. Information about the activity of the brain was presented recordings of local potentials (LP) of the four deep electrodes implanted in the thalamus and basal ganglia. The data were obtained at the Department of Neurosurgery, University of Cologne and the Institute of Neurosciences and Biophysics, Juelich (Germany). Our results show that the time series is resulted from the low-dimensional chaos. The embedding dimension for the time series is  $dN = 6$ . Also, the correlation dimensions were calculated using the algorithm of Grassberger and Procaccia. Our data show that the Kaplan-Yorke dimensions, which are also the attractor dimensions, are smaller than the dimensions obtained by the algorithm of false nearest neighbours. The first two Lyapunov's exponents are positive for the time series under consideration.

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### Non-linear prediction statistical method to forecasting evolutionary dynamics of environmental systems: Atmospheric pollutants dynamics

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During the last two decades, many studies treated chaos theory with respect to various dynamical systems. It is naturally that an attention of urban ecologists was attracted the new methodology. Nevertheless, the studies of chaos that was applied to time series of atmospheric pollutants are few in number, and their outcomes are ambiguous. In a whole, the use of chaos theory and non-linear short-range forecast of atmospheric pollutants are in principle possible, but time series of air constituents are by no means always chaotic. The aim of our present study is (1) to identify the concentration space-temporary evolution dynamics for CO<sub>2</sub>, CO, NO<sub>2</sub>, SO<sub>2</sub> in the atmosphere of industrial cities (2) to reveal the possible chaos in the hourly time series at several sites in Amsterdam (The Netherlands), Gdansk (Poland) and Odessa (Ukraine) during the 2003-2009 (3) to forecast the concentrations of these pollutants using the non-linear prediction method (4) to present a new, efficient approach to quite exact forecasting the atmospheric pollutants evolutionary dynamics (new Geomath technology). To identify the chaos in the time series, we combine the methods as follows: (1) To determine time delays, the concept of mutual information is used; (2) To determine attractor dimensions, we apply both the correlation integral method and the false nearest neighbours algorithm; (3) To refine the obtained results, we use surrogate data sets; (4) We evaluate Lyapunov exponents as the dynamic invariants of chaotic system. In spite of the fact that the correlation integral method provides the relatively small attractor dimensions, both the surrogate data method and the false nearest neighbours algorithm assert that the more reliable  $dE$  for all datasets is 6. Such a value for the embedding dimension is comparatively large, but still indicates the presence of low-dimensional chaos in the studied time series. Also, two positive Lyapunov exponents validate the previous outcome. The sum of positive Lyapunov exponents is the Kolmogorov entropy which is in turn inversely proportional to the predictability limits. In our case, these limits vary from about three to four days. Using the results of previous analysis, we apply the non-linear prediction method and compare the predicted values with both last one hundred data and nine hundred random data in the series. As an example, the real and predicted concentrations of CO<sub>2</sub>, CO, NO<sub>2</sub>, SO<sub>2</sub> etc in Gdansk, Amsterdam and Odessa regions are presented. Our results can be considered as an first example of quite satisfactory short-range forecast for the air pollutants in the above cited industrial cities.

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## Critical analysis of holographic critical systems

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Quantum many-body theory has long been seeking to expand its toolbox of computational techniques, thus allowing one to describe and classify a broad variety of non-Fermi liquid states of strongly correlated fermions. Of a particular interest are the examples of emergent critical behavior, as well as spectroscopic and transport properties of the incipient  $s$ -,  $p$ -, and  $d$ -wave charge/spin density waves and orbital current-type instabilities in itinerant (anti)ferromagnets, quantum spin liquids, compressible ('composite fermion') Quantum Hall states, etc. Recently, the focus has also been on the  $d > 1$ -dimensional zero density ('neutral') Dirac/Weyl systems characterized by the presence of isolated points ('nodes') or lines ('arcs') of vanishing quasiparticle energy. The intrinsic complexity of these systems has long been recognized, prompting the use of such sophisticated techniques as renormalization group,  $1/N$ - and  $\epsilon$ -expansions, Keldysh functional integral and quantum kinetic equation, supersymmetric diffusive and ballistic  $\sigma$ -models, etc. In spite of all the effort, however, the overall progress towards a systematic classification of various 'strange' metallic (compressible) states that are often indiscriminately referred to as 'higher dimensional Luttinger liquids' has been rather slow. In that regard, the recently proposed notion of a (broadly defined) holographic correspondence could provide a sought-after powerful alternative technique. Specifically, its widely used 'bottom-up' version could potentially offer an advanced phenomenological framework for discovering new and classifying the already known types of NFL behavior. However, despite a recent flurry of applications of the broadly defined ('non-AdS/non-CFT') holographic correspondence to a variety of condensed matter problems, the status of this intriguing, yet speculative, approach remains largely undetermined. We discuss the previously made holographic predictions and advocate for a compelling need to systematically contrast the latter against the results of alternate, more conventional, approaches as well as experimental data. It is also proposed to extend the list of computed observables and utilize the general relations between them as a further means of bringing the formal holographic approach into a closer contact with the physical realm.

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## Complete set of phase transition natures in generalized heterogeneous $k$ -core percolation on random networks

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We study heterogeneous  $k$ -core percolation with general mixture of the threshold  $k$  with  $k_{min} \geq 2$  on random networks to find the complete set of phase transition natures in the mean-field level. Based on the local tree approximation, the scaling behaviors of the percolation order parameter,  $P_\infty(p)$ , are analytically obtained for general distributions of

the threshold  $k$ . The analytic calculations predict that the generalized heterogeneous  $k$ -core percolation is completely described by the series of continuous transitions with order parameter exponents  $\beta_n = 2/n$ , discontinuous hybrid transitions with  $\beta_H = 1/2$  or  $\beta_{A4} = 1/4$ , and three kinds of multiple transitions, which first show the continuous transition with  $\beta_1 = 2$  or  $\beta_2 = 1$ , or discontinuous hybrid transition with  $\beta_H = 1/2$  and later the hybrid transition with  $\beta_H = 1/2$ . To decide the universality class of a continuous transition precisely and to confirm the analytically suggested transition natures numerically, the simulations of the generalized heterogeneous  $k$ -core percolations are also carried out. Since heterogeneous  $k$ -core percolation considers only the maximal cluster, we use the fluctuation of the order parameter as the susceptibility  $\chi(p)$ . We use the finite-size scaling ansatz  $P_\infty(p, N) = N^{-\beta/\bar{\nu}} F[(p - p^*)N^{1/\bar{\nu}}]$  and  $\chi(p, N) = N^{\gamma/\bar{\nu}} G[(p - p^*)N^{1/\bar{\nu}}]$ , where  $p^*$  is the heterogeneous  $k$ -core percolation threshold. Using the finite-size scaling ansatz of the order parameter and the susceptibility, the critical exponents  $\beta$ ,  $\gamma$ , and  $\bar{\nu}$  of a continuous transition are obtained to compare with the analytically predicted transition natures, and the analytically predicted transition natures are confirmed by numerical simulations. Especially, the exponents of the series of continuous transitions are shown to satisfy the hyperscaling relation  $2\beta_n + \gamma_n = \bar{\nu}_n$ .

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## A path integral formalism for non-equilibrium Hamiltonian statistical systems

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In the past two years a new approach to non-equilibrium Hamiltonian statistical systems has been proposed and tested by the author and co-worker B. Turkington. This assumes that the non-equilibrium probability density for coarse-grain variables can be well approximated by an element of the manifold of quasi-stationary maximum entropy densities in the manner of Zubarev. When subjected to the Liouvillian operator such approximate densities are transformed to a density generally outside the approximating manifold. The discrepancy between such a transformed density and a new approximating manifold member can then be quantified using their relative entropy.

Summing a series of such discrepancies in time results in an action functional for any trial path through the manifold. Curiously the associated Lagrangian has the same form as that of a particle moving in a curved space with a metric tensor given by the Fisher information matrix and also in an external electromagnetic field whose form can be calculated easily from the fine-grain Hamiltonian dynamics and the quasi-stationary density manifold.

The path action described above was used by Turkington to define a variational principle for identifying optimal approximating densities at a fixed time. The method has been tested using direct numerical simulations by Kleeman

and Turkington in detail using a spectrally truncated Burgers-Hopf turbulence model. It gives excellent results for states moderately removed from equilibrium. In particular it reproduces well the equilibration behaviour of all spectral modes of the system.

In the talk to be presented the theoretical development outlined above is extended and it is argued that a path integral approach is required. A generalized Boltzmann principle is used to weight manifold paths with the exponential of the path action. The likelihood of a manifold member at a particular time is termed a consistency distribution and is analogous to a quantum wavefunction. Interestingly the Lagrangian is not in general of Onsager-Machlup form and this difference is fundamental to the equilibration process and derives from the informational discrepancy mentioned above. At large times when the system is close to equilibrium however the path integral approaches one of Onsager Machlup form. The associated thermodynamics then is interestingly of a generic Oettinger form. As is well known such non-equilibrium thermodynamical formulations work well for a large range of practical physical systems.

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## Quantum Annealing of Hopfield Model

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Quantum adiabatic annealing algorithm minimizes Ising cost function: the ground state of  $-\frac{1}{2} \sum_{ij} J_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z - \Gamma \sum_i \hat{\sigma}_i^x$  is closely tracked as transverse field  $\Gamma$  is brought down to zero from a large value. Success is guaranteed by keeping the rate  $\dot{\Gamma} \ll \Delta E \Delta \Gamma$  near the bottlenecks of the algorithm, where the instantaneous ground state is separated from excited states by a small gap  $\Delta E$ ;  $\Delta \Gamma$  is the width of the region where the gap has comparable values. Initially optimistic (polynomial) empirical scaling of complexity for Exact Cover problem [1] was found to cross over to exponential behavior for large problem sizes in subsequent studies [2]. We perform a rigorous analysis of the complexity of a simple model: a Hopfield network (with finite rank  $J_{ij} = \frac{1}{N} \sum_{k=1}^p \xi_i^{(k)} \xi_j^{(k)}$ ) with only  $p = 2$  Gaussian-distributed patterns  $\{\xi_i^{(k)}\}$ . Mean-field analysis finds second order phase transition at  $\Gamma_c = 1$  with critical exponents  $\beta = 1/2$ ,  $\gamma = 1$ ,  $z\nu = 1/2$ . Hyperscaling relations predict scalings of the gap at the critical point and the width of the critical region as  $N^{-1/3}$  and  $N^{-2/3}$  respectively; also confirmed with rigorous methods. ‘Spin glass’ phase ( $\Gamma < 1$ ) is analyzed as follows: Using path-integral formulation, low-energy spectrum maps to that of a quantum-mechanical particle in a random potential on a ring. The key to the complexity of algorithm is the evolution of this random potential with  $\Gamma$  and its classical limit  $\Gamma = 0$  in particular. This classical potential is analyzed rigorously in ‘thermodynamic’ limit  $N \rightarrow \infty$ , where it is described by a continuous random process and maps to Langevin dynamics in the vicinity of the global minimum. The classical random potential increases on average as  $x^{3/2}$  away from the

global minimum but exhibits infinitely many ‘dips’. Finite transverse field creates ‘quantum’  $O(\Gamma^{3/2})$  corrections which give rise to a series of global bifurcations. Consecutive ratios  $\Gamma_{n+1}/\Gamma_n$  approach a universal distribution as  $\Gamma \rightarrow 0$ . A finite system will contain  $O(\log N)$  bottlenecks with the gap given by the tunneling exponent  $O(\exp[-c(\Gamma N)^{3/4}])$ . These tunnelings correspond to flipping  $O(\Gamma N)$  spins and become progressively easier towards the end of the algorithm as a consequence of the form of classical potential. This contrasts with the phenomenological model conjectured in [3]. This general mechanism may be relevant for NP-hard Sherrington-Kirkpatrick model having a polynomial gap at the critical point [4].

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## Methodology for Deployment of Optimally Distributed Solar Monitoring Networks

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We propose an adjustable framework to select locations for solar measuring stations based on spatial feature coherence, statistical criteria of optimality, and epistemic-based stochastic knowledge synthesis. Specifically, first the Optimal Cluster Selection (OCS) methodology is applied to yield the optimum—in terms of clustering validity metrics—number of irradiance measuring stations (MS). OCS improves MS placement by determining a structured station network scheme on the basis of cluster analysis, and adheres to statistical coherence features [1]. This stage provides us with a selection of possible different ways of configuring MS within the emerged optimal clusters. Consequently, the OCS results are wired to a Bayesian Maximum Entropy (BME; e.g., [2]) engine which applies advanced geostatistical methodologies to determine the optimal spatial MS distribution from the set of possible ones.

Our clustering algorithm of choice is Adaptive Affinity Propagation [3]; we focus on the derived clusters of coherent solar variability in the domain of Southern California. In an initial step, we compute the irradiance at all MS locations by using the surrounding values of the satellite modeled data grid. Using stochastic prediction, irradiance is then computed on all grid nodes on the basis of the MS modeled irradiance data from the first step. In a third step, we compute the mean square error R between the predicted node values and the satellite-derived ones. Given all possible MS spatial configurations, the optimal candidate for adaptation is selected as the one with minimum R. Scoring by R enables selection of the immediately next optimal candidate in case a selection is unfeasible due to factors such as natural obstacles or anomalies.

We use solar irradiance data from cloud segmented images

derived from satellite observations. The long term mean absolute error of the modeled data has been found to be around 13% with regards to ground true measurements. In the presence of nontrivial data uncertainty, BME is employed to analyze this information in a stochastic space-time analysis framework. BME is uniquely known among geostatistical methodologies for its ability to incorporate rigorously both accurate and uncertain (soft) types of data, in addition to a variety of information types (e.g., [4]-[5]).

In overview, our work combines the strength of new and established cross-disciplined research to benefit renewable energy monitoring and generation. By going beyond the narrow boundaries of a specialized application, we aim to help specialists across the spectrum of the renewable energy industry for planning, decision-making, and policy implementations.

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## Controlling the Dynamics of Herding Dominant Financial Markets

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Social cooperation, emergence of social norms, formation of financial bubbles and sudden flash-crashes, or even mass panic may be caused by endogenous dynamics and certain general features of human behavior [1]. These phenomena cannot emerge from the concept of ideally rational agent. In actual socio-economic systems agents are boundedly rational and have to rely on other agents. This gives a rise to the importance of coupling and interactions between the agents, which suggests that it might be possible to influence the collective behavior of the socio-economic systems.

One of the suitable frameworks to test this idea is known as agent-based modeling [2]. This framework introduces a generalized concept of an agent, which is meant to be used in place of the interacting parts of the modeled system. Interactions between those agents are also generalized and simplified. After this simplification only statistically relevant, in terms of zero-intelligence or bounded intelligence, behavioral details are retained. Consequently the resulting models appear to be simple, but are able to reproduce certain sophisticated features observed in real-life systems.

There is a simple agent-based herding model proposed by Alan Kirman, which is based on the herding behavior in ant colonies, but is actively used to model socio-economic interactions. An interesting extension of this model is to allow certain agents to be controlled, their state being set externally. We have already examined their influence in a generic scenario [3] and now we test the extended model in the agent-based financial market scenario. In doing so we first consider two distinct views of the financial markets - long term

fundamental vs speculative trading dynamics and also short term buy vs sell dynamics. After considering the two different views we explore the model, where both dynamics are present. In these possible artificial financial market scenarios we test different collective behavior control strategies. Namely we use inflexible agents who occupy only predetermined state (similar concept was used by Galam and Jacobs in modeling opinion dynamics [4]) and random agents who switch the occupied state randomly (this strategy was used by Biondo and others in different artificial financial market setup [5]).

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## Simulating Faulty Quantum Search Algorithm With Quantum Walks

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Faulty diffusion quantum search, is a form of quantum search that results when the diffusion operator of the original Grover's search algorithm is, under some realistic conditions, modified by a randomizing unitary matrix. The aim of this work is twofold: first to show that the faulty search can in fact be cast into a form of quantum random walk (QW), and second, to investigate the consequences of this equivalence between quantum search and QW, especially concerning the physical aspects of the resulting equivalent model, and its computational capabilities. The main tool for achieving such equivalence is the so called oracle algebra. Oracle algebra is a  $SU(2)$  Pauli matrix like algebra, determined generically for any characteristic function (oracle), which specifies the marked items (vectors) in the database vector space. On the other hand the necessary mathematical proof is to show that the faulty search map is equivalent to a QW map, with its "walker" and "coin" systems coupled in the appropriate way, so that each search query to be equivalent to a QW step. This proof is in fact provided by first identifying the faulty search map with a positive trace preserving map on density matrix, and then showing that this map can be turned into a unitary transformation  $U$ , however acting on an extended Hilbert space. This is a 2D dimensional extension and amounts physically to the introduction of an auxiliary spin system. The extra spin system identified as being the "coin system, together with the initial quantum search system identified now as being the "walker" system of a QW, provide the quantum search-quantum walk equivalence, that was aimed. The quantum entanglement developed between these two systems provide the ground for the quadratic speed up of the diffusion rate for the QW or the analogous quadratic reduction of the search time in the equivalent search problem. Further the unitary transformation  $U$  describing effectively the QW in the extended space is shown to be generated by a Hamiltonian operator physically describing a multi-particle long-range interaction quantum mechanical system. This system effectively simulates the initial faulty search. In more detail, the ensuing QW describes a process of Brownian motion, driven by a quantum channel type of transformation

with two unitary Kraus generators, which are generating erratic rotations of Bloch vector around an axis. Search queries are equivalent to collisions in the QW picture, while quadratically less queries imply quadratically less collisions, until the state vector of the quantum "walker" becomes the marked vector of search algorithm.

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### The Impact of Heterogeneous Threshold Distribution on Cascades in the Threshold Model with Multiple Initiators

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It has long been known through empirical studies that in a population of socially interacting individuals where each individual node holds an opinion from a binary set, a small fraction of *initiators* holding opinion opposite to the one held by the majority can trigger large cascades and eventually result in a dominant majority holding the initiators' opinion. Some recent studies have investigated such phenomena in the context of the adoption of scientific, cultural, and commercial products. One of the simplest models that capture adoption dynamics, irrespective of context, is the threshold model (TM) [1-5]. According to the threshold model, an individual changes its opinion only if a critical fraction of its neighbors have already adopted the new opinion. This required fraction of new adoptees in the neighborhood is designated the *adoption threshold*  $\phi$ .

However, the threshold model (TM) with uniform threshold does not capture the complex nature of social influencing when multiple initiators are present. We found that for sufficiently large spread in the threshold distribution, the tipping point in the social influencing process (i.e. the fraction of initiators needed for global cascades [5]) disappears and crosses over to a smooth transition governed by the size of initiators. Specifically, we studied cascades in the TM when nodes are assigned a threshold value drawn from the normal distribution with varying mean threshold  $\phi$  and standard deviation  $\sigma$ . We analyzed both synthetic and empirical networks using different sizes of initiators. We observed a non-monotonic change in the cascade size for varying  $\sigma$  that for small initiator sizes follows Watt's cascade condition. In addition, we found that, unlike the case of uniform thresholds, for large enough  $\sigma$ , a critical initiator size beyond which cascades become global ceases to exist. Thus, there is an important qualitative change from a sharp transition to a continuous change in the behavior of the cascade size vs. the fraction of initiators.

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### A kappa-modified Schamel equation for ion acoustic waves in superthermal plasmas

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The presence of energetic particles in plasmas, resulting in long-tailed distributions, is an intrinsic element in many space and laboratory plasma observations. The spectral index, kappa ( $\kappa$ ), for which the kappa distribution is named, acts to modify the effective thermal speed in the distribution function. At low values of  $\kappa$ , distributions exhibit strong superthermality - by this we mean that there is an excess in the superthermal component of the distribution compared to that of a Maxwellian. At very large values of kappa, the distribution function approaches a Maxwellian distribution. Another commonly observed phenomenon in both space and laboratory plasmas is that of particle trapping, whereby some of the plasma particles are confined to a finite region of phase space where they bounce back and forth. In space, for example, electron phase-space holes in both the upward and downward current region of the aurora have been observed [1]. In addition, many laboratory observations of the free expansion of plasma into a vacuum have recorded the propagation of holes, solitons, and rarefaction waves. Particle trapping is a nonlinear effect, and was first included in analytical models of electrostatic structures by Bernstein, Greene and Kruskal (BGK) [2], and later Schamel [3] developed a pseudopotential method for the construction of equilibrium solutions, and also described the dynamics of ion acoustic waves modified by resonant (trapped) electrons.

Motivated by these observations, we have undertaken an investigation of the propagation of ion acoustic waves in nonthermal plasmas in the presence of trapped electrons, both analytically and numerically. An unmagnetized collisionless electron-ion plasma is considered, featuring a non-Maxwellian-trapped electron distribution, which is modelled by a ( $\kappa$ ) distribution function combined with a Schamel distribution. The effect of particle trapping has been considered, resulting in an expression for the electron density. Reductive perturbation theory has been used to construct a kappa-Schamel equation, and examine its behaviour. A solitary wave solution is presented and its dynamics discussed. The chief modification due to the presence of particle trapping is stronger nonlinearity, while enhanced superthermality affects the amplitude and width of solitons adversely.

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### Modelling electrostatic solitary waves and shocks in non-Maxwellian plasmas: soliton existence and propagation characteristics from first principles

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Space plasmas are often characterized by the presence of energetic particles in the background, e.g. due to various electron acceleration mechanisms [1]. This phenomenon is associated with a power-law dependence at high (superthermal) velocity values, modeled by a kappa-type distribution function, which reproduces observed data more efficiently than the standard Maxwellian distribution approach [2]. It has been shown from first principles that this ubiquitous superthermal feature of plasmas may alter the propagation characteristics of plasma modes, and modify the plasma screening properties [3, 4].

The effect of excess superthermality on the characteristics of electrostatic nonlinear plasma modes is reviewed, from first principles. A kappa distribution function [1] is employed to model the deviation of a plasma constituent (electrons, in general) from Maxwellian equilibrium. An excess in superthermal population modifies the charge screening mechanism, affecting the dispersion laws of both low- and higher frequency modes substantially. Various experimental observations or/and Space observation data may thus be interpreted as manifestations of excess superthermality [2]. Focusing on the features of nonlinear excitations (shocks, solitons), we [5] have investigated the role of superthermality in their propagation dynamics (existence laws, stability profile) and dynamical profile [3], via a series of theoretical and numerical investigations. Recent advancements are briefly reviewed. The relation to other nonthermal plasma theories is briefly discussed.

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### An actin fractal compartmentalizes the surface of mammalian cells

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A broad range of membrane proteins display anomalous diffusion on the surface of mammalian cells. This behavior can be explained in terms of transient binding with a heavy-tail distribution of sojourn times, viscoelastic properties of the membrane, and obstructed diffusion induced by a fractal structure. Several studies show that a variety of membrane proteins are organized by intracellular components. Evidence for barriers to diffusion has been provided using both single-particle tracking and optical tweezers, which show that transmembrane proteins undergo hop diffusion, between

periods of temporary confinement in nanoscale domains. Even though these methods provide strong evidence for obstructed diffusion, a fractal structure inducing anomalous diffusion in the cell membrane has never been visualized due to experimental challenges. In particular, the membrane compartments exhibit a dynamic behavior which contributes to the complex diffusion of membrane proteins, and the resolution needed to observe such structures is beyond the optical diffraction limit. In this talk, I will describe recent experiments where we image the actin cortex with 40 nm resolution in 2 second time frames for continuous periods over more than one minute, while we simultaneously track individual membrane proteins that interact with the actin cytoskeleton. These experiments are performed with actin binding peptides labeled with photo-activatable fluorophores and different potassium channels labeled with quantum dots in live human embryonic kidney cells. Our results using dynamic super-resolution imaging and image processing show that the actin cortex introduces barriers to the motion of membrane proteins leading to a compartmentalization of the plasma membrane and antipersistent effects observable in the protein trajectories. We find that plasma membrane proteins are transiently confined within domains defined by the actin cortex. Furthermore, our results show that the actin-induced compartments are scale free and that the actin cortex itself forms a self-similar fractal structure. Thus, we see that this structure introduces compartmentalization in time scales from milliseconds to several seconds and leads to subdiffusion over a broad time scale as expected from diffusion in a percolation cluster. These results present a breakthrough in our visualization of the plasma membrane and our understanding on how the actin cortex interacts with the cell surface.

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### Communities and classes in symmetric fractals

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Networks constructed to describe complex systems are analysed in terms of community structure or other properties which may give some useful information about connections between particular system elements. In the presented work we analyse selected examples of Lindenmayer systems [1], which display some kind of symmetry. Namely, we construct the networks for the Sierpinski triangle and the Koch curve. The network symmetry allows us to indicate groups of similar nodes (classes), which may be used instead of original nodes to represent the analysed system [2, 3]. Any two nodes belong to the same class if they have the same number of neighbours which belong to the same classes. In general, classes identification procedure takes into account not only the number of in- and out-degrees, but also weights of the links. Even though we take explicitly into account only nearest neighbours of each node, in fact the procedure preserves information about connections with further neighbours. It is the case as the class of any node depends on the classes of

its neighbours, which in turn depend on the classes of their neighbours and so on. The representation of the network by classes allows to reduce the system size, while preserving information on any static measure defined on the nodes. Analysis in terms of community structure is made using the method proposed by our group [4], which is based on the assumption that any two nodes are likely to belong to the same community if they share a common neighbour. In this method a set of differential equations is used to determine the time evolution of the weights of the links. The latter evolve due to interactions between nodes in the whole network. As a result we obtain a division of the whole set of nodes into communities. As a criterion to accept a division we use the calculated value of the modularity [5]. Yet the method of the community indication for analysed systems, causes the overlapping nodes being classified as singular communities. Such a situation does not reflect the network structure, as those nodes can be tightly connected with the remaining nodes of the system. To remove this fault of the method we add some noise to the connectivity matrix. Then, for the obtained matrix, we repeat the calculation several times. The obtained histograms indicate the frequency of nodes originally identified as forming singular communities being connected with remaining communities of the system. The subsequent analysis of the obtained histogram profiles allows for a reconstruction of the connections between communities. Also, our method allows for proper identification of communities which differ in size. The question we address

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### What are the main drivers of the Bitcoin price? Evidence from wavelet coherence analysis

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Bitcoin is a potential alternative currency to the standard fiat currencies (US dollar, the Euro, Japanese Yen, etc.) with various advantages such as low or no fees, a controlled and known algorithm of the currency creation, and an informational transparency of all transactions. Of course, where an upside is there usually is a downside as well. During its increasing popularity and public attention, the Bitcoin system has been accused and labelled as an environment for organized crime and money laundering, and it has been a target of repeated hackers attacks causing some major losses to the bitcoins owners. However, it needs to be noted that none of these downsides is completely remote to the standard cash currencies either.

Here, we focus on various possible sources of the price movements, ranging from fundamental to speculative and technical sources, and examine how the interconnections behave in time but also at different scales (frequencies). To do so, we utilize continuous wavelet analysis and specifically the wavelet coherence which is able to localize correlations between series, its evolution in time and across scales. It

needs to be stressed that both time and frequency aspects are important for the Bitcoin price dynamics as the currency has undergone a wild evolution in recent years and it would be naive to believe that the driving forces of the prices have remained unchanged during its existence. We show that indeed both time and frequency characteristics of the dynamics are worth the investigation and various interesting relationships are uncovered.

First, even though Bitcoin is usually labelled as a purely speculative asset, we find that standard fundamental factors – usage in trade, money supply and price level – play a role in Bitcoin prices in the long term. Second, from the technical standpoint, the increasing prices of Bitcoin motivate users to become miners. However, the effect is found to be vanishing in time as the specialized mining hardware components have driven the hash rates and difficulty too high. Nonetheless, this is a standard market reaction to an obvious profit opportunity. Third, the prices of bitcoins are driven by investors' interest in the crypto-currency. The relationship is most evident in the long run but during the episodes of explosive prices, the interest drives the prices further up, and during rapid declines, it pushes them further down. Fourth, Bitcoin does not seem to be a safe haven investment. And fifth, even though the USD and CNY markets are tightly connected, we find no clear evidence that the Chinese market influences the USD market.

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### Optimal bounds on critical or tricritical points of nonlinear globally coupled systems with additive noise

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We consider an infinite array of globally coupled overdamped anharmonic oscillators subject to additive Gaussian white noise introduced in [1], which is closely related to the mean field  $\varphi^4$ -Ginzburg-Landau model. We prove the existence of a well-behaved critical manifold in the parameter space which separates a symmetric phase from a symmetry broken phase. Given two of the system parameters there is a unique critical value of the third [2]. There are no re-entrance transitions. Due to the global coupling a mean field description is exact. In the stationary state the mean field is determined by the stable fixed points of a self-consistency map (SCM) [3]. The phase transition condition can be expressed in terms of the first derivatives of the SCM. Derivatives of the SCM are, up to a factor, equal to cumulants of the probability distribution. Showing that the kurtosis is negative we prove that the critical control parameter  $a_c$  is bounded by its limit values for weak and for strong noise. In these limits the mechanism of symmetry breaking differs. For weak noise the distribution is almost Gaussian and the symmetry is broken as the whole distribution is shifted in either the positive or the negative direction. In this case the kurtosis reaches

asymptotically zero. For strong noise there is a symmetric double-peak distribution and the symmetry is broken as the weights of the peaks become different. The qualitative behavior is the same for higher order saturation terms  $\varphi^n$ , where  $n \geq 6$  is even. The critical point is calculated for strong and for weak noise, these limits are bounds for the critical point. Introducing an additional, competing, nonlinearity leads to richer behavior. Then the parameter space divides in three regions, a region with a symmetric phase, a region with a phase of broken symmetry and a region where both phases coexist. The region of coexistence collapses into one of the others via a discontinuous phase transition whereas the transition between the symmetric phase and the phase of broken symmetry is continuous. The tricritical point where the three regions intersect, satisfies an additional condition on the third derivative of the SCM. Exploiting inequalities on the cumulants we find optimal bounds for the tricritical point which are assumed in the limits of weak and of strong noise.

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## Two interdependent binary opinion networks

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We have proposed a model, which is a generalisation of our previous work ( see, for details Refs[1,2], where two binary social networks are strongly interacting. We study a time evolution dynamics of such a complex network, which may describe the process of the presidential election, where two party, A and B are competing. The complex network consisting of two sub-networks, A and B. The network A consists of agents who are voting for the party A, while the network B of agents voting for the party B. During the election process both agents, A and B, have a chance to influence the vote of the chosen agent. As the result of this interaction the agent makes a choice to vote for the party A or for the party B. The time evolution consists of many steps, where at each step all agents of the network have a chance to make their choice coming out of their interaction with other social agents. The process is continued until a some fixed point is reached.

We show that the evolution of such two competing parties system or two interdependent binary networks is well described by some class of models usually used to simulate strongly correlated electrons. In terms of these models the giving votes for the party A or the party B corresponds to the spin up or spin down flipping of orientations of the electrons which may sit on the nodes of the networks. Specifically we have considered the case when the nodes of the network are occupied or not. We fixed the connection between the nodes of the network and in general considered an arbitrary cases where there are K connections. Specifically, for more details we have focused on the case when each node is connected to three other nodes. We determined what is important to win the election campaign and define the criterion of the winning. It is mostly related to the initial stands of the votes before the election started and the type of the interaction between the

agents. However it is important to note that there is a strong diversity of opinions in such a complex networks although in many cases the result of the election is well predefined by the initial stand of the votes.

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## Temporal $\lambda$ -peak in 'dragon-king' dynamics on a real-life financial market

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For one-and-half decade, physicists have been intensively studying structural and topological properties of complex networks in order to understand the mechanisms responsible for the evolution of real-world complex systems and their miscellaneous consequences. Arguably, one of the most exclusive among these, is the condensation phenomenon, together with a  $\lambda$ -transition (or temporal  $\lambda$ -peak) associated with this phenomenon – which hitherto have never been observed in a real-world network. By the term 'temporal  $\lambda$ -peak' we understand here the temporal shape of a short-range order parameter resembling the Greek letter  $\lambda$ . This terminology is in analogy to the  $\lambda$ -peak of the heat capacity vs. temperature formed by the  $\lambda$ -transition between the normal I <sup>4</sup>He and superfluid II <sup>4</sup>He components. Here, we demonstrate the first evidence for such a real-life condensation phenomenon with an associated  $\lambda$ -transition, together with a preceding phase of nucleation growth. Furthermore, we here investigate in empirical and phenomenological ways, a complete phase diagram for these intriguing dynamic phase transitions in real-world complex networks. In other words, we fill a void in merging empirical and phenomenological characterisation of the dynamical phase transitions in complex systems by identifying three of them on real-life financial markets. We extract and interpret the empirical, numerical, and semi-analytical evidences for the existence of these phase transitions, by considering the Frankfurt Stock Exchange (FSE), as a typical example of a financial market of a medium size. Using the canonical object for the graph theory, i.e. the Minimal Spanning Tree (MST) network, we observe: (i) The initial phase transition from the equilibrium to non-equilibrium MST network in its nucleation phase, occurring at some critical time. Coalescence of edges on the FSE's transient leader is observed within the nucleation and is approximately characterized by the Lifshitz-Slyozov growth exponent; (ii) The nucleation, which accelerates and transforms to the condensation process forming, in the second phase transition, a logarithmically diverging  $\lambda$ -peak of short-range order parameter at the subsequent critical time; (iii) The third phase transition, where the peak logarithmically decreases over three quarters of the year, resulting in a few loosely connected sub-graphs. The  $\lambda$ -peak, resembling the continuous phase transition from the normal fluid I <sup>4</sup>He to the superfluid II <sup>4</sup>He, is reminiscent of a non-equilibrium superstar-like superhub or a 'dragon-king' effect, abruptly accelerating the evolution

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### Empirical symptoms of catastrophic bifurcation transitions on financial markets

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Discontinuous phase transitions in complex systems (much as in liquid-gas systems) together with critical phenomena are topics of canonical importance in statistical thermodynamics. During the evolution of the complex systems we observed various catastrophic breakdowns preceded by flickering phenomenon. This type of evolution is an example of a generic problem how small changes can lead to dramatic consequences. Overall, in this work we developed various metrics associated with catastrophic bifurcation transitions. All these things were considered on a well-defined example of notoriously significant and unpredictable financial markets. The significant contribution to explanation of mechanism of financial market evolution, in particular the settlement of whether the early-warning signals are visible there, is a generic challenge of our work. Notably, the problem of whether the early-warning signals in the form of a *critical slowing down* phenomena (observed in multiple physical systems) are present on financial markets, was clearly formulated in [1]. Nowadays, as well critical as catastrophic slowing downs seem to be the most refined indicators of whether a system is approaching a critical threshold or a bifurcation catastrophic tipping point, respectively [2]. The principal aim of this work is the evidence on empirical way that catastrophic bifurcation breakdowns or transitions, preceded by flickering phenomenon, are present on so much unpredictable financial markets. Overall, in this work we developed various metrics associated with catastrophic bifurcation transitions, in particular, the catastrophic slowing down (analogous to the critical slowing down). All these things were considered on a well-dened example of financial markets of small and middle to large capitalisations. The catastrophic bifurcation transition seems to be connected with the question of whether the early-warning signals are present in financial markets. This question continues to fascinate both the research community and the general public. Interestingly, such early-warning signals have recently been identified and explained to be a consequence of a catastrophic bifurcation transition phenomenon observed in multiple physical systems, e.g., in ecosystems, climate dynamics and in medicine (epileptic seizure and asthma attack). We provide an analogical, positive identification of such phenomenon by examining its several different indicators in the context of a well-defined daily bubble.

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### Can we estimate turbulence energy dissipation rate from PIV measurements?

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Understanding how kinetic energy is dissipated in turbulent flows has been a great challenge for many years and would have important implications in many areas such as fundamental research, aeronautics or industry. Recent advances in imaging have now made it possible to access to the three velocity components in a plane or in a small volume of a fluid, up to scales of the order of 1mm. This scale is 10 to 100 times larger than the dissipative scale in large Reynolds number (Re) experiment. Estimate of the energy dissipation rate is then possible using techniques borrowed from Large-Eddy-Simulation (LES) techniques. In the present contribution, we test these methods in a turbulent flow generated by two contra-rotating impellers (Von Karman flow) for Reynolds number ranging from 100 up to more than  $10^6$ . This closed flow geometry allows direct estimate of the energy dissipation by torque monitoring at the two impellers. We compare these measurements with dissipation estimates based on LES techniques using velocity field measured through stereoscopic particle image velocimetry (PIV). We are then able to compute the energy dissipation rate using its definition in terms of spatial derivatives of the coarse grained velocity fields, and to compare it with direct measurements of the injected energy rate over more than 6 decades in  $\log(\text{Re})$ .

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### Brain network modular structure of two opposite temperament groups in dimensions of novelty seeking and harm avoidance

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Introduction: R. Cloninger [1] has confirmed that psychobiological model of personality to account for individual variations in temperament and character. Temperament dimensions were originally proposed to be independent of on another. However, a meta-analysis by Miettunen et al. (2008) [2] found a significant negative correlation between temperament traits, especially significant negative correlation between harm avoidance (HA) and novelty seeking (NS). Based on this negative correlation, the current study revealed the whole-brain connectivity modular architecture for two contrasting temperament groups. Here we analyzed modular architecture for the functional and morphometric brain network and measured similarity between two groups as well as between functional and morphometric network. Subjects and Methods: 40 healthy volunteers (mean age = 25.23.3 years) took part in this experiment. High resolution

structural image and resting state fMRI were acquired. The Korean version of the TCI (including 140-items) was used to assess temperament and character factors. To classify 40 subjects into two groups, we used k-means clustering algorithm with HA and NS as input vectors. Average functional networks for each group were constructed for studying modular architecture. The positive edge weights of the group-averaged functional network matrix formed by Pearson's correlations were subjected to the graph theoretical modularity estimation algorithm, which characterizes the segregation of a network [3]. Similarity measure defined by normalized mutual information was used to compare the modular architecture between two groups.

**Results and Conclusion:** Using the k-means clustering algorithm, 19 subjects were clustered into a group having high HA and low NS, 21 subjects were clustered into a group having low HA and high NS. In the functional network module analysis, we found that the overall network modular organizations showed a similar pattern except for sub-network structure among the prefrontal cortex (PFC), basal ganglia (BG), and limbic system. In high HA and low NS group, the regulatory brain regions, such as the PFC, are clustered together with the limbic system. This finding suggest that the neural basis of inhibited, passive, and inactive behaviors in the high HA and low NS group was derived from the increased network association between the PFC and limbic clusters.

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### Thermostatistics of near-field thermal radiation and nanoscale energy harvesting

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Thermal radiation heat transfer in the near-field regime is considerably enhanced as compared to the blackbody limit. That is, the amount of energy exchanged between bodies separated by a submicron distance is notably higher than that for bodies separated by macroscopic lengths. The tunneling of evanescent electromagnetic waves is responsible for this enhancement, an effect that can rise only when the bodies are close to each other.

Due to the contribution of evanescent modes, the local density of states is modified near an interface separating two media. This implies that also the thermodynamic functions will depend on this contribution and will show a very different behavior from the corresponding one in the far field case. Furthermore, at the interface of polar materials, the coupling of phononic excitations with the electromagnetic fields results in the so-called surface phonon polaritons. When two planar sources supporting surface phonon polaritons are placed at a distance smaller than the thermal wavelength, the resonance of these modes is responsible for the considerable increase of the emitted radiation. These surface waves can be thermally excited at the nanoscale due to their existence in the infrared. Using the fluctuation-dissipation theorem, the correlations of electromagnetic fields can be computed, whence the average Poynting vector is obtained. The Poynting vector gives the

radiated energy flux, and from this quantity the entropy flux can be computed. Both energy and entropy fluxes depend on the spectral flux of modes, which contains the information about optical properties of the materials.

Here, we show that the maximum work that can be obtained from the thermal radiation emitted between two planar sources in the near-field regime is much larger than the one corresponding to the blackbody limit. This quantity as well as an upper bound for the efficiency of the process are computed from a thermostatic formulation in the near-field regime. The case when the difference of temperatures of the hot source and the environment is small, relevant for energy harvesting, is studied in detail. We also show that thermal radiation energy conversion can be more efficient in the near-field regime. Both maximum work flux and efficiency depend on the optical properties of the materials, and the explicit dependence on the frequency of the surface phonon polariton has been obtained.

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### Network Analysis of 3D complex plasma clusters in a rotating electric field

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Network analysis was used to study the structure and time evolution of driven three-dimensional complex plasma clusters [1]. The clusters were created by suspending micron-size particles in a glass box placed on top of the rf electrode in a capacitively coupled discharge [2]. The particles were highly charged and manipulated by an external electric field that had a constant magnitude and rotated uniformly in the horizontal plane. Depending on the frequency of the applied electric field, the clusters rotated in the direction of the electric field or remained stationary. The three-dimensional positions of all particles were measured using stereoscopic digital in-line holography.

The network approach was used to elucidate the structural changes in the cluster. The Analysis revealed an interplay between two competing symmetries in the cluster. Spherical and cylindrical ordering of the particles was examined by comparing network measures of the experimental data with null models. The well established network measures local connectivity, clustering coefficient and average path length were considered [3]. The null models were artificial data with a certain number of points in perfectly spherical order, and the rest in cylindrical order. Network analysis of the clusters showed that the rotating cluster was more cylindrical than the nonrotating cluster.

These findings were in agreement with the radial confinement computed by assuming a dynamical force balance. The radial

confinement was shown to be stronger in the case of cluster rotation, increasing the cylindrical of the cluster.

The emergence of vertical strings of particles was also confirmed by using a network analysis. While the traditional method of a fixed threshold has limitations such as erroneously including passing by particles and a somewhat arbitrary threshold, community finding algorithms [4] yield a more elegant approach of finding structures in complex systems. With the aid of *multislice networks* [5], it is possible to examine the whole time series at once and thus resolve the time evolution of the strings.

We demonstrated that network analysis is a powerful tool to analyze the structure of complex plasma clusters and may have numerous applications in other complex systems where the characterization of the spatial structure plays a vital role.

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### Population Of Simple Cognitive Agents Their Model And Performance In Dynamic Environment

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A cognitive agent is an abstraction of an autonomous entity capable of interacting with its environment and other autonomous entities. Such agent is capable of performing cognitive acts consisting of a sequence of the following activities: (1) perceiving information in the environment and provided by other agents; (2) reasoning about this information using existing knowledge; (3) judging the obtained information using the existing knowledge; (4) responding to other cognitive agents or to the external environment; (5) learning, i.e. modifying the existing knowledge based on the newly acquire information.

The goal of our work is to identify and discuss an example that could be used as an experimental platform in virtual reality to identify a simple cognitive agent unable to express concepts analytically and unable to use crisp values. In the development of the model of our cognitive agent, called naive creature, we pursue the route of biomimicry and steer away from formal methods and established learning algorithms. Our developed naive creature may experience fear and/or desire, is capable of evaluating if a strategy has been applied successfully and is capable of applying this strategy again with small changes to a similar but new situation. We describe how a simple agent born as *tabula rasa*; i.e. a blank slate, can be trained to learn how to safely cross a road; i.e. a unidirectional single-lane highway and later, when the situation changes, unidirectional multi-lane highway and bi-directional multi-lane highway. The highways are implemented as a modified Nagel-Schreckenberg model and each creature is provided with mechanism to reason to cross safely the highway but it does not have built-in knowledge base of its environment. The population of creatures as the result of the learning process builds this knowledge base. We have chosen a learning mechanism that belongs to what is called social observational learning, that is each agent learns

from the behaviour of other agents.

In the presented work we investigate how fear and/or desire, creature mobility along the edge of the highway, car traffic intensity, presence or absence of erratic drivers, may affect the creatures population success in learning to cross safely the highway. We investigate how the fact of acquiring a prior knowledge in one environment helps the population of creatures to learn to be more successful in another environment. We present selected simulation results, their analysis and outline future work.

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### Modeling the space plasma microstates with Kappa distribution functions

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The in-situ measurements of particle velocity distributions in the solar wind and terrestrial magnetosphere indicate that these space plasma systems are out of thermal equilibrium, even in the absence of beams and temperature anisotropy. This feature is revealed by the dual structure of the velocity distributions of all species of plasma particles (electrons, protons and heavy ions), always containing two principal components, a low-energy core well described by a standard Gaussian (Maxwellian) distribution function, and suprathermal (high-energy) tails, forming the halo populations and well fitted by the Kappa power-laws. Though the thermal core populations are dominant, kinetic effects of the highly anisotropic halo cannot be neglected. Collisions are indeed rare in space plasmas, but the long-range correlations between plasma particles are ensured by the plasma waves and instabilities as the principal mechanisms of relaxation, whenever plasma deviates from thermal equilibrium.

However, the attempts to predict the wave dispersion and stability properties are limited to simplified models, which either (a) consider the core of Maxwellian type and ignore suprathermal populations, or (b) minimize the core assuming this component cold and model suprathermal tails with Kappa power-laws, or (c) incorporates both the core and halo populations in a single (global) Kappa that is nearly Maxwellian at lower energies, and decreases smoothly as power-law at high speeds. Implying a reduced number of macroscopic plasma parameters used to describe the moments of the distribution as well as their time and space evolution, the idealized models are convenient for calculations, but worthwhile is to assess to which extent these models can explain the wave fluctuations and turbulence measured in space plasmas.

Here it is proposed a comparative analysis of the anisotropic models and the resulting wave instabilities. The anisotropic models include bi-Maxwellian distribution functions for the core populations, and a variety of anisotropic Kappa-type distribution functions for the energetic halo. These models have been introduced to parameterize the kinetic anisotropy, e.g., temperature anisotropy, beams (strahls), which are believed to be at the origin of wave fluctuations observed in space plasmas.

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### Chaos in ergodicity

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The author has developed the ergometric theory of the ergodic hypothesis[1]. It is a physical theory, differing from classic mathematical theories, by actually taking the time average over the autocorrelation function defined by linear response theory. The time dependence of the autocorrelation function is obtained formally by the recurrence relations method that had been earlier established and successfully applied[2]. The ergometric theory yields a physical condition called an ergometer by which a variable of a Hermitian system is ruled ergodic or not. Illuminating is information on physical mechanisms behind why some variables are ergodic and others are not.

The prior existing general theories of the ergodic hypothesis are all purely mathematical on classical phase functions by Birkhoff, Khinchin and others. They do not take the time average (as the physical theory does) but seek conditions for which the ergodic hypothesis can hold. These conditions are abstract math properties e.g. the transitivity of phase space, the existence of the time average. The thermodynamic limit never appears.

Although the ergometric theory is a quantum many-body formalism, it applies to classical ergodic theory by taking the classical limit. It allows one to compare it with the classical math theories. We have shown that ergodic or not ergodic by the ergometric theory is the same as by the abstract math conditions[3]. What has remained unexplained however is an important abstract condition that the time average is over a single trajectory which starts "almost" everywhere from an ergodic surface. That is, why just one trajectory and why not everywhere.

According to the theorem of Sharkovskii and also of Li Yorke, the existence of 3-cycle in a continuous map on an interval implies the existence of chaos. By analytically solving the 3-cycle problem in the logistic map, we have exhibited chaos[4]. In this study we find that a chaotic trajectory has exactly the same abstract properties that characterize an ergodic trajectory. Hence we can conclude that chaos is in ergodicity. Additional studies show that the converse does not necessarily holds.

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### Biofilaments as Annealed Semi-flexible Copolymers

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Biofilaments were long considered as structures Worm-Like Chains. In many in vivo or in vitro situations, however, biofilaments manifest some annealed heterogeneity and should be considered as annealed random copolymers. The building blocks of the filaments differ from each other, for example, by the internal structure of the monomer, by the presence of some adsorbed species or by the curvature. Based on the copolymer concept, we embed the description of these systems in a common formalism. Analogous to the propagator introduced by Edwards for Gaussian chain, we introduce the Schrödinger type equations for the filament. For the description of annealed heteropolymer with two switchable monomer states, we generalized the spinor representation introduced by Grosberg. We demonstrate how the annealed heterogeneous nature of the filament is reflected by statistical correlations like the tangent-tangent correlation function or the cyclization probability.

In particular, we demonstrated a number of consequences of the annealed copolymer filament switchable between two different states of stiffness. We showed that a properly averaged persistence length  $l_p^{eff}$  successfully describe thermal fluctuations which are dominated by small deflections. The end-to-end correlations of the annealed heteropolymers markedly deviate from that of the simple worm-like-chain. Further we found that the simple exponential decrease of the tangent correlation recovered for large filaments is not governed by the effective persistence length  $l_p^{eff}$ . The latter effect is most pronounced when the aspect ratio of the persistent segment is moderate and it is huge effect for most synthetic polymers. This originates from switches to shorter persistent segments (type-1) in the large scale random walk regime, which increase entropy. Systematic deviations from the WLC arise at large deflections. The tail of the angular deflection distribution is much larger and the closure probability can be dramatically enhanced. These effects reflect a profound alteration of the copolymer statistics to better accommodate the constraint.

Our results show that annealed filaments adapt cooperatively to external constraints. This contributes to explain anomalous elasticity manifested by biofilaments.

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### A New Molecular Dynamics Simulation Method of the Constant NPT Ensemble

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The traditional molecular dynamics simulation methods are for microcanonical ensemble systems that has conserved energy, and volume, and number of particles. However, a majority of experiments on physicochemical systems are carried out at constant temperature and pressure, in which

the energy and volume of the system fluctuate. To examine the dynamical properties of molecules in such systems, one often employ the molecular dynamics simulation methods with thermostatting and barostatting algorithms. Unfortunately, the well-established thermostatting algorithms such as Anderson thermostat, Berendsen thermostat, Nos-Hoover thermostat, as well as the barostatting algorithms such as Anderson barostat, Berendsen barostat, and their variants, are known to alter the dynamic properties of the molecules under consideration, because their equations of motion are modified by the coupling with the thermostat or the barostat. To circumvent this artifact, we propose a new molecular dynamics simulation method that disturb only the molecules near the wall of the simulation box. A similar thermostatting algorithm was proposed a long time ago by Ciccotti and Tenenbaum, who introduced boundary conditions to simulate thermal walls at a given temperature. However, they used reflecting boundary conditions and thus their method was influenced more severely by the size effect. As for the barostatting algorithm, our method has some similarity with that of Uline and Corti in that the volume fluctuation is controlled by a shell particle. We test the efficiency of the our algorithm in attaining the target temperature and pressure, and the conformity of the velocity, kinetic energy distribution, and other dynamic properties to the constant NPT ensemble. Although our method is not so efficient as the Berendsen thermostat and barostat in attaining the target temperature and pressure, it is at least as efficient as the Nos-Hoover thermostat and the Anderson barostat. Once the target temperature and pressure are reached, our method maintains the constant NPT condition perfectly. It is well known that Berendsens methods do not produce the simulation results conforming to the constant NPT ensemble. Therefore, one may use the Berendsens methods in the initial phase of the equilibration run, and thereafter employ our method to obtain reliable simulation results that conform to the constant NPT ensemble system and do not alter the dynamic properties as well as the equilibrium properties.

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### Manna universality class is distinct from the directed percolation class

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Recently, Basu *et al.* showed in one dimension that the known models in the Manna class, i.e., the discrete and continuous conserved Manna models, exhibited the critical behavior of absorbing phase transitions similar to that of the directed percolation (DP) when generated from the initial states prepared from the steady states by sweeping all particles by one Monte Carlo step to the nearest-neighbor sites [1]. Based on the numerical analysis, they claimed that an independent Manna class does not exist and all models known to belong to the Manna class were expected to show a DP critical behavior after a very long time.

We here studied the discrete conserved Manna model (referred

to as the Manna model) and the conserved lattice gas (CLG) model both on a square lattice, and the Manna model on a one-dimensional chain and on a strip of  $L_x \times L_y$  sites with  $L_x \gg L_y$ . It was found that the Manna model and the CLG model on a square lattice yielded consistent results within an error of less than 1% and that the exponents characterizing the dynamics of active particles differ considerably from the known exponents obtained using the random initial states while those associated with the steady-state quantities remain the same. The critical exponents for both models satisfied the known scaling relations and, thus, the known violation of the scaling relations for models with a conserved field was resolved by using the natural initial states. The results were also found to differ by 7%–12% from the corresponding DP values.

For the Manna model on a chain, the critical density of particles was found to be smaller than the recently reported value and, accordingly, the order-parameter exponent was considerably different from the DP value. With the estimates of the decay exponent and the order-parameter exponent, the off-critical scaling held nearly perfectly, and the finite-size scaling also held with the dynamic exponent inconsistent with the DP value. The influence of quenched disorder on a strip of  $10^5 \times 20$  sites was also studied and the results were compared with those of the prototype model in the DP class, i.e., the contact process (CP). It was found that whereas the CP showed the Griffith phase in which the active-particle density decreased following a nonuniversal power law, the Manna model showed a standard critical behavior. These results consistently suggested that the Manna universality class is a unique and independent class which is distinct from the DP class. This is in strict disagreement with the recent observation in Ref. [1].

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### Turbulent Diffusion and Pair Dispersion of Bright Points in the Solar Photosphere

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The study of diffusion and transport of magnetic field concentrations within the turbulent solar convection zone and atmosphere is very important for the understanding of several fundamental solar physics processes, such as dynamo effect, magnetoconvection, and energy release processes in the atmosphere. In this context, the motions of local magnetic flux concentrations and magnetic bright points represent one of the main sources of information. These magnetic elements represent the solar atmosphere signature of strong (kilogauss) magnetic flux tubes and undergo random walk motions driven by turbulent convection. These motions have been studied extensively as a diffusion process and in this framework they can be investigated through the analysis of the scaling properties of mean square displacements and

pair mean squared separations. The turbulent diffusion and pair dispersion of magnetic field concentrations in the solar photosphere is studied here through the analysis of bright points motions in observations of the solar granulation obtained with the New Solar Telescope of the Big Bear Solar Observatory. A quiet-Sun area, a coronal hole, and an active region plage are investigated. In all the three magnetic environments, evidence for the presence of a super-diffusion regime is observed. It is also found that the pair mean squared separation  $\Delta^2(t)$  follows a power law time scaling  $\Delta^2(t) \sim t^\eta$ , with the power law index  $\eta \simeq 1.5$ . It is suggested that these results can be explained in the framework of the Batchelor theory, under the hypothesis that the range of observed time scales corresponds to a non-asymptotic regime in which the photospheric magnetic bright points keep memory of their initial separations. Further studies of the pair dispersion of photospheric bright points, performed on longer time series, can provide information about the magnetic field transport in the solar atmosphere. Moreover, the diffusion properties of local magnetic field concentrations are closely related to the dynamics of solar atmosphere turbulence. The physical processes analyzed in the present work represent one of the possible ways to investigate the efficiency of the turbulent energy transfer to small scales, which may play an important role in the energy dissipation processes occurring in the upper atmospheric layers.

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### Emergent structures and effective thermodynamics in systems of active particles.

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In many instances of active matter, the relevant units which constitute the system are self-propelled particles: they transform energy from their environment into motion. Self-propelled particle systems are manifold in biology and appear at very different scales, ranging from bacteria to animal flocks. Only very recently, artificial active liquids made of Janus colloids have been realised in the laboratory. The large number of available results suggest that it might be some 'universal' features concerning the collective behaviour of active matter. Recent investigations have shown that different self-propelled particle systems share several generic features (clustering, phase separation, large fluctuations, etc.), hence motivating their study from a statistical mechanics perspective. In order to discuss how the introduction of self-propulsion affects the structure and dynamics of particle systems, I will introduce an extension of the hard-sphere model where particles perform a persistent random walk. I will describe the stationary phases of this 'minimal model' over a broad range of parameters. I will show that purely repulsive hard disks spontaneously self-aggregate into fractal clusters as self-propulsion is increased and eventually form a percolating one (gelation). This clustering can be rationalised by a solvable kinetic model of reversible aggregation. I will show that the addition of a finite amount of noise is needed to trigger a non-equilibrium phase separation. I will discuss the steady-state dynamics of the active liquid and show how

the Stokes-Einstein relation is broken even at low densities. The diffusion constant has a non-monotonic behaviour and the analysis of time correlation functions show that the dynamics are strongly heterogeneous. Then, I will show that self-propulsion induces an effective short-range attraction by making connections between the active system and a model of attractive particles. I will use this analogy to interpret the numerical data obtained close to the out-of-equilibrium glass transition that occurs in dense self-propelled particle systems. I will also discuss the possibility of defining an effective temperature from the fluctuation-dissipation ratio in this regime. Finally, I will compare my predictions with experimental studies of self-propelled colloidal suspensions and discuss which thermodynamic concepts can be extended to this non-equilibrium situation.

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### Undamped periodic oscillations in many-body systems

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It is well known (see [1, 2, 3]) that a system consisting of an arbitrary number of particles of equal masses interacting via an arbitrary potential of homogeneity degree  $-2$  and confined by an isotropic harmonic potential can sustain undamped periodic compressional oscillations. Since such a system has, in the general case, a chaotic dynamics, one might expect that it would always eventually reach thermal equilibrium. However, it turns out that these undamped compressional oscillations are *adiabatic*, and that correspondingly, the temperature varies with the size of the system (in the specific case stated above, this dependence is one of inverse proportionality). Since thermal equilibrium presupposes a constant temperature, it follows that such systems never attain it. A straightforward consequence of this behaviour is that such systems must have vanishing bulk viscosity. In this talk, we review and generalize these findings to more general confining potentials and extend the results further to time-dependent systems. In particular it is possible to describe thermodynamic cycles of such systems. This study is not restricted to adiabatic variations, so we are in the position to look at a great variety of different behaviours. It is seen, for example, that such standard "irreversible" processes, as free expansion, do not in fact lead to an increase of entropy in such systems and can be explicitly reversed. It can however be shown that the second law cannot be violated in those cases where the model can be studied explicitly, in spite of the fact that approach to equilibrium does not take place. We further discuss the concept of thermodynamic equilibrium for such systems. The underlying motive of these features is the existence of a hidden non-abelian symmetry. There is a conservation law apart from energy, the value of which must be specified in any description of the system. However, additional macroscopic quantities, which do not commute with the Hamiltonian, then generally have values which vary over a macroscopic range. These values must be specified to

describe the state, but this can only happen destroying the stationarity. It is readily seen that the structures defined above carry over to quantum mechanics, so that similar results can be stated for the corresponding quantum mechanical systems. In the case of harmonic confinement, the expectation value of the radius performs undamped oscillations, whereas in more general cases a more complex behaviour is found.

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### Effective-medium theory and simulations of under-coordinated random lattices with weak bending interactions

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Concepts associated with the rigidity percolation transition of random elastic networks have been used in many branches of science, such as amorphous solids, granular materials, mineralogy, and the mechanics of the cytoskeleton of living cells. Archetypes of this transition are crystalline lattices with elastic interactions, where a random variable is assigned to the probability  $\mathcal{P}$  of existence of each bond in the network. For small enough  $\mathcal{P}$ , the shear modulus vanishes due to local floppy regions that prevent rigid regions from percolating. There is a phase transition at  $\mathcal{P} = \mathcal{P}_c$ , above which the shear modulus is different from zero. For network glasses,  $\mathcal{P} = \mathcal{P}_c$  might also mark the transition from an amorphous solid (with high average coordination) to a polymeric glass (with low average coordination) [1]. Homogeneous honeycomb and diamond lattices with only nearest-neighbor interactions are under-coordinated, in the sense that their average coordination number  $z$  is smaller than the isostatic limit of mechanical stability  $z_c = 2d$ , where  $d$  is the spatial dimension [2]. They have floppy modes, whose zero frequencies span the entire Brillouin zone. In this work, we use numerical simulations and an effective-medium theory to study the rigidity percolation transition of the diluted honeycomb and diamond lattices when weak bond-bending forces are included. We construct bending potentials that are a manifestly rotationally invariant combination of displacement vectors, with no dependence on stretching. As a result, our model predicts a sensible bulk modulus that does not depend on the bending stiffness. We show that both shear and bulk moduli vanish at the rigidity percolation transition point. From the effective-medium theory, we extract scaling and asymptotic relations for the behavior of the elastic moduli in the limits of small disorder, and near the rigidity percolation critical point. For small disorder, these results are in good quantitative agreement with the numerical simulations. We also find overall qualitative agreement. Our findings are specially relevant to characterize the transition of covalent glasses from amorphous solid to polymeric glass. We also discuss generalizations of this model when next-nearest-neighbor bonds are randomly added.

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### Systemic price cojumps

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Instabilities in the price dynamics of a large number of financial assets are a clear sign of systemic events. By investigating high cap stocks traded in different stock exchanges, we find that there is a large number of multiple cojumps, i.e. minutes in which a sizable number of stocks displays a discontinuity of the price process. We find strong evidence on individual stocks that jumps are clearly not compatible with a Poisson process description, since we measure that a jump changes the probability of observing another jump in the near future. We are able to describe this time clustering feature by modelling the jump series with a class of self-exciting point processes, namely the Hawkes processes. We introduce a one factor model approach where both the factor and the idiosyncratic jump components are described by a Hawkes process. We introduce a robust calibration scheme, which is able to distinguish systemic and idiosyncratic jumps. A series of questions naturally arises, about whether the advent of High Frequency Trading and changes in regulatory policies might affect the systemic behaviour of markets. Do these systemic events show common properties across different markets? Have markets become more unstable in recent years? If so, can this be explained just by higher levels of jumpiness of single assets, or is it due to an increased cojumpiness, i.e. a higher level of synchronisation? By considering the last fifteen years, which have experienced the introduction of High Frequency Trading, we show that the number of jumps has slightly declined, but the number of cojumps, especially when involving a large number of stocks, has significantly increased. This is a clear sign of an increased synchronisation in financial markets, especially for what concerns their extreme behavior. Finally, we study the exogenous (i.e. news driven) and endogenous nature of systemic price cojumps. Quantifying news impact as the fraction of systemic cojumps that occurred within a 3-minute window after a news, we find that only roughly a 30% of the systemic events can be ascribed to a macro news. Interestingly, when lowering the time window to exact synching at 1-minute level, the fraction reduces considerably; instead, allowing for time windows larger than 3 minutes, the fraction trivially increases, but in a negligible way. This suggests that the time scale of news digestion is of few minutes, significantly larger than our 1-minute data resolution. These findings confirm that the majority of systemic events cannot be associated with an exogenous market-moving news and leave open the possibility for an endogenous explanation.

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## Random Graphs in Opinion Dynamics

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Networks and their statistical dynamics have been the focus of much study in the last 10 years. In the last 5 years, progress have been made on the calculations of average or bulk properties of social opinion dynamics on large random networks. These new results include a more accurate calculation of the expected times to consensus and the tipping point effects of minority mavens / leaders or committed agents, as the average degrees of the underlying random networks changes and / or the network model is given by a well-known continuum stochastic model known as the random geographic graphs (RGG). In this talk, we summarize recent results obtained by Lim, W. Zhang, Korniss and B. Szymanski. We will give some comparisons to actual examples of real world networks in this talk.

We also give a thematical framework comprehensive enough to represent many of the multi-agent signalling networks in recent works such as the generalized Naming Games (NG) and the Voter models used in modeling of ad-hoc wireless networks, agreement on a small list of tags in the WWW and social opinion dynamics in forums and elections.

Our aim is to show that the asymptotic behaviour of a proper subset of these signalling games on very large networks (where the effects of demographic noise can be safely neglected), can be described rigorously as a system of nonlinear coupled ODEs with the Monotonicity Property (MP) in phase-space consisting of the coarse-grained population fractions of each type. The nonempty complement of the monotone signalling systems include two main types, namely, (I) the signalling systems where the associated random walk models are Martingales, that is, the deterministic drifts are zero everywhere in phase space; in such systems, which includes all the Voter models, the asymptotic behaviour is diffusion - driven, and (II) signalling systems which have nonzero drift almost everywhere in phase space but whose nonlinear ODEs are not monotone; we will give such a counterexample below. Naming game, the prototypical case of opinion dynamics, on 2-dimensional random geometric graph is investigated. Study of this model helps to understand the spatial distribution and propagation of social opinions. A main feature of this model is the automatic emergence of spatial structure called opinion domains which are geographic regimes sharing the same opinion with clear boundaries. We provide the mean field equation for this dynamics and discuss several properties of the equation such as the stationary solutions and two-time-scale separation.

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## On the resolution of a chaotic phase space

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We deal here with the problem of evaluating long-time averages, such as diffusion coefficients, decay of correlations, escape rates, or Lyapunov exponents of chaotic systems, in the presence of white, additive noise. In the context of chaos, the ideal dynamical average is weighed with the natural

(invariant) measure of the state space, which is however prohibitive to compute directly in most cases. One valid, established alternative is then to weigh the average by properly partitioning the state space, and defining a suitable symbolic dynamics, as a means to recognize the most important (i.e. most often visited) regions. In particular, unstable periodic orbits constitute the skeleton of the dynamics, and can be a valuable tool to chart the state space.

Additional background noise, always present in any realistic model, calls for a substantial change in the way we think partitions due to the element of randomness: fractals are washed away, periodic orbits no longer exist, precious symmetries are broken. Thus novel ideas, formalisms, and algorithms are to be developed accordingly.

We find it most sensible to follow the evolution of densities in the state space via Fokker-Planck equation, whose smooth, locally invariant solutions around periodic orbits of the deterministic flow constitute the building blocks of our partition algorithm.

In a recent spew of publications, this novel partition algorithm has been fully developed and tested in the framework of one-dimensional hyperbolic and non-hyperbolic maps. Due to the asymptotic balance between deterministic, exponential shrinking of neighborhoods around longer and longer periodic orbits, and the smearing effect of noise, a *finite* resolution is found for the state space. As a consequence, Fokker-Planck equation is replaced by a *finite-dimensional* evolution operator that yields estimates of long-time observables (escape rate and Lyapunov exponent), found to be in good ( $\sim 1\%$ ) agreement with blind-grid ('Ulam) discretizations. Subsequently, the formalism has been extended to higher dimensions, where we must account for both expanding and contracting directions of the deterministic system, and their interplay with isotropic noise.

We finally report on the latest advances of our optimal partitioning technique, applied to a two-dimensional hyperbolic map with background noise.

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## Entropic correlations in metals and astrophysical plasmas: their role in nuclear fusion reactions

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We discuss the important contribution of particle correlations both to the entropy and to reaction rates in different kind of plasmas leading to interesting theoretical and experimental relationships. We focus on fusion reaction in metals, in astrophysical plasmas, and in artificially confined and strongly correlated plasmas.

Strong enhancements of the rates at low-energy have been observed in several of these environments. Standard screening effects are not sufficient to explain these experimental data. We show that generalized statistical distributions, energy-momentum quantum uncertainties, and two-body entropic

correlations can play an important role for understanding these phenomena.

We consider fusion experiments in deuterated metals, underground measurements of low-energy fusion cross sections at LNGS, fusion reactions in the solar core, neutral and non-neutral plasmas, Debye-Huckel electron screening, fusion in liquid metals, melting entropy, stopping-power determination, Galitski Yakimets energy-momentum uncertainty, and non-Maxwellian distributions.

## Understanding kappa distributions in space plasmas

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**Topical Area:** A

**Workshop:** 2

Numerous independent analyses on collisionless space plasmas have revealed a peculiar statistical behavior of this category of plasmas: Particle populations of space plasmas reside in stationary distributions which are out of thermal equilibrium. Thermal equilibrium is a special stationary state; systems at thermal equilibrium have their distribution function of particle velocities stabilized into a Maxwell distribution, which is connected with the classical framework of Boltzmann-Gibbs statistical mechanics. However, Maxwell distributions are extremely rare in space plasmas, with the vast majority of these plasmas residing at stationary states out of thermal equilibrium, which are described by kappa distributions. The kappa distribution provides an unambiguous replacement of the Maxwell distribution for systems out of thermal equilibrium such as space plasmas. Kappa distributions do not embody Boltzmann-Gibbs statistics, but instead, they are connected with the generalized statistical framework of non-extensive statistical mechanics, provided the continuum of the energy spectrum, and the key-relationship between the entropic  $q$ -index and the kappa distributions'  $K$ -index,  $q = 1 + 1/K$  [1]. Understanding the origin of kappa distributions was the starting point of all the further development and investigation, which involves [2] (i) the possible physical mechanisms that create the kappa distributions and the framework of non-extensive statistical mechanics; (ii) the physical meaning of thermodynamic parameters such as temperature, kappa index, entropy, etc; (iii) the multi-particle description of kappa distributions; (iv) the kappa distribution of a Hamiltonian when a non-zero potential is given; (v) the possible description of distributions as superposition of kappa distributions. The kappa distributions and the proven tools of non-extensive statistical mechanics have been successfully applied to a variety of space plasmas throughout the heliosphere, from the inner heliosphere, e.g., the solar wind and planetary magnetospheres, to the outer heliosphere, e.g., the inner heliosheath and beyond [3,4]. These analyses led to the determination of the thermodynamic variables and the understanding of the underpinning physical processes of these plasmas, as well as to more fundamental findings, such as the new quantization constant that characterizes collisionless space plasmas [5].

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## Pricing in a complex financial market: instability from local measures

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A huge number of financial instruments (bonds, futures, swaps, options, etc) are priced every day. Each of such instruments is generally priced according to models which approximate market dynamics and need to be calibrated on market data. In spite of the complexity of market dynamics, models used in pricing are often very simple: they depend on few parameters so as to allow for a very efficient and fast calibration on a small set of observed prices and rates. This implies unavoidable approximations, both in the model and in the calibration, that are sometimes corrected in ad-hoc manners.

Different institutions price different financial instruments according to different models, calibrating them against different data. As a consequence, not only the results are intrinsically approximated, but approximations are different for different financial instruments. In financial jargon, this is a situation where each financial instrument is priced using a local market measure, which may be specific of that instrument and/or of that institution. In addition, in some cases misaligned prices induce trading activity that enhances further misalignment, which in turn causes price instability (e.g. crashes). It has been argued that this practice can potentially lead to the emergence of arbitrage opportunities, i.e. the possibility of making a riskless profit, because the difference in the approximations used may give rise to a system of inconsistent prices. Thus, in this talk I shall interchangeably talk about financial instabilities and arbitrage opportunities in what follows. I shall discuss the emergence of instabilities in a stylized model of a financial market, where different market actors calculate prices according to different (local) market measures. Typical properties for ensembles of large random markets will be derived using techniques borrowed from the statistical mechanics of disordered systems. I shall show that, depending on the number of financial instruments available and on the heterogeneity of local measures, the market undergoes a sharp phase transition separating an arbitrage-free phase from an unstable one, where the complexity of the market, as measured by the diversity of financial instruments, increases, and arbitrage opportunities arise. Focusing on two different classes of local measures inspired by real market strategies, I shall discuss analytical and numerical evidence to corroborate such findings.

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### Noise can enhance stability in Si bulk?

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In recent years, an increasing interest has been directed towards the possible constructive influence of noise and fluctuations on the dynamical response of different nonlinear systems, focusing on cooperative effects between the external noise and the intrinsic interactions of the systems. Indeed, previous theoretical investigations have shown that, under specific conditions, the addition of external fluctuations to systems, characterized by the presence of intrinsic noise, may affect the dynamics of the system in a counterintuitive way, inducing an enhancement of stability and resulting in a less noisy response [1]. This possibility has been successfully explored, for example, in the dynamics of magnetic spin systems [2] and in the electron spin relaxation process inside semiconductor structures [3]. The possibility of reducing the electronic diffusion noise, by adding a random fluctuating contribution to the driving oscillating electric field, has been found in III-V semiconductor crystals [4], whereas this opportunity, to the best of our knowledge, has not been yet tested in covalent semiconductor materials. Aim of the present work is to investigate the noise-induced effects on the electron transport dynamics in low-doped n-type Si crystals. To simulate the dynamics of electrons in the bulk, by taking into account the band structure, the scattering processes, as well as the heating effects, a Monte Carlo approach is used. The effects caused by the addition of an external noise source are investigated by studying the modifications in the correlation function of electron velocity fluctuations and in its spectral density. The system is driven by a high-frequency periodic electric field in the presence of two different kind of external fluctuations: a Gaussian correlated or a Random Telegraph noise source. Our findings show that, critically depending on the external noise features, the dynamical response of electrons driven by the periodic electric field can benefit from the constructive interplay between the fluctuating field and the intrinsic noise of the system. In particular, we found that also in Si bulk the presence of the fluctuating component can reduce the total noise power up to 10 % in a wide range of noise amplitude. Furthermore, we found a non-linear behaviour of the integrated spectral density with both the noise amplitude and the noise characteristic times.

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### Active and Passive Faults Detection by Using the Page Rank Algorithm

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Here we try to find active and passive places for earthquakes in the geographical region of Iran. The approach of Abe and Suzuki is adopted for modeling the seismic history of Iran by a complex directed network. By using the Page rank algorithm we assign to any places in the region an activity index then the most active places are determined. Recently, scientists have great interest in studying the earthquake within the context of complex systems theory [1,2]. The faults are the main components of a geological system that their motion are responsible for most of the spatiotemporal phenomena in a geological region. When the activity of a fault occurs abruptly it can produce an earthquake. Earthquakes have influence on their surrounding faults by transferring the stresses to them and may trigger subsequent events [3,4,5]. Seismic waves are generated by sudden releasing of the energy in an earthquake and propagate through the earth's crust. Fluctuation of stresses associated with seismic waves may temporarily exceed the local failure threshold in the far field faults and leading to other earthquakes in the places that are not close to the original earthquake's epicenter [4,5]. Many evidences support the fault-fault interaction because of the earthquake stress changes. The nature of such interactions are more complex than often thought and the complex network method seems to be good candidate for studying and understanding the geophysical phenomena in a system of faults belonging to an area. In this paper we continue our study about the Iran earthquakes network and try to find out information about the active and passive regions in Iran by calculating the centrality of nodes in the corresponding network. Centrality of a vertex can be measured via different ways Degree, Betweenness, closeness and PageRank to determine the relative importance of it within a graph. PageRank is a link analysis algorithm to assign a numerical weighting to each element of a hyperlinked set of documents with the purpose of measuring its relative importance within the set. The rank value indicates an importance of a particular earthquake's regions. In other words, this parameter calculates the passive regions in the network. The CheiRank is constructed for a directed network with the inverted directions of links. It is similar to the PageRank, which ranks the network nodes in average proportionally to a number of incoming links. Due to inversion of link directions, the CheiRank ranks the network vertices in average proportionally to a number of outgoing links. This quantity measures the passive regions in earthquake networks.

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### Complex fluids at finite concentration: A theoretical approach.

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A general theoretical approach to study simple and complex homogeneous or inhomogeneous fluids, at finite concentration, through simple models is presented. New effects are found. Some are briefly discussed, but focus on those for concentrated bulk colloidal dispersions. In the past, theoretical studies of colloidal dispersions, at infinite dilution, have been made. Here are presented integral equations results for concentrated colloidal dispersions, for a colloidal primitive model (CPM). In the CPM the colloidal particles are charged hard spheres, with their charge at the center of the particle, immersed in a primitive model electrolyte. The radial distribution functions for all the fluid components are calculated and, thus, also the  $\zeta$ -potential, induced charge,  $\sigma(r)$ , and other functions of interest. The results are compared with those from the CM, other simpler approaches and computer simulations. Important deviations from the widely used CM are reported. New effects, attributed to the large size and charge and to the finite concentration of colloidal particles, are found. In particular, the theory predicts very long range particle-correlations and a polarity inversion of  $\zeta$ -potential for several conditions, always associated with charge reversal. However, if different charge distributions on the colloidal particles are considered, for example if the charge is located on the surface of the colloidal particles, polarity inversion of the  $\zeta$ -potential may not be observed. We refer to this new model as the Modified Colloidal Primitive Model (MCPM). We discuss the possible implications of our results in relation to the electrophoretic mobility of colloidal particles, and the appearance of voids, as has been experimentally found. A good qualitative agreement between the theoretical predictions and computer simulations is observed. As another test of our theory, the adsorption of a model phosphotungstic acid ( $\text{H}_3\text{PW}_{12}\text{O}_4$ ) + HCl solution on a eicosylamine  $\text{CH}_3(\text{CH}_2)_{19}\text{NH}_3^+$  monolayer, is compared with experimental data.

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### Phenomenological model of thermoelasticity of solids

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The thermodynamics of oxide compounds plays an important role in many area of applied physics and engineering, including heterogeneous catalysis, sensors, high temperature superconductors and solid state batteries. The geometric nonlinear theory of oxide compound has been investigated. It predicts the reversibility of the transitions as manifested under thermal and elastic stress based only on crystal symmetry and geometric compatibilities between phases. The thin-film composition-spread technique was devised to rapidly map

the lattice parameters and the thermal behaviour of oxide compound systems. Moreover where materials undergo to high temperature stress, phase stability might be an important factor in determining material performance: a knowledge of the solid thermodynamics is fundamental for successful applications. Studies on the interaction between mechanical and thermal effects in solid bodies have therefore received greater attention. The thermal behaviour of solids under elastic deformations is analysed in order to obtain a working mathematical model which combines the molecular dynamics under high temperature stress and the elastic reversible transformations. The thermal behaviour has been described by Parrinello-Rahman model: Parrinello-Rahman Lagrangian is obtained by assuming that the cell inertia tensor is spherical and constant in time and the cell fluctuation motions are irrotational. Its outputs, that depend upon temperature, have been used as inputs for a molecular dynamic model that takes into account the applied elastic perturbation. Investigations into the atomistic-to-continuum coupling have been pursued and a hierarchical modelling in terms of a macroscale treated by continuum mechanics and the microscale governed by statistical mechanics may be a good fruitful compromise. If the microscale is computed with the help of molecular dynamics, the isostress-isoenthalpic ensemble as proposed by Parrinello and Rahman becomes a statistical ensemble where the equations of motion are directly derived from a Lagrangian. This Lagrangian could be situated into a continuum mechanics setting. The behavior of this continuum-related Lagrangian in a kinetics-driven setting, obtained by imposing an external stress, and a kinematics-driven setting, by imposing the shape of the molecular dynamics cell, is investigated. The results give useful information about the variation of the mechanical stress and strain limits of the oxide compound when applied in hostile high temperature environment.

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### Brownian motors: Enhancement of efficiency by noise

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Transport in micro-world is strongly influenced by fluctuations and random perturbations. The influence has seemed to be usually destructive, i.e. transport is weakened with respect to such quantifiers like averaged velocity or current. However, a constructive role of both equilibrium and nonequilibrium fluctuations is also observed in many systems and phenomena such as Brownian ratchets, stochastic resonance, molecular motors and machines, genetic and biochemical regulatory systems, to mention only a few. Fluctuations can enhance the averaged velocity, can reverse transport direction and can induce anomalous transport properties. Studies of this subject can (directly or indirectly) explain unusual properties as e.g. bidirectionality of kinesin and dynein transport inside living cells.

We show that biased non-equilibrium noise  $\eta(t)$  can be more effective than deterministic constant force  $F$  transporting inertial Brownian particles in a *symmetric* periodic substrates and driven by unbiased time-periodic (simple harmonic) force.

Within selected parameter regimes the noise  $\eta(t)$  can *solely* induce anomalous transport process of the same kind as the deterministic force  $F$ , however with efficiency several times greater than when the particle is transported by  $F$ . As an example of noise  $\eta(t)$  we apply white Poissonian shot noise for which we demonstrates very surprising effect of a solely white Poissonian noise induced enhancement of the Stokes efficiency of the anomalous transport process occurring in generic, biased systems and devices. This phenomenon has its roots in the fact that replacement of the deterministic static load with the carefully adjusted non-equilibrium noise allows for simultaneous amplification of the negative current and minimization of its fluctuations. In particular, it can be observed for moderate values of the spiking frequency  $\lambda$  of the white Poissonian noise. Furthermore, in the limiting case of very large frequencies  $\lambda \rightarrow \infty$  with fixed mean  $\langle \eta \rangle = F$  the transport process induced by this non-equilibrium noise is both quantitatively and qualitatively equivalent to the one caused by the constant force. Moreover, we demonstrated the possibility of manipulating the direction of transport just by tuning the parameters of the white Poissonian noise.

As an application of the theoretical study we consider the resistively and capacitively shunted single Josephson junction for which the presented anomalous transport process can be measured.

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## Coagulation, gelation, random structures

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A review of recent achievements in the theory of sol-gel transitions in coagulating systems is given. The basic ideas of the stochastic approach to the dynamics of coagulating systems are outlined and its possible application as a basis for a theory of gelation in coagulating systems is described. This approach allows for a consideration of *finite* coagulating systems, i.e., those containing a finite number of particles. In parallel, the classical (Smoluchowski's) approach to the coagulating systems with instant sinks serves as a tool for studying the sol-gel transition. The latter approach allows for a correct explanation of the gelation scenario where the gel does not serve as an active sink of the sol particles. The new results that I plan to report are:

- Sol-gel transitions in evolving random networks and coagulation.

Thermodynamically large systems of particles (e.g., molecules) are normally described in terms of extensive variables. For example, the total number of coagulating particles in a volume  $V$  is proportional to  $V$  and the proportionality coefficient is known as the particle concentration. It is silently assumed that the total occupation numbers of the particles are proportional to the total volume. The particles whose occupation numbers grow slower than  $V$  as  $V \rightarrow \infty$  are considered to be "off side" and are not taken into account in studying the properties of the systems. Meanwhile, it is not possible to deny the possibility for the existence of very low concentrated ob-

jects that still affect considerably the fate of the system. A very good example is the sol-gel transition in coagulating systems or the emergence of the giant components in evolving random graphs. The analogy between the evolution of random graphs (see Lushnikov, 2005) and gelation in coagulating systems shed a new light to the kinetics of formation gels in physicochemical systems, and vice versa, the use of the kinetic approach adopted from the theory of coagulation helps in the consideration of the evolution of random graphs.

- Exact expression for the critical times of the sol-gel transition.

The gelation process depends strongly on the structure of the coagulation kernel, in particular, on the homogeneity exponent  $\lambda$  of the coagulation kernel. Almost nothing is known on the dynamics of the sol-gel transition except for one exactly solvable model with  $\lambda = 1$ . Fairly recently I was able to find some exact results for the critical times in the coagulating systems. The results are found for initially monodisperse particles.

- Source-enhanced gelation

The gelation processes in source-enhanced systems is an extremely difficult task. The analogous task in the theory of random graphs is the evolution of the random graph with variable number of edges and vertexes. I show how this problem is solved in the theory of coagulation and how the results can be applied to graphs.

- Gelation in coagulating systems with arbitrary initial conditions. (Lushnikov 2012, 2013)

In thermodynamically large gelling systems it is possible to find the general solution for arbitrary initial conditions and to study the dependence of the transition kinetics on the types of the initial conditions. It is shown that the transition always has a scaling regime whose exponents however, are not universal if the initial conditions are algebraic functions of the masses of coagulating particles.

- Gelation in mixed systems (including K-partite random graphs)

Interestingly, that for mixed gelling systems it is possible to obtain many results analogous to those in monocomponent systems. In particular, it is possible to find the exact expression for the particle composition spectrum for arbitrary initial conditions.

I also plan to discuss the hot problems of the theory of sol-gel transition.

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## Anomalous diffusion from particle interactions

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Several microscopic mechanisms can give rise to anomalous diffusion. Perhaps the most well-known example is based on the continuous time random walk: when the distribution of waiting times is algebraic, at long times, the system obeys the fractional-time diffusion equation which in turn leads to

sub-diffusive behavior. In this contribution, we describe an alternative mechanism based on the interaction between a population of particles undergoing a random walk. The probability for a given particle to remain in its current position or to jump to a new position is taken to depend, in an arbitrary manner, on the fraction of the population at its current position and at the new position. Using a multiscale analysis, we show that this leads to a Fokker-Planck, or generalized diffusion, equation for the probability to find a particle at a give location. When we demand that this equation possess diffusive - that is, self-similar - solutions, we find that the interactions must give rise to a power-law dependence of the jump probabilities on the local concentration of particles. We note that the generalized diffusion equation has self-similar solutions that take the form of so-called q-exponential distributions. When the model is completely local, so that the probability for a particle to jump depends only on the concentration at its position, sub-diffusive behavior results. Traveling solutions are also possible and are investigated numerically. Finally, in the general, non-local, case that the jump probabilities depend on the concentration at the particle's location and at the terminus of a jump, both sub- and super-diffusive behavior is possible. In all cases, we make comparisons between the mesoscopic predictions and microscopic simulations of the underlying discrete model. For the sub-diffusive case, convergence to the diffusive limit is rapid whereas in the case of super-diffusion, considerable care is necessary although good agreement is eventually achieved in all cases. It is remarked that the sub- and super-diffusive behavior can be understood as a manifestation of effective attractions and repulsions between the walkers. Our model represents an alternative microscopic mechanism (i.e. particle interactions) for the generation of anomalous diffusive behavior at the macroscopic level.

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### Generalization of Classical Nucleation Theory to Include Multiple Order Parameter

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Nucleation is one of the most important processes in many areas of physics, chemistry and engineering. Today, there is a vast resurgence in interest in the subject due to the development of new experimental techniques that allow us, for the first time, to probe the very small and the very fast, as well as the tremendous computational power complemented with the recently-developed rare-event techniques that allow for the first time for atomistic simulation of realistic models of nucleation. Yet, our understanding of it has progressed only incrementally since the development of Classical Nucleation Theory (CNT) in the 1920's-1940's. CNT is based on intuitively plausible but fundamentally phenomenological ideas such as the capillary approximation for free energies and the idea of quasi-stationarity and of detailed balance. Recently, it has been proposed that a more fundamental

description is possible by recognizing the fundamentally non-equilibrium nature of the process of nucleation and, so, by making use of the techniques of non-equilibrium statistical mechanics. Specifically, a new approach based on fluctuating hydrodynamics has been developed which avoids the most severe assumptions of CNT. It allows for the use of modern developments, such as Density Functional Theory for the calculation of free energies, and yet reproduces CNT in appropriate limits. In this talk, show how this approach on the one hand reduces to CNT in the appropriate limits while, on the other hand, that certain ambiguities that preclude the application of CNT to the case of small critical clusters (i.e. high supersaturations), related to a lack of covariance of CNT, are automatically eliminated by our approach. We then use this approach to explore the issues involved in generalizing CNT to include multiple order parameters. Specifically, we discuss a generalization of the description of liquid-vapor nucleation which allows for more than just the change in size of a sub-critical cluster (which is all that is possible in CNT). Instead, we allow more realistically for the dynamical change of both density and size of a developing cluster. Although straightforward in principle, numerous difficulties are encountered requiring careful development of the generalized model. We present the results of semi-analytic calculations of the nucleation pathway in the two-dimensional order parameter space and of the nucleation rate and compare to numerical simulations.

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### Action at the distance

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Can correlation effects in a fluid confined in big but finite compartments linked by small openings, such as shallow channels, occur over distances much larger than the bulk correlation length? Recently, Gasparini and co-workers [1,2] have demonstrated this rather striking "action at a distance" effects in a two-dimensional array of microscopic pockets filled with superfluid <sup>4</sup>He. These pockets are boxes with side length  $L$  either 1 or 2  $\mu\text{m}$  linked by either a channel or a uniform film of thickness in the range of  $\sim 10 - 30$  nm. The measurements of several responses such as the enhancement of a specific-heat and in the amount of superfluid of a <sup>4</sup>He film show that under certain conditions these boxes can be strongly coupled to neighboring ones. What seems to be crucial in this work is the size of boxes and connectors and the vicinity of the critical point [3]. Perron et al [3] suggested that action-at-a-distance effects might be a more general feature of systems with phase transitions than is usually supposed, a view which we confirm in our work for uniaxial classical ferromagnets and their analogs (simple fluids or binary mixtures in the lattice gas approximation, all belonging to the Ising model universality class). Our theory has a key ingredient: the Fisher-Privman theory of finite size effects in first order phase

transitions for cylinders in  $d$ -dimensions specified  $L_{||} \times L_{\perp}^{d-1}$  [2,3]. We have extended this theory to capture correctly the asymptotic behavior of the pair correlation function in strips. We apply our extended Fisher-Privman theory to a uni-axial Ising ferro-magnet in which three-dimensional blocks of dimension  $L_0$  (in units of the lattice spacing) are coupled together by two-dimensional  $(2-d)$  strips of width  $M$  and length  $L$ , with ferro-magnetic coupling  $K_0$  (in units of  $k_B T$ ) for each nearest-neighbor pair of spins. In this "network" Ising model constructed from the  $2d$  array of boxes and connecting strips, we see that the parameters can be tuned to produce long-range order, in itself not perhaps surprising, but with *extraordinarily long connecting links*; this diverges exponentially with system width, on a scale of the inverse surface tension. Thus, ordering between boxes is feasible over length scales of many thousands of molecular diameters. We have tested the application of extended Fisher-Privman theory by MC simulation.

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## Dynamics of a dissipative multi-state quantum system

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We study the relaxation dynamics of a multi-state quantum system, characterized by the tunneling through a potential barrier, in interaction with a dissipative environment.

Among the physical and chemical systems (both natural and engineered) displaying dissipative tunneling, superconducting devices with Josephson junctions have recently collected a vast interest, as they can be used for implementing the quantum computation hardware. In these devices it is observed the macroscopic quantum tunneling of a flux quantum associated to the state of the current trading the superconducting circuit. The flux quantum can be modelled as an effective particle in a biased double well potential, subject to the linear interaction with a thermal bath of mutually independent harmonic oscillators, according to the Caldeira-Leggett model. A static bias creates a metastable state from which the particle can decay.

If the bath's temperature is low enough it is sufficient to consider the first two energy levels. By this way the problem reduces to that of the celebrated *spin-boson model*, whose dynamics is given by the quantum mechanical tunneling through the potential barrier. At higher temperatures or in the presence of external time-dependent driving, it is appropriate to consider more than two levels, so that one can observe a richer dynamics, with coherent oscillations inside a single potential well, tunneling events or even thermally induced barrier crossing.

In this work the time evolution of the populations in the discrete variable representation (DVR) is obtained, using a real time path integral technique. The time evolution of the system is obtained in terms of a set of coupled integro-differential equations, or generalized master equation (GME), for the populations in the DVR. In this context, various approximation schemes for the kernels of the GME

are used, depending on the system-bath coupling and bath temperature. Under appropriate conditions a Markovian master equation with constant rates is extracted.

Introducing a suitable definition of escape time from the metastable state, we calculate this quantity as a function of the environmental parameters, with emphasis on the role of the initial condition, obtaining a non-monotonic behaviour as a function of the damping strength.

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## The intensity of the random variable intercept in the sector of negative probabilities.

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The authors discuss the properties of the intensity  $\rho$  of measurement of a real random variable  $X$  with marginal density distribution  $pdf(x) := \frac{W(x,y=const.)}{\int_{-\infty}^{\infty} W(x,y=const.)dx}$  restricted to domain  $x \geq a$ .  $W(x,y)$  is the Wigner distribution of a certain quantum state for which function  $W(x,y)$  is not positively defined. The definition of the functional  $\rho$  is as follows:

$$\rho = \rho(pdf, a) := \frac{\int_a^{\infty} x \cdot pdf(x)dx}{1 + \int_a^{\infty} pdf(x)dx} . \quad (1)$$

It is naturally related to the intensity of measurements in the context of transactions (buying/selling). Here parameter  $a$  denotes the price below which a seller will always decide not to sell a given good, and the above quotient represents the rate of profit in one cycle (buying/selling) [1]. It is assumed that the duration of consecutive buying and selling cycles is a random variable and quotient (1) expresses, in the language of probability density, the quotient of the average value of the profit rate in a whole cycle divided by the average duration time of this cycle.

The classical definition of random variable implies monotonicity of its distribution function. This fact leads to an interesting property of intensity  $\rho$ : i.e. this function has a (global) maximum at its fixed point [2]. The assumption that function  $pdf(x)$  is non-negative is used in the proof of this property. It is therefore interesting how extreme property  $\rho$  will change when this assumption is violated (so-called negative probability). For the purpose of the discussion an oldest interpretation of negative probabilities was adopted. This definition has been known since the times of Captain Robert Giffen ('40s of the 19th century [3]) who was the first to notice a non-monotonic market demand for so-called Giffen goods (this modern description of the discovery is the result of formulating supply/demand curves for the logarithms of prices).

Previous analysis of such cases of  $\rho$  was limited to the first

excited state of the quantum harmonic oscillator. This state plays a particular role in an information-theoretic measurement which entails analysing distributions that minimise the Fisher information function (see subjective supply and demand curves in [4,5]). The new properties of  $\rho$  that are characteristic of quantum models can, for example, be used as a test for the existence of states with negative probabilities. They also suggest a strategy for maximising the profit earned on transactions in Giffen goods.

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### Competing contact processes in the Watts–Strogatz network

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We investigate two competing contact processes on a set of Watts–Strogatz networks [1] with the clustering coefficient tuned by rewiring. The base for network construction is one-dimensional chain of  $N$  sites, where each site  $i$  is directly linked to nodes labelled as  $i \pm 1$  and  $i \pm 2$ . So initially, each node has the same degree  $k_i = 4$ . The periodic boundary conditions are assumed as well.

For each node  $i$  the links to sites  $i + 1$  and  $i + 2$  are rewired to two randomly selected nodes so far not-connected to node  $i$ . The rewiring procedure occurs with probability  $q$ . Increasing rewiring probability  $q$  influence the nodes degree distribution and the network clusterization coefficient.

For given values of rewiring probability  $q$  the set  $\mathcal{N}(q) = \{\mathcal{N}_1, \mathcal{N}_2, \dots, \mathcal{N}_M\}$  of  $M$  networks is generated. The clusterization coefficient  $C_j$  for  $j$ -th network is defined as the average over nodes  $i = 1, \dots, N$  of the local coefficient  $c_i$ , where

$$c_i = \frac{2y_i}{k_i(k_i - 1)}, \quad (2)$$

and  $k_i$  is the degree of  $i$ -th node, i.e. the number of nodes linked to  $i$ , and  $y_i$  is the actual number of links between these  $k_i$  nodes [1].

The network's nodes are decorated with spin-like variables  $s_i \in \{S, D\}$ . During simulation each  $S$  node having a  $D$ -site in its neighbourhood converts this neighbour from  $D$  to  $S$  state. Conversely, a node in  $D$  state having at least one neighbour also in state  $D$ -state converts all nearest-neighbours of this pair into  $D$ -state [2]. The latter is realized with probability  $p$ . We plot the dependence of the nodes  $S$  final density  $n_S(t = T)$  on initial nodes  $S$  fraction  $n_S(0)$ . Then, we construct the surface of the unstable fixed points in  $(C, p, n_S(0))$  space. The system evolves more often toward  $n_S(T) = 1$  for  $(C, p, n_S(0))$  points situated above this surface while starting simulation with  $(C, p, n_S(0))$  parameters situated below this surface leads system to  $n_S(T) = 0$ . The points on this surface correspond to such value of initial fraction  $n_S^*(0)$  of  $S$  nodes (for fixed values  $C$  and  $p$ ) for which their final density is  $n_S(T) = \frac{1}{2}$ . In contrast to the previous studies [3,4] the initial fraction  $n_s(0)$  of nodes in  $S$ -states influences the final state  $n_S(T)$  of

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### Influence Of Maximum Wind Speed In Wind Power Use

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This work shows the influence of maximum wind speeds in wind power use, therefore comparisons of the mean wind velocities and maxima speeds for two consecutive semesters were done by using the classical Weibull formulation in numerical simulation. The potential given by the winds need to take in account the instantaneous potential due to the inertial discharge by maximum speed in the machines, which effect is ignored in the numerical formulation of the wind power potential.

The statistical models has a better representation of the variation velocity wind is the Weibull distribution that is controlled by two parameters, this distribution has been used in several research papers, among the most recent, Kuhnen et al. (2004), Burton et al. (2002) Sucharov & Marques (1992).

For the occurrence of calms  $\nu = 0$  Sucharov & Marques (1992) used the hybrid formulation proposed by Brown & Takle (1978) where a probability of calm was introduced to calculate the Weibull parameters and it was found that there is little difference between the results obtained by the classical and hybrid formulation in the calculation of the potential of the wind. In Marques et al. (2010) was compared the values of wind speed measured at anemometer tower installed next to small wind turbines in the park Energias Alternativas UENF, in Campos dos Goytacazes, where we showed that there is a large temporal variability, detected by accurate wind measurements at intervals of 20 minutes and, consequently, the values of wind energy to be produced.

In this work, aiming at a more accurate estimate of wind power than currently available in the literature, from the analysis of wind data and the analysis of the computational simulations Weibulls models to verify the influence of potential energy due to speed maximum, since the in usual simulations was used the values of average speeds, which produces only partial results of the amount exact of energy to be produced. The results show that the energy varies strongly due to gusts of wind maximum speeds and it was found the influence this has on the generation of energy since a significant amount of energy due to the inertia of the turbine are not considered in the simulation computing. Thus the energy obtained by the maximum speed is of the order of approximately three times more than the energy calculated by averaging speeds. It was shown also that the scale parameter Weibull has a strong influence on the calculation.

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## Dynamical systems and the direct transition to turbulence in subcritical flows

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Dynamical systems theory has been successfully applied to understand the transition to turbulence in closed flows, e.g. convection, for which the existence of a linear instability mechanism leads to globally supercritical scenarios, with chaos theory found relevant especially in the presence of strong confinement effects. In open flows, the situation is more complex because such a mechanism is not always present or efficient, while upstream-downstream advection makes the confinement question somehow ill posed. Wall-bounded flows, with non-inflectional base flow profiles, indeed become turbulent according to a specific direct scenario in which the nontrivial turbulent branch remains disconnected from the trivial base-flow branch, implying wild sub-criticality and hysteresis. The most emblematic cases are Hagen-Poiseuille (pipe) flow (HPF) and plane Couette flow (PCF), both being linearly stable for all Reynolds numbers. Sub-criticality, associated with the coexistence of limit sets in state space, translates to laminar-turbulent coexistence in physical space in the form of turbulent plugs in HPF, turbulent spots in boundary layer flow, PCF or plane channel flow, for example. This specific transition problem, of great practical importance will be reviewed, starting from the low-dimensional dynamical systems viewpoint. This approach relies on the study of the dynamics in periodic domains just sufficient to observe nontrivial dynamics called minimal flow units (MFUs). Such a reduction permits the elucidation of mechanisms sustaining turbulent flows, the determination of exact solutions to the Navier-Stokes equations with different symmetries and time dependence. These solutions are all unstable but their stable and unstable manifolds build a skeleton in phase space that constrains typical trajectories. Unstable periodic orbits (UPO) are of particular interest since the intersection of these manifolds can generate homoclinic tangles supporting unstable chaotic behavior as envisioned by Poincaré (1890). This framework has been instrumental in explaining the decay of turbulent puffs in HPF. The (low-dimensional) dynamical system approach can however not be the whole story and genuine spatiotemporal dynamics has to be reintroduced at some stage, further analyzed using concepts and tools borrowed from the theory of phase transitions in statistical physics, in particular direct percolation and extreme value theory, following an early suggestion of Pomeau (1986).

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## A novel finite-size scaling method for the calculation of critical exponents of very low values found in explosive percolation

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Percolation is a topic that attracts the interest of many scientists dealing with statistical physics problems. The phase transition related to percolation is a geometric one, i.e. the appearance of an infinite connected cluster. The critical exponents  $\beta$ ,  $\nu$  and  $\gamma$  describe the critical behavior associated with the percolation transition and are universal. The basic task when dealing with critical phenomena is to define these exponents with sufficient accuracy. There is a variety of methods used to estimate the values of  $p_c$ , and the exponents. The most widespread one, and generally considered as reliable, is that of finite-size scaling analysis. According to this method, a quantity  $S$  scales as  $X = N^x F[(p - p_c)L^{1/\nu}]$ , where  $X$  is an arbitrary quantity of interest (e.g. size of the largest cluster  $S_{max}$ , susceptibility  $\chi$ ),  $x$  is a critical exponent characterizing this quantity and  $1/\nu$  is the correlation length exponent. This general relation is only valid when a system undergoes a continuous transition at  $p_c$ . In fact, according to finite-size scaling theory, the scaling function tends to zero for  $p < p_c$  and takes large values for  $p > p_c$ . Thus, if  $p = p_c + \epsilon$ , and  $\epsilon$  is small, we expect that the pairwise differences will be very sensitive to the value of  $p_c(N)$ . This makes the method of estimating the actual critical point and the respective exponent values questionable. On the other hand, an equivalent procedure could be to take the inverse function  $1/Y_{N_i}(p) = 1/S(p)N_i^{-x} = 1/F[(p - p_c)N_i^{1/\nu}]$ . In this case, the function behaves in an opposite way. As such, it gives large values for  $p < p_c$  and tends to zero for  $p > p_c$ . Thus, for  $p < p_c$  we expect to encounter the same problem as before. We can overcome this problem by simply combining the two variants, i.e.  $H_{N_i} = Y_{N_i} + 1/Y_{N_i}$ . In this case, the minimization function acquires the form:  $\Lambda = \sum_{i \neq j} (H_{N_i}(p) - H_{N_j}(p))^2$ . We will illustrate the superiority of the proposed method in the case of explosive percolation where exponent values are very low (i.e.  $\beta \approx 0.055$  in ER). Furthermore, our results will show that even with a relatively small amount of statistical averaging, we can acquire exponent values at least as accurate (if not more) than previous methods. Indeed, our results will provide similar values for the exponents even with 2-3 orders of magnitude smaller statistical sample than many published manuscripts.

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## Intercellular Communication Between Human Lens Epithelial Cells Analyzed By Means Of Complex Network Methodology

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Tools developed in the framework of complex network theory are nowadays frequently used for the analysis of the structure and function of complex biological systems. So called small-world and scale-free network properties have been identified at all scales of functioning of living organisms [1]. Moreover, complex network analysis has not only proven to reliably quantify the networks organization, but is also very suitable for discovering connectivity abnormalities in terms of pathological conditions. In the present study we extend these ideas to the human ocular lens epithelium. In particular, we construct and analyze the intercellular communication network of human lens epithelial cells attached to surgically isolated anterior lens capsules. It has been shown that this system represents a reliable source for the studies on lens epithelial cells functioning and can potentially provide an important insight into the pathophysiology of cataract development, which is the leading cause of visual impairment worldwide. It is believed that intracellular calcium homeostasis plays a crucial role in the process of cataract development [2,3]. The network connectivity is defined by means of detected interactions between individual cells, which are determined on the basis of intracellular  $\text{Ca}^{2+}$  dynamics in response to stimulation with acetylcholine. Changes in cytosolic  $\text{Ca}^{2+}$  concentration were obtained with an inverted fluorescent microscope by measuring the temporal changes of Fura-2 fluorescence in lens epithelial cells. We were able to extract and analyze anterior lens capsules from patients with cortical and nuclear cataracts as well as cataracts caused by other pathologies [4]. Our results reveal that activation of lens epithelial cells after stimulation, characterized by onsets of cytosolic  $\text{Ca}^{2+}$  increase, varied in time and space. Furthermore, lens epithelial cells form relatively efficient and highly modular networks with well-expressed communities, which indicates formation of cells in local clusters. Hence, the networks from cortical and nuclear cataract capsules exhibit very similar topological features, while cataract capsules accompanied with some other pathologies, such as uveitis, have different network structure in terms of higher segregation and lower efficiency. Presented results shed novel insight into the functional mechanisms and organization of human lens epithelial cells, which could not be obtained with conventional methodological tools.

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## Phase behavior of liquid-crystal monolayers of rod-like and plate-like particles

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The difference between the phase behaviours of molecularly thick films (Langmuir monolayers) and that of colloidal monolayers confined between two parallel planes is mainly due to the fact that the amphiphilic molecules are allowed to rotate out from the air/liquid interface, while the colloids can rotate only in the plane parallel to the confining walls. The consequence is that the phase behaviour of Langmuir monolayers can be much richer than that of 2D colloidal systems. For example, upon compression, only a few phases (isotropic, nematic, solid) may occur if the confined particles have only 2D orientational freedom [1,2], while several additional, tilted or not tilted, phases can be present in the case of Langmuir monolayers with out-of-plane rotational freedom due to their intermediate quasi-two-dimensional character [3] (i.e. 2D in translations and 3D in orientations).

In the present work the orientational and positional ordering properties of liquid crystal monolayers are examined. Particles forming the monolayer are modeled as hard parallelepipeds of square section and their shapes are controlled by the aspect ratio. The particle centers of mass are restricted to a flat surface and three possible and mutually perpendicular orientations of their axes are allowed. In the case of rod-like shapes, particles align along the layer normal in order to achieve the lowest possible occupied area per particle. This phase is a uniaxial nematic even at very low densities. In contrast, for plate-like particles, the lowest occupied area can be achieved by random in-plane ordering in the monolayer, i.e. planar nematic ordering takes place even at vanishing densities. It is found that the random in-plane ordering is not favorable at higher densities and the system undergoes an in-plane ordering transition forming a biaxial nematic phase or crystallizes. For certain values of the aspect ratio, the uniaxial-biaxial nematic phase transition is observed for both rod-like and plate-like shapes. The stability region of the biaxial nematic phase enhances with decreasing aspect ratios for plate-like particles, while the rod-like particles exhibit a reentrant phenomenon, i.e. a sequence of uniaxial-biaxial-uniaxial nematic ordering with increasing density if the aspect ratio is larger than 21.34. In addition to this, packing fraction inversion is observed with increasing surface pressure due to the alignment along the layers normal. At very high densities the nematic phase destabilizes to a nonuniform phases (columnar, smectic or crystalline phases) for both shapes.

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## Collective behaviors of market participants during special offer quote in stock market crashes

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In the earlier paper (Maskawa, 2012), the author analyzes the multivariate time-series of stock returns of the constituents of the FTSE100 listed on the London Stock Exchange for the period from May 2007 to January 2009 to study precursors to the global market crashes in 2008. He reported that a sharp rise in a measure of the collective behavior of stock prices was observed before the market crash. It was shown to be associated with news including the words "financial crisis." They did not impact stock prices severely alone, but they exacerbated the pessimistic mood that prevailed among stock market participants. Such news increased after the Lehman shock preceding the market crash. It was followed by the paper (Maskawa et al, 2013) studying market-wide price co-movements around crashes by analyzing a dataset of high-frequency stock returns of the constituent issues of Nikkei 225 Index listed on the Tokyo Stock Exchange for the three years during 2007–2009. Results of day-to-day principal component analysis of the time series sampled at the 1 min time interval during the continuous auction of the daytime reveal the long range up to a couple of months significant auto-correlation of the maximum eigenvalue of the correlation matrix, which express the intensity of market-wide co-movement of stock prices. They also study the market mode (the first principal component) in the framework of Multi-fractal random walk model, and find that a measure of the collective behavior of stock prices grows before almost all large intraday price declines of less than -5%. The large intraday stock price changes around crashes occurred intermittently, which are mainly created during the opening of trading of the morning and afternoon sessions. If a large imbalance between asked and bid quotation creates at a certain moment transaction is interrupted by the market rule, and the market control the quotation price. The market send down (or up) the quotation price (special quotation price) gradually at a fixed time interval. If the buyer (or seller) accept the special quotation price, transaction resumes. This process is called "special offer quote". In this paper, we analyze the collective behaviors of market participants during special offer quote in stock market crashes. We analyze the quotations of market participants in the framework of a decision making model using Bayesian estimation using the personal information and the observation of the decisions of the other participants as a social information (A. Pe'rez-Escudero and G. G. de Polavieja, 2011). This model well describe the collective behavior of market participants in the market crash of October 2008.

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## Nonlinear propagation of drift waves in rotating planetary environments with nonthermal population of electrons

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Spokes were one of the most intriguing features observed both by Voyager 1 and 2. They were nearly radial, wedge shaped features in the Saturn's B ring and were found to become wider towards the planet and were given the name Spokes owing to their radially elongated shape. The B ring, spanning 25000 km in radius, contains most of the mass of the ring system. Its optical depths were found to range from 0.4 to nearly 2. Its substantial radial structure (i.e., spokes) is irregular and contains a great deal of variation in its density and brightness, nearly all of it unexplained. The precise mechanism generating the spokes is still unknown. Among other ideas, it was also suggested that the collective interactions of particles, that are characteristic of a plasma-like media, led to unique disturbances in the ring (1). The electromagnetic aspects have further been elucidated (2). Previously suggested causes of the grains charging do not agree with all spoke characteristics, which include their rapid generation, localized formation primarily in Saturn's midnight-dawn sector, the seasonality of their apparitions, and, crucially, their morphologies. The rotating planetary magnetospheres and ionospheres contain dust particles, and if they are in sufficient quantity, they can become magnetized and consequently drift waves appear. It is a fairly well established idea that the planetary rotation about the axis could produce corotating electric fields that are believed to exist in the magnetospheres of Jupiter and Saturn. Rotation induced dispersive dust drift waves were suggested as the possible cause of the formation of spokes in the Saturn's B ring by Masood et. al. (3). Using the plasma parameters found in the Saturn's B ring, it was shown that the theoretically predicted spatio-temporal scalelengths agreed well with the satellites and HST observations of the spokes. In this study, we use the Kappa distributed ions and electrons as well as Cairns distributed ions and Boltzmannian electrons to investigate the effect of the non-Maxwellian ions and electrons on the propagation characteristics of the dust inertia driven drift waves in the rotating Saturnian environment. It is shown that the non-Maxwellian ions and electrons affect the spatio-temporal scales of the nonlinear solitary structures. It is observed that the dust charge, mass, and number density also affect the soliton structure significantly. The modifications in the earlier results owing to the inclusion of Kappa and Cairns distribution are also highlighted with special reference to spokes in the Saturn's B-rings.

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## Statistical manifolds on deformed exponential families and their applications

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In geometric theory of statistics, it is known that a statistical model naturally admits a Riemannian manifold structure with dualistic torsion-free affine connections, and such geometric structures play important roles in statistical inferences. Nowadays, this geometric theory for statistics is called information geometry, and it is applied various fields of statistical sciences (cf. Amari and Nagaoka 2000). A statistical manifold is a differential geometric formalism of such geometric structures. Let  $(M, g)$  be a Riemannian manifold, and let  $\nabla$  be a torsion-free affine connection  $\nabla$  on  $M$ . The triplet  $(M, \nabla, g)$  is called a *statistical manifold* if  $\nabla$  and  $g$  satisfy a suitable compatible condition (cf. Matsuzoe 2013). In the case of exponential family, e.g. Gaussian distributions, gamma distributions, this geometric structure is induced from the relative entropy, which is also called the Kullback-Leibler divergence, for the given exponential family. A deformed exponential family is one of generalizations of exponential families, which has been studied in anomalous statistical physics and in machine learning theory (cf. Naudts 2011). It is known that a deformed exponential family naturally admits several deformations of mathematical structures. For example, two kinds of expectations for random variables can be defined on deformed exponential family, the one is the standard expectation, and the other is the *escort expectation*. The difference in these two expectations brings two different statistical manifold structures.

In this presentation, we give a survey about geometry of deformed exponential families and elucidate geometric and statistical meanings of deformed mathematical structures. For example, two different expectations on a deformed exponential family can be regarded as the choice of dual affine coordinates on statistical manifolds (cf. Matsuzoe and Henmi 2013). From the viewpoint of statistics, these two expectations are characterized by properties of estimating functions. Divergence functions on deformed exponential families can be constructed from these estimating functions (cf. Matsuzoe 2013).

As an application, we study generalizations of maximum likelihood estimators. It is known that a deformed exponential family admits deformed algebras, e.g., the  $\kappa$ -sum and the  $\kappa$ -product in  $\kappa$ -statistics case (cf. Kaniadakis 2013). These deformed algebras cause generalization of independence of random variables, and we find that the maximum likelihood method should be modified according to this generalization of algebras. We also find that this generalization of maximum likelihood method is quite natural from the viewpoint of differential geometry.

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## Linear and nonlinear photonic rogue waves in complex transparent media

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Rogue waves (RWs) have for long triggered the interest of scientists because of their intriguing properties. They are extreme coherent waves with huge magnitude which appear suddenly from nowhere and disappear equally fast. RWs firstly appeared in relatively calm seas but recent works have demonstrated that rogue wave-type extreme events may appear in various of physical systems such as microwaves, nonlinear crystals, cold atoms and Bose-Einstein condensate, as well as in non-physical systems such as financial.

RW pattern formation emerges in a complex environment and it still unclear if their appearance is due to linear or nonlinear processes. Intuitively, one may link the onset of RW pattern formation to a resonant interaction of two or more solitary waves that may appear in medium, subsequently it has been tacitly assumed that extreme waves are due to nonlinearity. However, large amplitude events may also appear in a purely linear regime; a typical example is the generation of caustic surfaces in the linear wave propagation.

In this work we investigate optical wave propagation in a strongly scattering optical media that consist of Luneburg-type lenses randomly embedded in the bulk of transparent glasses. Spherical or cylindrical Luneburg lens (LL) has very strong focusing properties directing all parallel rays impinging on them to a single spot on the opposite side surface. Both in the simulation and in the experimental configuration used in this work, we use "Luneburg holes (LH)" or anti-Luneburg lenses instead of original LLs; the LHs have a purely defocusing property. The index of refraction for LLs as well as for LHs is very large, viz. of the order of 40% and thus a medium with a random distribution of LHs can be characterized as a strongly scattering random media. Finally, to confirm the broader validity of our results we investigated numerically the propagation of EM plane waves through other random systems with different scatterer types such as original LL and ordinary lenses with constant refractive index (flat lenses).

By analysing the EM wave propagation in the linear regime we observe the appearance of RWs that depend solely on the scattering properties of the medium. Interestingly, the addition of weak nonlinearity does not modify neither the RW statistics nor the position where a linear RW appears. Numerical simulations agree nicely with the experimental findings and altogether prove that optical rogue waves are generated through the linear strong scattering in such complex environment.

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### Non-classical regimes of colloid-facilitated impurity transport in statistically homogeneous double porosity media

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A description of impurity transport basing on dual porosity model implies that the medium of migration can be represented as a superposition of two overlapping homogeneous subsystems [1]: 1) fast subsystem, which is the ensemble of well-conducting channels providing impurity transport over large distances, and 2) slow subsystem which retard the motion of impurity particles. Local relaxation time of impurity distribution between subsystems and inside slow subsystem as well may be sufficiently large. This leads to the appearance of a set of non-classical transport regimes [2], when the evolution of impurity concentration is not described by the Gaussian profile. For example, sub-diffusion and quasi-diffusion take place in certain time intervals. On the other hand of no less importance is the possibility for active impurity (impurity in the first subsystem) to transfer into a phase, which does not interact with slow subsystem (e.g., to be adsorbed on colloidal particles [3]). In this case the action of traps may be temporarily suppressed that leads to an effective acceleration of impurity transport. In the present work we consider a model of colloid-facilitated impurity transport in statistically homogeneous double porous (fractured-porous) medium. If there are no any colloids then a retardation of transport of dissolved impurity over fractures is determined by impurity diffusion into weakly permeable porous matrix. Colloids do not leave into matrix due to their comparatively large size. The impurity adsorbed onto colloidal particles also stay in the fast subsystem and is transported with the velocity of colloids. At large times colloids with the adsorbed impurity come into the regions far from the source, which are depleted with the dissolved impurity. Here a re-resolution of impurity from colloids into liquid phase and its final leaving into the matrix take place. Thus the behavior of impurity is described by a set of transport regimes. Consequence of these regimes and boundaries of time intervals, where these regimes are realized, depend on characteristic times of separate processes (for sorption/desorption rate and leaving of solute into matrix) and on capacity of subsystems (power of traps and partition coefficient between solute and adsorbed fractions) as well. A type of transport regime in a given time interval is determined by a self-consistent action of all pointed above mechanisms. In the present work the consequences of transport regimes and corresponding time intervals have been described for different values of medium parameters.

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### Influence of mobile second phase on impurity behavior in double porous media: application to colloid-facilitated transport

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A description of impurity transport basing on dual porosity model implies that the medium of migration can be represented as a superposition of two overlapping homogeneous subsystems [1]: 1) fast subsystem, which is the ensemble of well-conducting channels providing impurity transport over large distances, and 2) slow subsystem which retard the motion of impurity particles. Local relaxation time of impurity distribution between subsystems and inside slow subsystem as well may be sufficiently large. This leads to the appearance of a set of non-classical transport regimes [2], when the evolution of impurity concentration is not described by the Gaussian profile. For example, sub-diffusion and quasi-diffusion take place in certain time intervals. On the other hand of no less importance is the possibility for active impurity (impurity in the first subsystem) to transfer into a phase, which does not interact with slow subsystem (e.g., to be adsorbed on colloidal particles [3]). In this case the action of traps may be temporarily suppressed that leads to an effective acceleration of impurity transport. In the present work we consider a model of colloid-facilitated impurity transport in statistically homogeneous double porous (fractured-porous) medium. If there are no any colloids then a retardation of transport of dissolved impurity over fractures is determined by impurity diffusion into weakly permeable porous matrix. Colloids do not leave into matrix due to their comparatively large size. The impurity adsorbed onto colloidal particles also stay in the fast subsystem and is transported with the velocity of colloids. At large times colloids with the adsorbed impurity come into the regions far from the source, which are depleted with the dissolved impurity. Here a re-resolution of impurity from colloids into liquid phase and its final leaving into the matrix take place. Thus the behavior of impurity is described by a set of transport regimes. Consequence of these regimes and boundaries of time intervals, where these regimes are realized, depend on characteristic times of separate processes (for sorption/desorption rate and leaving of solute into matrix) and on capacity of subsystems (power of traps and partition coefficient between solute and adsorbed fractions) as well. A type of transport regime in a given time interval is determined by a self-consistent action of all pointed above mechanisms. In the present work the consequences of transport regimes and corresponding time intervals have been described for different values of medium parameters.

[1] H.H. Gerke, and M.T. van Genuchten, *Water Resour. Res.* **29**, 305 (1993).

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## Molecular Dynamics study of the elasticity of a lipid membrane

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The pulmonary surfactant system is a lipid-protein complex that surrounds the pulmonary alveoli. Assembled in an air-liquid interface, it is the largest surface contact that air breathing vertebrates have with its surrounding [1]. About 80% of the pulmonary surfactant is made of phospholipids, 5-10% of neutral lipids, mainly cholesterol, and 8-10% of proteins. Its structure results from the amphiphilic property of the phospholipids. Phospholipids own a polar head (hydrophilic) and a hydrophobic tail such that in an aqueous interface, phospholipids usually self-organize in the form of a bilayer. In the case of an air-liquid interface, they form a monolayer with the head group next to the water and the tail pointing toward the air [1].

It is also known that the principal active molecule in the surfactant epithelium is the dipalmitoylphosphatidylcholine (DPPC). This component can be highly packed reducing the surface tension below 2 mN/m although as a result of its high packing at physiological temperature, it is found in a gel phase. Gelation hinders the reduction of the surface tension and in addition reduces its lateral mobility. It is here when a second relevant molecular species, palmitoyl-oleoylphosphatidylcholine (POPC), plays a crucial role. This new type of molecule, unlike DPPC, has an asymmetry in one of its tails, consequently reducing the packing of DPPC in membranes made of DPPC and POPC. Besides, the presence of POPC makes the membrane be in a fluid phase due to its low melting temperature ( $-3^\circ$ ) and helps to reduce the surface tension even further.

In this work we use Molecular Dynamics simulations to obtain values of the bending and diffusion coefficients for DPPC membranes mixed with a second species of molecules, POPC. The goal of this study is the characterization of the bending and diffusion coefficients as a function of the concentration of POPC molecules in the lipid. We use the MARTINI force field for phospholipids [2] and an effective model for water molecules. Our preliminary conclusions indicate that the asymmetry coming from the POPC molecules makes packing more difficult and, as a result, the density of the membrane decreases as the concentration of POPC increases. We also show how the bending modulus increases with the concentration of POPC. We believe that the asymmetry coming from POPC molecules acts as a hook so that molecules are entangled harder, and hence the cost to bend the bilayer rises

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## Structural properties of complex networks

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Many complex systems, both natural [1], and man-made [2, 3], can be represented as multiplex or interdependent

networks. Multiple dependencies make a system more fragile: damage to one element can lead to avalanches of failures throughout the system [4, 5]. Recent theoretical investigation of two [6] or more [7] networks in which vertices in each network mutually depend on vertices in other networks has shown that indeed small initial failures can cascade back and forth through the networks, leading to a discontinuous collapse of the whole system. Damage in one network propagates along edges and leads to damage in the other network. This is an individual stage of a cascade in back-and-forth damage propagation. Son et al. showed that this approach can be simplified and is equivalent to considering damage propagation in multiplex networks. They proposed a simple mapping between the model used in [6] in which a vertex in one network has a mutual dependence on exactly one vertex in the other network, and a multiplex network with one kind of vertex but two kinds of edges. The mapping is achieved by simply merging the mutually dependent vertices from the two networks. In this talk I will revisit a number of well-studied problems concerning structural properties of complex networks. Some concepts like percolation, k-core organization, bootstrap percolation and avalanche collapse of the giant viable component in multiplex networks are well well-known to the audience but I will present them in a different perspective showing the recent analytical advances from a network theory point of view. In contrast, we study the avalanches of damage triggered by the removal of randomly chosen single vertices. These avalanches increase in size approaching the critical point, signaling the impending collapse of the giant viable cluster. At the critical point the mean avalanche size diverges. Below the transition, on the other hand, there is no precursor for the appearance of the giant viable cluster. The transition is thus asymmetric. I will show that however different problems they share some common features like a hybrid phase transition.

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## New Physics learned from single particle tracking

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In 1905 Albert Einstein formulated the laws of diffusion, and in 1908 Jean Perrin published his Nobel-prize winning studies determining Avogadro's number from diffusion measurements. With similar, more refined techniques the diffusion behaviour of submicron tracer particles or even single molecules is now routinely measured in the complex environment of living biological cells and similarly complex systems. It is frequently observed that the passive diffusion of such particles deviates from Einstein's laws.

This talk will discuss the basic mechanisms leading to such anomalous diffusion as well as point out its physical

consequences. In particular, it will be discussed how we can understand the experimental observation of weakly non-ergodic behaviour (time and ensemble averages of physical observables behave differently) and ageing (physical observables depend on the time span between initial preparation of the system and start of the measurement).

More specifically, it will be shown how within the continuous time random walk model with scale-free waiting time distribution the weakly non-ergodic behaviour influences time averaged physical observables and their irreproducibility. Interestingly, quite similar features are observed for multiplicative Markovian processes with space-dependent diffusion coefficient. It is therefore important to have available a range of diagnostic tools to analyse the stochastic character of anomalous diffusion processes. These include the velocity-velocity correlation function, higher order moments, the  $p$ -variation method, the distribution of the amplitude scatter of individual realisations of the stochastic process, the ageing dependence, and others. The results will be rationalised for experimental data from single particle tracking in living biological cells and extensive molecular dynamics simulations. Ageing, the explicit dependence of a physical observable on the time span between the initial system preparation and the start of the measurement, is the second main effect of non-stationary processes such as the continuous time random walk model. We will investigate how time averaged observables are affected by the ageing of the system. Intriguingly, it turns out that in time averaged the ageing simply factorises, i.e., the dynamics of the system is just corrected by a prefactor. This is much simpler than ageing effects in ensemble averaged observables, where crossovers between different scaling regimes emerge.

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### Coupled genetic circuits: a control mechanism for the onset and arrest of collective oscillations

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First we consider a system of only two genetic circuits, coupled with mutual repression. Each of these circuits is composed of a positive and a negative feedback loop in a motif, which is frequently found in genetic and neural networks. We analyze the behavior of this coupled system as a function of the coupling strength and the maximal growth rate of one of the species, which serves as a bifurcation parameter also for a single unit. The phase diagram shows eight regimes of qualitatively different behavior such as collective limit cycles or fixed points and different forms of multistable states. Depending on the choice of the initial conditions and the variation of coupling and bifurcation parameter, the system of four variables can switch between a multitude of states. Moreover we present a detailed bifurcation analysis and identify the birth mechanisms of limit cycles such as super- and subcritical Hopf bifurcations, fold and pitchfork bifurcations. Our synthetic system may serve as a toggle switch and work as a cyclic nano-device if the bifurcation parameter and the

coupling are accordingly tuned. In particular, in case of multistable states we show how to control the return to the initial state once one cycle in bifurcation-parameter space has been performed.

Next we couple a larger set of these units of the order of a few thousands, again with repressive bonds; we then observe collective oscillations for an intermediate parameter interval between collective fixed-point behavior. In this way, by monotonically varying a single bifurcation parameter, we can control the onset and arrest of these oscillations, which result from bifurcations of the same system, and therefore the duration of the oscillation period. This mechanism may be of interest for the oscillations in a segmentation clock which last over a finite interval of time, where the underlying mechanism for their arrest has not been identified so far. Depending on the coupling strength, the network topology and the tuning speed of the bifurcation parameter, our numerical simulations reveal a rich dynamics out-of-equilibrium with multiple inherent time scales, long transients towards the stationary states and interesting transient patterns. Most interesting is a self-organized pacemaker that corresponds to dynamical symmetry breaking, since all oscillators are equipped with a uniform set of parameters. If we zoom into the transition regime between oscillatory and fixed-point behavior, we can pursue the arrest of oscillations as it proceeds along the arms of spirals. Here it is challenging to understand what determines the spatial dependence of the phase conversion until all units cease to oscillate. Familiar concepts like a competition between the free energy and the interface tension between the different phases are not applicable to our dynamics, which is defined by a set of differential equations.

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### Ageing of Classical Oscillators during a Noise-Driven Migration of Oscillator Phases

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We consider classical nonlinear oscillators like rotators and Kuramoto oscillators on hexagonal lattices of small or intermediate size. When the coupling between the elements is repulsive and the bonds are frustrated, we observe coexisting states, each one with its own basin of attraction. Depending on the parameter choice, these are fixed points, limit cycles, or quasi-periodic solutions. States, which correspond to collective limit cycles, differ in particular by their degree of synchronization and by their patterns of phase-locked motion. For special lattices sizes the multiplicity of stationary states gets extremely rich. When disorder is introduced into the system by additive or multiplicative Gaussian noise, we observe a non-monotonic dependence of the degree of order in the system as a function of the noise intensity: intervals of noise intensity with low synchronization between the oscillators alternate with intervals where more oscillators are synchronized. In the latter case, noise induces a higher

degree of order in the sense of a larger number of nearly coinciding phases. This order-by-disorder effect is reminiscent to the analogous phenomenon known from spin systems. The non-monotonic evolution of the degree of order is found not only for a single interval of intermediate noise strength, but repeatedly as a function of increasing noise intensity. So we observe a noise-driven migration of oscillator phases in a rather rough potential landscape, presumably with hierarchies in the potential barriers. Upon this migration, a multitude of different escape times from one metastable state to the next is generated. The histogram of escape times characterizes what may be called the stationary state. Based on these observations, it does not come as a surprise that the set of oscillators shows physical aging. Physical aging is more specific than the generic phenomenon of aging which refers to a slower recovery from a perturbation the older the system is. It is characterized by nonexponential relaxation after a perturbation, breaking of time-translation invariance, and dynamical scaling. When our system of oscillators is quenched from the regime of a unique fixed point toward the regime of multistable limit-cycle solutions, the autocorrelation functions depend on the waiting time after the quench, so that time translation invariance is broken, and dynamical scaling is observed for a certain range of time scales. These are typical indicators of aging also in glassy systems like spin glasses or aging materials. In this way we uncover the rough potential landscape along with a multitude of inherent time scales as a common mechanism behind aging in quite different realizations.

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### Towards an understanding of the Maximum Entropy Production in climate toy models.

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The Principle of Maximum Entropy Production (MEP) states that a non-equilibrium stationary system selects the state that maximizes entropy production while taking into account some constraints. Though many attempts to demonstrate this principle did not lead to convincing results, the MEP principle has been successfully used in many physics fields and notably in climate science. It allows to obtain well suited approximations of global temperature and heat fluxes distributions very fast as compared to conventional climate models. However a rigorous definition of the MEP principle is still needed. We tackle this problem by using climate toy models for which dynamical analogues of the thermodynamical quantities can be defined.

More precisely, we studied two classical models coming from statistical physics that can be considered as similar to those used in climate science and that use the MEP principle: the Asymmetric Simple Exclusion Process (ASEP) and the Zero Range Process. They are 1D stochastic systems on which particles can move following probability transitions. They are put out of equilibrium by a coupling between two reservoirs at

the edges. After a certain amount of time the system reaches a stationary state. Analogously to the equilibrium situation, we consider that the dynamical rules are not exactly known. Allowing for the transition rate to vary, we show that the dynamical rules that maximize the entropy production and those that maximize the rate of variation of the dynamical entropy, known as the Kolmogorov-Sinai entropy, coincide with good accuracy. We show this numerically in the ASEP model and in some of its variants like the Langmuir Kinetics-ASEP and the Constrain-ASEP models. We then demonstrate this result analytically in first order of the deviation from equilibrium in the Zero Range Process. Moreover the dynamical rules chosen by MEP are physically coherent, meaning that close to equilibrium the rules are diffusive whereas the more out-of-equilibrium the system is the more convective the rules become. These results point to an existing link between these two empirical principles suggesting some basis for the improvement of the MEP principle formulation.

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### Exogenous shocks in financial markets: Statistical analysis using business news

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Recently, we showed clearly that frequency of financial news has same statistical laws as financial market activity [Mizuno, et. al. 2012]. We investigated what kind of news topic moves the market in the long run [Hisano, et. al. 2013]. Market prices often move in response to news [Joulin, et. al. 2008].

In this presentation, we use raw texts of more than 30 million news records provided by Thompson Reuters and tick data in NYSE from 2003 to 2014 in order to study their impact on market price. We focus on "novelty and topicality" of news. First, we define the "novelty and topicality" by estimating the difference between news. The news including many words which appeared in past news probably has low novelty, because we may be able to predict the contents of news from the past news. We measure the similarity between news at time  $t$  and news at time  $t + \tau$  using mean of cosine similarity. The cosine similarity slowly decays with time interval  $\tau$ . The decay function has a power law. We define novelty of news by the reciprocal of cumulative cosine similarity with past news. We also define topicality of news. News that are published by about 300 news agencies are included in the news database. Each news agency has different subscribers. Probability, the topicality of news reported by many news agencies has high. We measure topicality of news from cosine similarity of news at same time between different news agencies.

Next, we focus on "volatility", "number of transaction" and "trading volume" in order to measure market activity, and show change of market activities (volatility, number of transaction, trade volume) depend on size of novelty and topicality of news. When news has low novelty and low topicality, the market activities hardly respond. On the other hand, when news has high novelty and high topicality, market activities go up rapidly just after the news. The tail of distribution

of market activities grows fat. For example, the tail of volatility distribution usually has a power law with exponent -3. However, the tail follows power distribution with exponent -2 just after the news. The effect of news shock exponentially decays after the news. Finally, we investigate relationships between sign of price change and terms in news article. The terms which increase stock prices exist statistically only when the news has high novelty and high topicality. Many positive words are contained in these terms for price increase. We also will talk about characteristics of these terms in this conference.

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### Discretized kinetic theory on a network as a tool for the study of economic interactions

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The network of interpersonal connections is one of the main factors which affect the income distribution emerging from micro-to-macro models. In particular, the assortative or disassortative nature of the network could play a role. The insertion of a network structure into the evolution equations of kinetic theory is not trivial and turns out to impose constraints on the network itself. Our model (as discussed in Bertotti and Modanese, 2012) is based on a system of differential equations of the kinetic discretized-Boltzmann kind. Society is described as an ensemble of individuals divided into a finite number of income classes; the individuals exchange money through binary interactions, leaving the total wealth unchanged. The interactions occur with a certain predefined frequency, and several other parameters can also be adjusted. For instance, we can fix the probability that in an encounter between two individuals the one who pays is the rich or the poor; we can make the exchanged amount depend on the income classes (variable saving propensity), etc. After a sufficiently long time the solution of the equations reaches an equilibrium state characterized by an income distribution, which depends on the total income and on the interaction parameters, but not on the initial distribution. A kinetic model can be equipped with a network structure in a probabilistic way, through the introduction of correlation coefficients  $P(\beta|\alpha)$ , which give the conditioned probability that an individual with  $\alpha$  links is connected to one with  $\beta$  links. An approach of this kind has been proposed by (Boguna et al. 2003), who have described the diffusion of epidemics through differential equations giving the probability for each individual to be infected and infect in turn others. It is well known that in this case the network structure is crucial, and that the hubs have a fundamental role in the transmission of the disease. Also in our model a network can be introduced in a similar way. However, some important differences naturally arise in the structure of the equations and in the conditions satisfied by the correlation coefficients. The reason is that in our model describing economic exchanges, money is conserved, while in a contagion process, unfortunately, the disease can multiply for free. We analyze these issues in general form and give results concerning some particular numerical solutions for the asymptotic distributions  $x_i^\alpha(t)$ , where  $t \rightarrow \infty$  and  $x_i^\alpha(t)$  denotes the density of individuals belonging to the  $i$ -th in-

come class and having  $\alpha$  economic links with other individuals.

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### Density distribution of the molecules of a liquid in a semi-infinite space

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The variation of the number density in a liquid, as a consequence of a boundary discontinuity, is often disregarded in many applications. Nevertheless, this subject is of growing interest in many fields such as in the study of thin films, or in biology, where investigations are becoming more and more detailed with decreasing dimensions (Plech 2001, Tadmor 2001). The aim of this work is to obtain an estimate of the number density distribution in a liquid as a function of the distance from a discontinuity when the molecules of the system are in the state of thermodynamic equilibrium (the velocity distribution function is the Maxwellian distribution) and the average velocity is equal to zero. The physical situation refers to a liquid composed of certain molecules surrounded by a medium of different molecules that can be in various phases (solid, liquid or gas). We will also consider the case of a liquid neglecting the effects of the surrounding medium (in other words a liquid bordering vacuum). The problem of a liquid in contact with its vapour has been treated in (Frezzotti 2005). Our study is performed in the framework of kinetic theory to obtain some liquid properties as a consequence of discontinuities. In the case of a liquid phase, the effects of the interaction between molecules cannot be disregarded. An estimate of this effect is obtained resorting to inclusion of a self-consistent field (the Vlasov field). The self-consistent field is appropriate to describe the dynamic behaviour of a system where every molecule interacts simultaneously with a large number of surrounding molecules and the correlation between pairs of molecules can be disregarded since its effect becomes negligible with respect to the collective interaction. This physical situation occurs when the range of the intermolecular force is large in comparison to the average distance between molecules, as is the case of a plasma or of liquid state. From this point of view, interactions in a liquid system are treated somewhat like is done in plasma physics when there are enough particles in the so-called Debye sphere and simultaneous multiple collisions prevail. To calculate the Vlasov self-consistent field, several phenomenological potential models have been proposed which, more or less, describe the real multiple force interaction mechanism. In this work the Sutherland pair potential model is used because it is mathematically simple and the predictions on the liquid properties are expected to be qualitatively consistent with the behaviour of many real fluids.

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## Free Energy Evaluation in Polymer Translocation via Jarzynski Equality

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The process of polymer translocation occurs in many biological and biotechnological phenomena. It has received great attention in both experimental and theoretical studies in recent years due to its important role in many crucial biological processes, such as mRNA translocation across a nuclear pore complex, drug delivery, injection of DNA from a virus head into a host cell and gene therapy. However due to the complexity of the interactions involved, especially between the pore and the membrane, computer simulations have been widely used as a fundamental research tool. Most of the numerical studies can be classified into the topical issues of (i) translocation driven by chemical potential gradients (ii) translocation driven by external forces, and (iii) unbiased translocation. The first observation of polymer translocation was done experimentally by Kasianowicz *et al.*, using ssDNA fragments driven through a narrow  $\alpha$ -hemolysin nanopore in a biomembrane by application of an external voltage. Kasianowicz *et al.* observed that the mean translocation time,  $\langle\tau\rangle$ , scales linearly with polymer length,  $N$ , and inversely with the potential difference across the pore. Putting together the Fokker-Planck approach and classical nucleation theory, Sung and Park and Muthukumar theoretically predicted that the translocation time scales with polymer chain length  $N$  as  $\langle\tau\rangle \sim N$  for driven translocation and  $\langle\tau\rangle \sim N^2$  for the case of nondriven translocation. In 2002, Chuang *et al* pointed out that the estimate of the translocation time,  $\tau_{trans} \sim N^2$ , based on the Fokker-Planck equation for the first-passage time over the membrane entropic barrier couldn't be correct since  $\tau_{trans}$  cannot be smaller than the Rouse time, which scales as  $\tau_R \sim N^{1+2\nu}$ . We perform, with the help of cloud computing resources, extensive Langevin simulations which provide free energy estimates for unbiased three dimensional polymer translocation. We employ the Jarzynski equality in its rigorous setting, to compute the variation of the free energy in single monomer translocation events. In our three-dimensional Langevin simulations, the excluded-volume and van der Waals interactions between beads (monomers and membrane atoms) are modeled through a repulsive Lennard-Jones (LJ) potential and consecutive monomers are subject to the Finite-Extension Nonlinear Elastic (FENE) potential. Analysing data for polymers with different lengths, the free energy profile is noted to have interesting finite size scaling properties.

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## Model of Air Traffic Control as a Local Optimization on a Navigation Point Network

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The importance of air transport has considerably grown in time, being nowadays an essential fast mean to connect

national and international locations. Although in recent years the competition with other means of transportation, like high-speed railways, and the recent economical crisis have slowed down the air traffic growth that had been forecast in the first years of the new millennium, an increase of traffic load is still expected due to the competitiveness of new Eastern and Middle Eastern economies [1]. This growth of traffic could bring the actual Air Traffic Management system (ATM) over its capacity limits mainly leading to airport congestions, but also to a situation in which safety standards and performances during the en-route phase of a flight might not be guaranteed anymore. Thus, it is important to study and understand the limits and the criticality of the current ATM system during this phase of a flight, seeking for new solutions aimed at improving its capacity. Complex Systems Physics has already proven to be useful in dealing with the criticality of many transportation systems [2,3].

In the current ATM system each aircraft is supposed to fly over predefined airways between some geographical references called navigation points, while safety standards are guaranteed by air traffic controllers, whose duty is to prevent aircraft from getting too close each other. To do that, controllers can easily perform the required redirections without the need of relying on the established preexistent airways. In this paper we present a model of Air Traffic Control (ATC), where air traffic is regulated by controllers who provide the necessary safety separation between aircraft while, at the same time, trying to minimize flight delays. We model the action of the controllers as a local search algorithm between the nodes of a Navigation Point Network, i.e. the network of geographical references over which aircraft fly. Each time a dangerous loss of separation is detected, controllers look for nodes toward which to reroute the aircraft so to provide a safe trajectory with the least possible delay. As the traffic load increases, the model shows a transition from a phase in which all safety conflicts are resolved irrespective to the traffic pattern injected in the system, to a phase in which conflicts cannot be resolved anymore. The model is tested onto a synthetic regular airspace with periodic boundary conditions and then applied to the actual airspaces. The model is then validated by reproducing some regular days of operation within a national airspace in Europe and by comparing the results with historical data.

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## Entanglement stabilization assisted by a nonlinear oscillator coupled to a qubit through a conditional displacement Hamiltonian

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Quantum entanglement is an essential feature in quantum theory, and it is recognized of paramount importance in quantum technologies and quantum information science. Stimulated by this, the aim of this work is to exploit and improve this fragile resource in a qubit-oscillator system, both

components corresponding to the two most basic building blocks in quantum information.

Specifically, we investigate a qubit coupled to a quartic (undriven) nonlinear oscillator (NLO) through a conditional displacement Hamiltonian. In the last decades, quantum NLO have been implemented in several settings, including trapped ions [1], optomechanical systems [2], etc. Furthermore, the inclusion of nonlinearities in the oscillator potential allows new possibilities to generate non-classical states [3, 4]. Motivated by this, we consider an initial separable state composed of a qubit superposition state and a coherent state for the oscillator. Throughout this work we use two relevant parameters, namely, the qubit-NLO coupling and the nonlinearity for the NLO potential. We first recalled the results for the case in absence of nonlinearity. Here, the entanglement generation is due to the superposition principle of the hybrid system and it shows a periodic dynamics. On the other hand, when the nonlinearity is present in the system and in the weak qubit-NLO coupling regime, we analytically—under the rotating-wave approximation—show that a new Kerr-like term appears in the dynamics leading to i) quadrature squeezing of the oscillator state for short times, ii) the suppression of the entanglement decay by the appearance of a stabilization region, and iii) an enhancement of the entanglement negativity compared to the linear case.

The most interesting case corresponds to the strong qubit-NLO coupling; here we see that two- and four-phonon transitions play a relevant role both in the entanglement stabilization and in its enhancement. In this case i) the entanglement stabilization region is achieved faster than in the weak coupling case, ii) the entanglement negativity can reach its maximal value by virtue of the orthogonalization of the oscillator states relevant to the present dynamics, where the quadrature squeezing is lost even for short times. In fact, the oscillator shows an intricate behavior exhibiting negative values in the Wigner distribution.

Finally, as it is very well-known the quantum entanglement is a fragile resource and quite susceptible to decoherence, hence we solve numerically the corresponding master equation, concluding that these effects remain robust to the presence of decoherence in the oscillator system.

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## Potential Magnetic Field Extrapolation

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We demonstrate available modules of the MPI-AMRVAC open source code (Porth et al. submitted ApJS 2014, Keppens et al. JCP 2012) of frequently used models for global spherical (PFSS) models, as well as for local Cartesian box models (Green function based), and make some observations on their accuracy. We can routinely use synoptic magnetograms as inputs from GONG observations at resolution  $180 \times 360$  and from MDI at resolution  $1080 \times 3600$ . We make use of these

magnetograms after performing the magnetogram remeshing technique, similar to Chebyshev collocation method to interpolate a  $\cos(\theta)$  grid, with denser grid points at the poles than in the middle, onto a uniform  $\theta$  grid. Here we present a study for the solar Carrington rotation number *CR2029* in 2005, using observations from the space telescope instrument MDI. We focus on two active regions within *CR2029*, one located at the North hemisphere *AR10759* and one at the South hemisphere *AR10756*. These two dominant active regions on each hemisphere will be used to (1) compare the global potential field source surface spherical extrapolation approach and a local potential field Cartesian one, and (2) to understand the influence of raising the number of spherical harmonics. For the latter, we will take active region *AR10756* as the photospheric region on which we examine the radial magnetic field variation over a line crossing the active region's opposite polarities, as influenced by the number of spherical harmonics used.

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## Wave propagation and collisionless damping in quantum liquids

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A quantum liquid is a system of strongly interacting molecules at temperature sufficiently low that the effects of quantum mechanics becomes important, and this with varying degrees of degeneracy. Quantum degeneracy is evaluated by the Sommerfeld parameter

$$\frac{n\hbar^3}{\gamma(2\pi mK_B T)^{\frac{3}{2}}}$$

the value of 1 being the limit between high and low degeneracy. The theory of quantum liquid may be said to have had its origin in 1956 when Landau introduced the concept of “quasi-particles” to describe the macroscopic behaviour of  $\text{He}^4$ .

Many substances become liquid at moderately low temperatures, so that the Sommerfeld parameter is only slight less than one and then the degeneracy is present, but low. Aim of the present work is to investigate the problem of longitudinal wave propagation in a quantum liquid, emphasising, in particular, the existence of wave damping without energy dissipation by collisions, akin to the phenomenon of Landau damping in plasmas - both classic or degenerate. The dispersion equations will be obtained together with the expressions for wave damping, for fully and slightly degenerate Fermi and Bose liquids, starting from the quantum kinetic equation (QKE) as can be derived in the framework of the quantum Bohm potential. The effects of interactions between the molecules of the system will be accounted for through a self-consistent field: this approach, leading to a quantum Vlasov equation, is appropriate for liquids, since molecules therein are subjected to simultaneous interactions with a large number of surrounding molecules, and hence correlation is negligible - an approach often referred to as mean-field, or central-field model. The self-consistent field is derived from a modified Lennard-Jones model

$$U_{1,2}(r) = \begin{cases} \infty & \text{for } r \leq \sigma \leq r_0 \\ 4\epsilon \left[ \left(\frac{r_0}{r}\right)^{12} - \left(\frac{r_0}{r}\right)^6 \right] & \text{for } r > \sigma \end{cases}$$

Calling  $F_L$  the self consistent field and  $F_Q$  the force due to the Bohm potential, the governing quantum Vlasov equation is

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} + \frac{\mathbf{F}_L + \mathbf{F}_Q}{m} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

Starting from the above QVE and considering small perturbations, dispersion relations are obtained for longitudinal waves in quantum liquids.

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## Dispersion Relation of Longitudinal Waves in Liquid He-4 in the framework of the Bohm interpretation of QM

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At the temperature of 2.18 K liquid helium undergoes an abrupt transition, well seen experimentally, e.g., in specific-heat experimental measurements: the shape of the  $(C_V - T)$  curve resembles the greek letter  $\lambda$  and the transition temperature is often called the  $\lambda$ -point. This anomalous behaviour of liquid helium has been explained with a two-fluid model: the normal liquid helium, known as He-I, and a new state called He-II that appears at the transition temperature. Tisza was the first to put forth a phenomenological theory using the two-fluid concept, in 1938; in 1941 Landau developed in another form a theory founded on this model. Landau also investigated the problem of the propagation of ordinary sound waves, in which the two fluids move together to create a pressure disturbance moving at about 240 m/s. Landau proposed an energy-momentum spectrum of the elementary excitations in liquid helium at temperatures under the  $\lambda$ -point that has a striking resemblance to what is found experimentally. This spectrum passes through a maximum that is followed by a minimum and then rises again, approximately linearly; to represent this point of the spectrum Landau introduced ad hoc characteristic excitations that he called rotons, and proposed an energy-momentum spectrum of the elementary excitations in liquid helium at temperatures below the  $\lambda$ -point, that was later substantially confirmed experimentally by Yarnell et al., who determined the dispersion relation of waves (the so-called first sound) in superfluid He<sup>4</sup> at 1.1 K: the dispersion relation shows a non monotonic behaviour - with a maximum and a minimum - usually explained in terms of Landau's rotons. In some authors' view, rotons are thought to be phonons of a wavelength close to interatomic spacing; others propose quantized vortices as a model for rotons. The aim of the present work is to obtain the energy spectrum of the elementary excitations in liquid helium at temperatures below the  $\lambda$ -point, within the framework of the quantum kinetic Vlasov equation as can be obtained in the Bohm potential approach. This is complemented by a Lennard-Jones type potential, modified to account for a distance of minimum approach depending, even though very slowly, from wavelength, following Feynman's line of thought. This yields a self-consistent field that, together with the force due to the Bohm potential, leads to an energy-momentum spectrum that reproduces quite interestingly the experimental

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## On the Gap and Time Interval between the First Two Maxima of Long Random Walks

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Random walks (RW) are a very good laboratory to study extreme and order statistics of strongly correlated random variables. Indeed, although extreme value questions are very well understood in the case of independent and identically distributed (i.i.d.) random variables, much less is known in the case of strongly correlated variables. Random walks and Brownian motion, together with some rare examples like random matrix theory, are scarce physically relevant models where exact analytical solutions have been obtained.

In this context, we have investigated the statistics of the gap,  $G_n$ , and the number of time steps,  $L_n$ , between the two rightmost positions of a Markovian one-dimensional random walker, starting from  $x_0 = 0$ , after  $n$  time steps. The jumps are i.i.d. random variables drawn from a symmetric probability distribution function (PDF),  $f(\eta)$ , the Fourier transform of which has the small  $k$  behavior  $1 - \hat{f}(k) \propto |k|^\mu$ , with  $0 < \mu \leq 2$ . For  $\mu = 2$ , the variance of the jump distribution is finite, and the RW (properly scaled) converges to a Brownian motion. For  $0 < \mu < 2$ , the RW is a Lévy flight of index  $\mu$ .

First, we show that the joint PDF of  $G_n$  and  $L_n$  converges to a well defined stationary bi-variate distribution  $p(g, l)$  as the RW duration  $n$  goes to infinity. Then, we present some results of a thorough analytical study of this limiting joint distribution  $p(g, l)$ , as well as of its associated marginals  $p_{\text{gap}}(g)$  and  $p_{\text{time}}(l)$ . Among the rich variety of behaviors we have found, depending on the tail of  $f(\eta)$ , we focus on the two particular cases where this tail is (i) algebraic and (ii) super-exponential. In Case (i), we find that  $p(g, l)$  takes the scaling form  $p(g, l) \sim g^{-1-2\mu} F_\mu(lg^{-\mu})$  when both  $l$  and  $g$  are large, where  $F_\mu(y)$  is a  $(\mu$ -dependent) scaling function. From this result we show that  $p_{\text{time}}(l)$  has an algebraic tail  $p_{\text{time}}(l) \sim l^{-\gamma(\mu)}$  for  $l \gg 1$  with  $\gamma(1 < \mu \leq 2) = 1 + 1/\mu$ , and  $\gamma(0 < \mu < 1) = 2$ . (For  $\mu = 1$  one gets  $p_{\text{time}}(l) \sim l^{-2} \ln(l)$ ). It follows in particular that  $\langle |l| \rangle = +\infty$  for all  $0 < \mu \leq 2$ . In Case (ii), we find that  $p(g, l)$  for large  $g$  concentrates onto the two symmetric values  $l = \pm 1$ , which corresponds to configurations where the first two maxima are next to each other. Finally, we reconsider the problem for a random bridge where the random walk starts and ends at the origin after  $n$  time steps. Our analytic results are borne out by numerical simulations that will also be presented.

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## Quantum Walks on Complex Networks

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The dynamics of excitations on complex structures is at the heart of every transport process in science, ranging from the natural sciences (physics, chemistry, biology) to the social sciences. In a basic approximation the systems of interest can be described by a network of  $N$  of nodes (e.g., atoms, proteins, individuals, etc.). In order to describe the dynamics of an excitation over such networks, one distinguishes two limiting cases, the pure diffusive case, described by so-called Continuous-Time Random Walks (CTRW), and the pure coherent case, described by Continuous-Time Quantum Walks (CTQW), a variant of Schrödinger's equation. If the Hamiltonian  $\mathbf{H}$  of the CTQW is identified with the transfer matrix  $\mathbf{T}$  of the CTRW, there is the obvious advantage that both operators have the same spectrum and the same eigenmodes / eigenstates. The global transport efficiency is captured by the average probability to return to (or remain at) the initial node  $j$ , which has a lower bound given by the Cauchy-Schwarz inequality [1]. The topology of the network will be of crucial importance. As a first example of the strikingly different dynamical behavior of CTQW and CTRW, we show that very symmetric networks like the star graph or the complete graph display low transport efficiencies of the CTQW, much worse than in the CTRW case. This can be overcome by topological disorder, where bonds are added to the star graph, eventually arriving at the complete graph [3]. The reason for this behavior is found in the change of the spectrum of  $\mathbf{H}$ , determining the transport efficiency. As a second example where striking differences between CTQW and CTRW arise, we consider the transport *through* finite two-dimensional lattices of different aspect ratios with randomly placed bonds (note the obvious relation to percolation) [2]. For transport from, w.l.o.g., left to right, we place in the right-most column traps for the excitation, such that the (survival) probability to remain within the network can decrease [1,4]. In the CTRW case, one obtains the intuitively expected behavior that more bonds are necessary to facilitate transport from left to right in the landscape configuration than in the portrait configuration. Surprisingly, for CTQW we obtain similar behavior for landscape and for portrait configurations. In particular, for portrait configurations when the two ends are only a single bond apart, roughly the same number of bonds as in the landscape configuration is necessary to support transport through the network. We show that this effect can be understood by analyzing the eigenstates of  $\mathbf{H}$ .

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## Griffiths phases in complex brain networks

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Griffiths phases in complex brain networks.

Hallmarks of criticality, such as power-laws and scale invariance, have been empirically found in cortical networks and it has been conjectured that operating at criticality entails functional advantages, such as optimal computational capabilities, memory, and large dynamical ranges. As critical behavior requires a high degree of fine tuning to emerge, some type of self-tuning mechanism needs to be invoked. In this talk we show that, taking into account the complex hierarchical-modular architecture of cortical networks, the singular critical point is replaced by an extended critical-like region which corresponds –in the jargon of statistical mechanics– to a Griffiths phase. Using computational and analytical approaches, we find Griffiths phases in simple models of neural activity propagation running on top of synthetic hierarchical networks as well as in empirically obtained brain networks such as that of the human connectome. Similarly, the spontaneous emergence of coherent behavior through synchronization plays a key role in neural function and performance, and its anomalies often lie at the basis of pathologies. We study analytically and computationally the synchronization (Kuramoto) dynamics on the actual human-brain connectome network. Between the synchronous and asynchronous regimes that can be achieved by tuning details in any generic network, here we elucidate the existence of a novel, broad intermediate state, stemming from the hierarchical modular organization of the connectome. Where one would expect a hierarchical synchronization process, we show that the interplay between network structure and quenched node frequency heterogeneity gives rise to frustrated synchronization, metastability, and chimera states. We uncover the spectral origin of the dynamic freezing behind such phenomenology, and discuss how such complex synchronization patterns relate to the need for the brain to access large dynamical repertoires without *ad hoc* fine-tuning to a critical point. Thus, for activity propagation as well as for synchronization dynamics running on the human connectome network we find strong evidence of a novel broad phase, placed at the between order and disorder. Stretched critical-like regions, stemming from structural disorder, yield enhanced functionality in a generic way, facilitating the task of self-organizing, adaptive, and evolutionary mechanisms selecting for criticality.

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### Comparative Discussion On Models Of Evaluation For Optical Properties Of $\text{Bi}_2\text{O}_3$

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Theoretical research methods have been presenting a growing importance in the last decade. The investigations of the optical properties of various types of samples are, by this, a great area of continuous research, because of the significant applicative impact of the optical properties in the fields of industry and health. As a straight proof, we should be considered the rising impact factor of related literature journals. The aim of this paper is the studying from a statistical point of view a series of global effective representations, in order to obtain a general model that includes temperature dependence of the optical properties. The main method is based on the Kramers-Kronig transformation type. This is an extension of our previous studies, in order to obtain better and the most simplest models. In this respect, the optical properties of  $\text{Bi}_2\text{O}_3$  were estimated by Kramers-Kronig analysis using different analytical approaching methods for a series of changed configurations. A novelty of the present study is due to the implementation of an entire MAPLE software approaching tool in order to succeed in reaching the huge volume of complex computations. The numerical fit of the reflectance spectrum is applied, accompanied by a careful extrapolation, was followed by a recursive optimisation phase in order to obtain a better statistical approximation result. A second novelty aspect is due by using a different approaching method, based on neural network. This phase was implemented for comparison. By adapting the learning rate and by considering a series of different layers structure, we succeed in reaching better results than the previous. Bismuth trioxide wasn't studied so far in a systematic forthcoming and a theoretical approach is welcomed. We consider such a technique, since this powerful mathematical relation was proved productive for the determination of optical properties and is frequently used in the extraction procedures of optical constants, mainly in the mid-IR spectral region [14]. The use for these methods above presented implies a series of specific problems. Besides the advantage of increase the accuracy of approximation by using neural networks or spline functions on the interpolation process, there is the great disadvantage of increasing difficulty in building a global model that includes temperature dependence. This difficulty is even greater of their optical properties change is greater, as in the case study chosen for this article. This advance involves misassumptions regarding the spectrum in the spectral range beyond the measurement limits. The efficiency of this method is than proved and the good results recommends for further

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### An algorithm for optimizing the urban selective waste collection activity. A case study in Galati

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The Waste Management (WM) activity is an urban action becoming more and more important in the municipal economy. In many cities, this activity is linked to significant revenues from the selective waste collection services. Since the current economic situation requires an increasing efficiency and profitability in order to reduce operational costs, and an important part of revenue comes from this selective waste collection activity, effectiveness must be improved. As regards our specific case study in the city of Galati (250 000 inhabitants), WM activity has been redesigned. The main aim of this study is to suggest ways for the successful implementation of the selective collecting waste management system in order to facilitate a decrease in the operation costs. From this point of view, the vast majority of studies identified that the transport operations should be optimized. Several papers in the literature proposed a series of new algorithms for routes optimization through different approaching strategy. In this stage it could be concluded that there are two major tactics: a) basing on the traditional statistical forecasting methods like time analysis correlation and Regression analysis and b) basing on the System Dynamics as Modelling:Fuzzy, not-well defined environments, Causal loops, etc. The innovative character of the present paper results from two important aspects which extend the previous results: a) the best results is identified by comparing three of the most often methods used in literature, basing on a systematic approach. b) the optimization algorithm is based on the dynamic elimination of specific collecting points from the daily visiting routes so that the minimum cost path could be calculated. The considered network for this study contains 226 collecting points distributed in the Galati city. By implementing this model for each collecting point individually, the related algorithm for less cost routing can accomplish a true dynamic optimization method. Thus, the studied algorithm which is presented in the last part of this article is based on the determination of minimum cost routes, by removing from the visiting schedule list all the collecting points with non-significant amounts of waste. In this respect, by setting at each time the list of collecting points which have to be visited and based on a least cost route procedure, the operating costs in the waste management activity can be optimized. This solution, valid in a non-stationary minimum cost route procedure, is a novelty in the field.

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## A Hydrodynamic Model for Silicon Carbide Semiconductors including crystal heating

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The recent considerable advances have allowed Silicon (Si hereafter) semiconductor technology to approach the theoretical limits of this material. Moreover, advanced optoelectronic and microelectronic devices, as well as of power devices cannot be handled by the present Si-based devices, e.g. higher blocking voltages, switching frequencies, efficiency, and reliability. To overcome these limitations, new semiconductors like Silicon Carbide (SiC hereafter) and compounds, with their superior electrical properties (such as wide band gap, high electron saturation velocity, high breakdown electric field, high thermal conductivity) are likely candidates to replace Si in the next future. The drift diffusion model currently available for SiC and compounds is not able to describe accurately the physics because it is based on fitting parameters and ad hoc assumptions on the transport coefficients and the constitutive equations, whose validity is often restricted to specific devices and regimes close to thermodynamic equilibrium.

The kinetic theory is a powerful tool to understand transport phenomena of charged particles at a mesoscopic level. This theory is the natural framework to describe off-equilibrium phenomena in sub-micrometric semiconductor devices, where the onset of high electric fields, the influence of the thermal heating of the carriers and crystal lattice on the performance of semiconductor devices increase as the miniaturization is more aggressive and the density of the transistor grows.

The electro-thermal transport can be described by using the Bloch-Boltzmann-Peierls (BBP) kinetic equations for the phonon and the electron distribution functions, with the appropriate collisional operators. Since the solution of these kinetic equations is generally an uneasy task, by using the moment method, we have introduced a new hydrodynamic model where all the main interactions between electrons and phonons, as well as the scatterings of phonons among themselves are considered. By taking advantage of the Maximum Entropy Principle, the constitutive equations for such material have been determined without any need of introducing ad hoc approximations or phenomenological coefficients. Simulation results for the bulk case have been obtained.

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## Novel Concepts (and their Connection) in Percolation of Networked Systems

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I will focus on recent advances in the theory of discontinuous connectivity transitions in networked systems and the prediction of the position (time point, critical link density) and the type (1st or 2nd order) of the connectivity transition both in

complex environments and regular topologies such as lattices. Specifically, I will attempt to expose deep connections between the concepts of discontinuous connectivity transitions, non-self-averaging and discrete scale invariance in networked systems. The framework has applications in the prediction of tipping points, magnetic systems and evolutionary dynamics. An analytical line of arguments help toward the prediction of the phase transition type, both on the lattice and in networked systems, given a connection rule.

Because this abstract needs more than 2300 words find here more information:

PRL 2014 topic: I will report the discovery of a discrete hierarchy of microtransitions occurring in models of continuous and discontinuous percolation. The precursory microtransitions allow us to target almost deterministically the location of the transition point to global connectivity. This extends to the class of intrinsically stochastic processes the possibility to use warning signals anticipating phase transitions in complex systems.

PRX 2012 topic: Complex networks are a highly useful tool for modeling a vast number of different real world structures. Percolation describes the transition to extensive connectedness upon the gradual addition of links. Whether single links may explosively change macroscopic connectivity in networks where, according to certain rules, links are added competitively has been debated intensely in the past three years. In a recent article [O. Riordan and L. Warnke *Science* **333** 322 (2011)], O. Riordan and L. Warnke conclude that (i) any rule based on picking a fixed number of random vertices gives a continuous transition, and (ii) that explosive percolation is continuous. In contrast, we show that it is equally true that certain percolation processes based on picking a fixed number of random vertices are discontinuous, and we resolve this apparent paradox. We identify and analyze a process that is continuous in the sense defined by Riordan and Warnke but still exhibits infinitely many discontinuous jumps in an arbitrary vicinity of the transition point: a Devils staircase. We demonstrate analytically that continuity at the first connectivity transition and discontinuity of the percolation process are compatible for certain competitive percolation systems.

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## Key-phrase Detection using Fractal Patterns of Words in a Text

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Language is an adaptive complex system which has attracted the physicists attention recently. A growing interest exists for exploring the regularities as the characteristic of complexity in language and particularly in its written part namely the texts. In this work, we examine the fractal structure of texts as one of the existed regularities. We attribute a fractal pattern to every word within a text. The fractal dimension of these patterns allows us to distinguish between the important and the irrelevant words. The irrelevant words in a text appear to have dimension close to one and the important words considerably differ from one. Based on this finding,

we propose a novel method for ranking the vocabularies in text. A huge number of written documents are available on the internet or on the offline sources. Finding the favorite subject among these large amount of data seems difficult. Knowing the keywords of the texts can be helpful in finding the documents with the same subject. Each text contains two type of words; irrelevant words and important ones. Irrelevant words are common words which do not carry over any concept. In contrast, Important words are responsible for transferring the information from the writer to the reader. Hence they should distributed with a specific order through the text. In this paper, we examine the fractal structure as the specific regularity for the words. We attribute a fractal pattern, so a fractal dimension, to every word in the text and take advantage of these dimensions to extract the keywords of the text. We find that there is a difference between fractal dimensions of important words and irrelevant words of a text. The irrelevant words in a text appear to have dimensions close to one and the important words considerably differ from one. Therefore if the fractal dimension of a word differs more from one, the word is more important. Based on this finding, we define a degree of fractality for every word within the text and ordered the text due to this parameter. The words with higher degree of fractality are the keywords of the text.

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### Quantitative effects of RF and magnetic pulses on drosophila model

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Nowadays, radiofrequencies (RF) are a very important part of our world, especially due to the spreading of the mobile communications in the last decades.

This trend gave sensibly a boost to the study of the interactions of electromagnetic fields with biological matter and their related effects. That's why an always increasing scientific community tried to give an answer to the simple question: "Are the RF dangerous for the human beings? Why and how?"

A good candidate to test RF effects is the *Drosophila melanogaster* (fruit fly). *Drosophila* represents an ideal genetic model to study biological functions that are conserved in the evolution and, more recently, it is used as a model for human diseases from neurodegeneration to cancer. We analyzed the effects on the control of specific DNA repetitive sequences that generally are activated after different environmental stresses. The activation of these causes genomic instability and hence this analysis can give us evidence of particular dangers occurring in presence of RF.

The RF induced the activation of repetitive sequences, as *Stellate* sequences that cause the formation of crystalline aggregates in male germ cells. In addition, tests on the fertility of the treated individuals have been performed, where we

noticed a change in both the genders of *Drosophila*.

In order to get more detailed information about the effects of a magnetic field on the *Drosophila* samples, we tried to get them in some limit conditions, very rare to find regularly in nature, to test the dependence of the found results on the magnetic field strength. Using a fast circuit connected to an high - voltage power supply, we generated a pulsed magnetic field of moderate intensity (0.4 T), getting again informations from the spermatocytes and on the chromatin states.

Surprisingly, there's no trace of the alteration in the formation of crystal aggregates found in the RF exposition. Interestingly, the chromosomes presented some particular regions of decondensation or puffs. Further analysis are still work in progress and will be presented.

These overview of phenomena opens an interesting window in terms of explanation of interactions of electric molecular dipoles and magnetic dipoles (*diamagnetic anisotropy*) and their orientation in the cells. These molecular interactions, orienting the molecular dipoles, give rise to genetic consequences as well.

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### Large deviation estimates involving $q$ -deformed exponential functions

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From the Markov inequality  $\text{prob}(Y \geq y) \leq EY/y$  follows  $\text{prob}(X \geq x) \leq e^{-ax} Ee^{aX}$ , by choosing  $Y = e^{aX}$  and  $y = e^{ax}$  ( $a \geq 0$ ). It is obvious to replace in the latter substitution the exponential function by the  $q$ -deformed exponential function. The substitution  $Y = \exp_q(aX)$  then yields

$$\text{prob}(X \geq x) \leq \exp_{q^*}(-ax) E \exp_q aX, \quad (1)$$

where  $q^* = 2 - q$ . The latter inequality is our starting point for studying large deviation properties of fat-tailed distributions. The large deviation properties of the  $q$ -deformed exponential distribution can be obtained from results found in the mathematical literature.

Let  $\eta_n(x)$  denote the probability that the sum of  $n$  identically distributed variables is larger than  $nx$ . Then it is known that asymptotically  $\eta_n(x)$  behaves as  $n\eta_1(n(x - Ex))$ . In combination with the above inequality this yields a large deviation estimate for a large class of fat-tailed probability distributions. It is obtained by comparing these distributions with the  $q$ -deformed exponential distribution. The result is then that the probability that the sum of  $n$  identically distributed variables is larger than  $nx$  is asymptotically bounded above by  $n \exp_{q^*}(-nI(x))$  for some rate function  $I(x)$ . This rate function takes on the usual form of a Legendre transform.

As an example we consider the student's  $t$ -distribution. We show that for suitably chosen but non-optimized values of the parameters analytic upper bounds follow.

The large deviation principle is an essential property of models of statistical mechanics because a rigorous proof of the equivalence of ensembles (the so-called equipartition theorem) relies on it. We expect that our result can be used to study models where the individual degrees of freedom have fat-tailed probability distributions.

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## Quantum optimization

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A novel quantum optimization technique is developed. The developed algorithms are tested with a number of multi-dimensional nonlinear benchmark functions.

In many branches of science the optimization of continuous non-linear functions is a pertinent issue. Especially higher-dimensional problems remain challenging. In the last two decades novel techniques mimicking biological evolution have emerged. These evolutionary algorithms are quite different from the traditional gradient-based or simplex methods. Examples are particle swarm optimization [1] and simulated annealing [2].

Particle swarm optimization has been modified such that the individual particles are described quantum mechanically [3]. Further extensions of quantum particle swarm optimizations have been developed and applied [4]. The global search ability is claimed to be improved in the quantum version.

For simulated annealing, based on the introduction of a fictitious temperature, an analogous development has taken place [5]. This has led to the concept of quantum annealing, cf. [2] and references therein. The basic idea is to avoid getting trapped in local minima by means of quantum mechanics.

Inspired by these concepts, we have devised an optimization procedure which is also based on quantum theory. It neither involves a particle swarm nor annealing (in the sense of time evolution). In this presentation, our concept is put forward and elucidated. To this end, we start by introducing the necessary quantum mechanics.

Our optimization technique exploits many-particle quantum mechanical wave functions. Harmonic oscillator potentials and delta function potentials are used. The developed algorithms are implemented in Fortran 95 code. A number of benchmark functions, typically nonlinear and multidimensional are used to test our optimization method. For low dimensions, say up to four, all multi-dimensional problems are correctly solved. The apparently easier functions can be minimized for much higher dimensions; we have found twenty-dimensional solutions. The two-dimensional test functions like Easom and Goldstein-Price are also successfully minimized.

Detailed aspects of the proposed technique may be improved in the future. In particular, the dynamic scale or variance is probably not optimal yet. Also the computational load has not been addressed at all. Further comparison to other evolutionary approaches is in order as well.

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## Fractal globules: new approach to handmade molecular machines

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The over-damped relaxations of elastic networks constructed by contact maps of hierarchically folded fractal (crumpled) polymer globules are investigated. It is found that the relaxation dynamics of highly anisotropic fractal globules are very similar to the behavior of biological molecular machines. Being perturbed, the system quickly relaxes to a low-dimensional manifold with a large basin of attraction and then slowly approaches the equilibrium not escaping this manifold. We show that, despite the obvious structural differences between fractal globules and proteins, their dynamic behaviors are very similar. By these properties, synthetic fractal globules (FG) are suggested to be artificial molecular machines (MM) able to transform the perturbations into directed quasi-mechanical motion along a well-defined path [1]. The non-phantomness of a polymer chain in a three-dimensional space leads to bulk interactions vanishing for infinitely thin chains, and topological constraints preserved even for chains of zero thickness. At temperatures below a certain critical value (in a poor solvent), a polymer chain collapses into a weakly fluctuating drop-like globule of size  $R$ . In the ordinary globule formed by a chain with open ends, all subchains of length  $l$ , for  $l \ll R$ , look as mutually entangled Gaussian coils, since the volume interactions are screened in the melt. As a consequence, the ordinary globules have almost random elastic networks with many local metastable states and small basins of attraction. The soft and rigid relaxation modes of their elastic networks are poorly separated meaning that such globules do not possess the generic properties of molecular machines. For unknotted polymer rings, the equilibrium globular structure is quite different [2]. Topological constraints inherent in a polymer ring result into the so-called crumpled (or fractal) globule with self-similarly folded crumples (folds) embedded into each other. The blow-up interest to the FG has been launched by the experimental work [3] where it has been shown that the compact DNAs in the human genome shear some statistical properties of FGs. The current promising development of these ideas in genomics allow us to understand better the generic structure of interphase chromosomes. The ability of FGs to act as MMs provokes a new angle on the eternal problem of the origin of life concerning producing and selection of complex molecular structures at prebiotic evolution stages. The conclusions of our work permit us to put forward the conjecture about the possibility for the "primary MMs" to be a sort of the FGs formed under prebiotic conditions. These primary FG-MMs being produced from abiogenous materials, could perform some elementary functions typical for true biological MMs. The diversity of primary FG-MMs concerns mainly the number of principal soft modes manifested in the dimensionality of the attracting manifold on which the MM's action is performed.

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## Distance versus time scaling in human travel

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Human travel revealed many interesting scaling laws [1-3]. Such laws are valid for the trip length distribution, costs or flux intensities. Interestingly the relation between the trip length ( $d$ ) and travel time ( $t$ ) leads to a novel non-trivial scaling:  $d \propto t^\alpha$  with  $\alpha \in [0, 1]$ . This scaling law is universally valid for many different traveling modes: terrestrial, air, or even virtual ones. The scaling exponent,  $\alpha$  is not universal however, and depends on the studied transportation mode. Here, we present experimental data proving the validity of this scaling on distances that span more than three orders of magnitude. In order to understand this fascinating and seemingly universal feature of human travel, a simple network model is considered. The model is based on a random and undirected graph at criticality, embedded in a two-dimensional Euclidean space. Nodes represent cities while edges are transportation channels connecting them. Between each two node of the graph, a direct link (edge) is created with a probability  $P_{1,2} \propto (W_1 W_2) / [(W_1 + W_2) r_{1,2}]$ , where  $W_i$  is the size of node  $i$  and  $r_{i,j}$  is the Euclidean distance between node  $i$  and  $j$ . The size distribution of the nodes was taken as the entropy maximizing distribution for a fixed mean value, which is the exponential distribution. Edges are added consecutively after their  $P_{i,j}$  probabilities until the percolation threshold is reached. We assume that transport between any two nodes is realized on the shortest distance taken along the existing edges. The main parameters of the model are the  $L$  linear size of the embedding Euclidean plane, the average node size,  $\langle W \rangle$ , and the total number of nodes,  $N$ . The model successfully reproduces the experimentally observed distance versus time scaling law. The  $\alpha$  exponent can be tuned by varying  $L$  and  $\alpha$ . The topological properties of this simple random graph is studied, and the influence of the  $L$  and  $\alpha$  parameters are revealed. Acknowledgment: This research was supported in the framework of TAMOP 4.2.4. A/2-11-1-2012-0001 'National Excellence Program - Elaborating and operating an inland student and researcher personal support system' project. The project was subsidized by the European Union and the State of Hungary. We thank Tamas Dusek for data provided on air transportation.

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## Geometric and kinetic aspects of nonequilibrium steady states

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Open thermodynamic systems out of equilibrium can be modeled as stochastic (often Markovian) networks. In

contrast to thermal equilibrium with Boltzmann stationary distribution, zero currents and dynamics satisfying the (global) detailed balance condition, the nonequilibrium steady states (NESSs) reveal to be very sensitive to local details of the dynamics. In general, a NESS can be completely described in terms of stationary occupations of the microscopic states, local currents, and dynamical activity landscape, where the latter quantity refers to the time-symmetric component of the dynamics, i.e., the noisy motion irrespective of its direction.

In my talk I will focus on two topics: (i) how the complementary characteristics of NESS depend on local thermodynamic versus kinetic properties of the system; (ii) how geometry of the Markovian transition network determines dominant low-temperature patterns. Concerning the first issue, it will be argued that the non-Boltzmann weights emerge from a mutual frustration between transition channels with different dissipation properties. As a consequence, we provide a simple necessary condition for the population inversion of states to occur, this way refining the so called Landauers blowtorch theorem. Concerning the second, I will show that a large class of non-equilibrium stochastic systems at low temperatures can conveniently be described via dominant states and their excitations, dominant currents, and attractors with the largest dynamical activity. Topology of the attractors enables to classify possible low-temperature patterns according to how large fraction of the overall dynamical activity contributes to the dissipation. As a whole, this proposes a geometric point of view for nonequilibrium processes and provides a complementary approach to standard methods, mostly relying on spectral properties of stochastic transition operators.

Possible applications include the asymptotic analysis of current in discrete-state ratchet models and the construction of "zero-temperature phase diagrams for driven particle systems. I will finish with an outlook to work in progress on building a low-temperature asymptotic framework for slow thermodynamic processes between far-from-equilibrium steady states. Its objective is to connect the present framework to the non-equilibrium linear response theory and to provide an asymptotic scheme for reversible (or "excess) work and heat.

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## Phase transitions in protein folding

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The process of protein folding from an initial random chain phase to the collapsed native state, is a very complex physical phenomenon. It engages a multitude of disparate temporal and spatial scales, and during the process new collective variables become excited while old ones recede and fade away. In particular there are high energy barriers that the protein can overcome only by relatively slow and collective multi-molecule motions. The presence of many different scales causes severe computational bottle-necks in classical all-atom molecular dynamics approaches. As a consequence a detailed molecular dynamics simulation of an entire folding process

remains a formidable computational task. We describe how the folding and unfolding processes of a protein can be modeled effectively, directly in terms of biologically relevant time and distance scales. Our approach is based on the concept of universality : We derive an effective Landau-level energy function, from a combination of geometrical and physical principles. We conclude that the infrared limit of the energy function is essentially uniquely defined, and involves only terms that can be interpreted in terms of the integrable hierarchy of conserved charges that characterize the discrete nonlinear Schrodinger (DNLS) equation. Our generalized version of the DNLS equation admits stable soliton solutions, and we explain how these solitons correspond to the super-secondary modular building blocs such as helix-loop-helix of strand-loop-strand that are commonplace in practically all natively folded proteins. In the energy function there are only seven free parameters. As such these parameters are global, they are not amino acid specific. Here, we show how to compute the parameters from the structure, by demanding that the native state is a minimum energy configuration i.e. solves the generalized DNLS equation. The small number of parameters enables us to predictively address dynamical issues, such as the folding and unfolding dynamics of particular proteins. For this we introduce a framework of non-equilibrium dynamics, that allows us to repeatedly fold and unfold the proteins, by a change of ambient temperature. The approach that we adopt is a version of the Glauber protocol that is widely utilized to describe finite temperature relaxation dynamics in a statistical system which is out of thermal equilibrium. We present several examples of simulations of the folding and unfolding process by repeated heating and cooling cycles between a given low temperature i.e. bad solvent environment where the protein is collapsed, and various different high temperature i.e. good solvent environments.

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### Analysis of Flow Networks by the Maximum Entropy Method

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A “flow network” can be defined as a set of nodes connected by flow paths. This representation unites many different disciplines, including electrical, communications, pipe flow, fluid flow, transportation, chemical reaction and ecological, as well as human economic and social systems. Historically, flows on networks have been analysed by node and flow path conservation equations (Kirchhoffs laws) and/or network mappings (e.g. Tellegen’s theorem), and more recently by various optimisation methods and dynamical simulation.

A relatively unexplored approach, however, is the use of Jaynes’ maximum entropy (MaxEnt) method [Jaynes, 2003]. This has been applied previously to transportation [Ortúzar and Willumsen, 2001] and pipe flow (hydraulic) networks [Tanyimboh and Templeman, 1993], but without considering the frictional (dissipative) behaviour of such flows, and so in general is not correct. Existing methods, especially in hydraulics, also apply to fully constrained networks, in which the number of unknowns cannot exceed the number of equations.

We present a generalised MaxEnt method to predict the mean steady-state properties of a flow network, subject to observable constraints on particular nodes or paths, and also to physical constraints on the network (Kirchhoffs laws and resistance equation). The method adopts an entropy defined over all uncertainties within the system, including flow rates, potentials, frictional and capacity properties, and if necessary the network structure itself. The analysis can be applied to both fully constrained and under-constrained flow networks, and is sufficiently general to allow the inclusion of multiple connections between nodes, sources/sinks at each node, multiple species flows, and uncertainty in the network structure itself. The approach builds upon a previous MaxEnt formulation for the analysis of infinitesimal flow systems, based on an entropy defined on flux and chemical reaction states [Niven, 2009]. Of particular importance for under-constrained networks is the handling of the prior probabilities in the entropic formulation.

We further demonstrate the application of the MaxEnt method to pipe flow networks, which require a non-linear formulation of the Kirchhoff loop constraints and Lagrangian [Waldrip et al. 2013]. The integral equations are solved by quasi-Newton and Newton methods using multidimensional quadrature, involving inequality constraints and graph-theoretic techniques.

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### Consistent Thermodynamic Framework for Interacting Particles by Neglecting Thermal Noise

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A system of interacting vortices is currently used in the literature to model flux lines, and their associated vortices, in disordered type-II superconductors. Recent studies have shown that this system is related to nonextensive statistical mechanics [1,2,3]. Lately, the concept of an effective temperature  $\theta$  was introduced for a system of interacting particles, and more particularly, for vortices in type-II superconductors [4]. The quantity  $\theta$  was shown to represent an appropriate definition of effective temperature for this

system, exhibiting properties very similar to those of the usual thermodynamic temperature  $T$ , being: (a) A positive quantity by definition; (b) Thermodynamically conjugated to a generalized entropy per particle,  $s_q$  with  $q = 2$ , characteristic of nonextensive statistical mechanics, leading to the definition of a heat contribution,  $\delta Q = \theta ds_2$ ; (c) Proportional to the density of vortices  $n$ , yielding the desirable possibility for varying  $\theta$ , since recent experimental researches in type-II superconductors led to considerable advances in the ability of controlling many properties of these vortices, including their density; (d) Characterized by values that are much higher than typical room temperatures ( $\theta \gg T$ ), so that the thermal noise can be neglected as a good approximation ( $T/\theta \simeq 0$ ); (e) Physically interpreted in terms of the variance of the vortex positions,  $\theta \propto \langle x^2 \rangle^{3/2}$  [5]. In this presentation we will show a consistent thermodynamic formalism for this system, based on the effective temperature  $\theta$ . In order to achieve this, an infinitesimal-work term  $\delta W$  is introduced, leading to a proposal for the first law of thermodynamics. A Carnot cycle is constructed and its efficiency is shown to be  $\eta = 1 - (\theta_2/\theta_1)$ , where  $\theta_1$  and  $\theta_2$  represent the effective temperatures associated with the isothermal transformations of the cycle, with  $\theta_1 > \theta_2$  [5]. Moreover, we consolidate the first-law proposal by following the usual procedure for obtaining different potentials, i.e., applying Legendre transformations for distinct pairs of independent variables. From these potentials we derive the equation of state, Maxwell relations, and define response functions. All results are shown to be consistent with those of standard thermodynamics for  $T > 0$ .

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### Two-dimensional microphase separation of single-component homopolymer brushes

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The structural phase behavior of polymer brushes, single-component linear homopolymers grafted onto a planar substrate, is studied using the molecular Monte Carlo method in 3 dimensions.

Polymer chains grafted onto substrates, i.e. polymer brushes, have been extensively studied in physics, chemistry, materials science, and other scientific and industrial fields. Polymer brushes are widely utilized for applications such as wetting, adhesion, surface patterning, and colloidal stabilization. Recently, sophisticated polymer brushes, such as brushes composed of diblock copolymers and of binary mixture of homopolymers, have drawn significant attention. Polymer brushes on spherical substrates, which are referred to as polymer-grafted colloidal particles, have also attracted wide interest in various scientific and industrial fields.

The phase behavior of these complicated brushes is significantly dependent on the molecular architecture, shape of the substrate, ratios between the numbers of polymers of each polymer species, and other characteristics of each system. In the present work, the most basic and simplest polymer brush, i.e. single-component linear homopolymers homogeneously grafted onto a planar substrate, is simulated using the molecular Monte Carlo method to determine the universal and structural phase behavior of polymer brushes.

When simulation parameters of the system are set in regions of macrophase separation of solution for the corresponding non-grafted homopolymers, the grafted polymers also prefer segregation. However, macrophase separation is disallowed due to the spatially-fixed grafting points of the polymers. Such constraints on the grafting are similar to connecting points between blocks of non-grafted diblock copolymers at the microphase separation in the melt state. This results in microphase separation of the homopolymer brush in the lateral direction of the substrate. Here we extensively search the parameter space and reveal various lateral domain patterns that are similar to those found in diblock copolymer melts at microphase separation. These results demonstrate that topological constraints, i.e. the grafting points of the brushes and connecting points between blocks of each block copolymer, universally yield the microphase separation and various domain patterns, independently of the unique characteristics of each system.

We show that the simulation results are consistent with experimental results, which illustrates the validity of our theoretical work.

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### Burgers dynamics in random matrix models

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We obtain several classes of non-linear partial differential equations for various random matrix ensembles undergoing Brownian type of random walk. These equations for spectral flow of eigenvalues as a function of dynamical parameter ("time") are exact for any finite size  $N$  of the random matrix ensemble and resemble viscid Burgers-like equations known in the theory of turbulence. In particular, the viscosity is proportional to the inverse of the size of the diffusing matrix. In the limit of infinite size of the matrix (inviscid limit), these equations reduce complex inviscid Burgers equations, proposed originally by Voiculescu in the context of free processes. Complex Burgers-like equations for the spectral flow represent matrix-valued analogue of the Smoluchowski-Fokker-Planck equation of standard stochastic dynamics. Even in the case of the simplest Brownian walk, these equations are non-linear, contrary to well-known Fick equation for a single random walker. We identify spectral shock waves for these equations in the limit of the infinite size of the matrix, and then we solve exact, finite  $N$  nonlinear equations in the vicinity of the shocks, obtaining in this way universal, microscopic scalings

and critical exponents. In particular, we consider the cases of Gaussian Unitary Ensemble, Laguerre Unitary Ensemble (known also as complex Wishart ensemble) and Circular Unitary Ensemble. Finally, we show that similar but hidden Burgers-like structures appear (surprisingly) also in non-hermitian random matrix models, e.g. in the Ginibre-Girko ensemble, where they govern the concurrent evolution of both eigenvalues and eigenvectors. We conjecture that such concurrent evolution is the general feature of all non-hermitian random matrix models and has to be observed in physical open systems. We briefly outline applications and verifications of the proposed matrix-valued diffusions. In particular, we demonstrate an agreement of the critical exponents in the vicinity of the shock with numerical simulations of phase transitions in the lattice theory of strong interactions. Finally, we present the analogy between spectral shock waves in random matrix theory and caustics in so-called diffraction catastrophes in non-linear optics, considered, e.g. by Berry and Howls.

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### Pore network model of electrokinetic transport through charged porous media

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In charged porous media, the coupling between different transport phenomena arises from the excess electric charge of the fluid which compensates that of the solid walls. This charge may result both in the acceleration of the whole fluid under an applied electric field (electro-osmosis) and in the transport of charge if the fluid flows under an applied pressure gradient (streaming potential). In geophysics, the electroseismic effect, by which an electro-magnetic wave is generated from the motion of underground fluids under an applied acoustic wave, is exploited to determine the properties of geological formations. Streaming potentials and electro-osmotic flows can be measured in the laboratory to characterize the properties of porous media. Electrokinetic phenomena also play an important role in biology, membrane technology, microfluidics with electrokinetic pumps and more recently nanofluidics.

We introduce a method for the numerical determination of the steady-state response of complex charged porous media to pressure, salt concentration and electric potential gradients. The macroscopic fluxes of solvent, salt and charge are computed within the framework of the Pore Network Model (PNM) [1], which describes the pore structure of the samples as networks of pores connected to each other by channels. The PNM approach is used to capture the couplings between solvent and ionic flows which arise from the charge of the solid surfaces. For the microscopic transport coefficients on the channel scale, we take a simple analytical form obtained previously by solving the Poisson-Nernst-Planck and Stokes equations in a cylindrical channel [2]. These transport coefficients are upscaled for a given network by

imposing conservation laws for each pores, in the presence of macroscopic gradients across the sample. The complex pore structure of the material is captured by the distribution of channel diameters.

We investigate the combined effects of this complex geometry, the surface charge, and the salt concentration on the macroscopic transport coefficients. The upscaled numerical model preserves the Onsager relations between the latter, as expected [3]. The calculated macroscopic coefficients behave qualitatively as their microscopic counterparts, except for the permeability and the electro-osmotic coupling coefficient when the electrokinetic effects are strong. Quantitatively, the electrokinetic couplings increase the difference between the macroscopic coefficients and the corresponding ones for a single channel of average diameter.

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### Reliability of cell fate decision using a simple multiple cell model with inhibitory interaction

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Cell differentiation is one of the most fundamental ability of biological cells. The differentiation process is considered to be controlled by the expression levels of certain genes. Recent experimental studies indicate that cell-cell interaction plays an important role in the cell differentiation process and the cell fate decision. The reliability of cell differentiation ratio should be important properties for biological organisms to maintain homeostasis.

To understand the basic mechanisms of cell differentiation, we introduce the following simple multiple model with lateral inhibitory cell-cell interaction. The state of each cell, which is supposed to determine its cell fate, obeys a dynamical equation  $\frac{\partial u_i}{\partial t} = \alpha u_i - k \sum_{j \in \mathcal{N}_i} u_j$ , where  $u_i$  ( $i = 1, \dots, n$ ) and  $k$  denote the state variable of cell  $i$  and the intensity of cell-cell interaction, respectively. The  $\alpha$  denotes curvature of the potential  $\alpha = 1$  for plus side ( $u_i > 0$ ) and  $\alpha < 0$  for minus side ( $u_i < 0$ ) optimal interaction range (number) to gain the reliability of the proportion  $R$ . Since the cell-cell interaction occurs on the basis of random configuration of cells, the reliability gains with increasing interaction range in a short interaction case. When the interaction range is longer, the reliability decreases and converges to a finite value. Moreover, this system produces several different spatial patterns when the dynamics of cell states includes "reset" operation, which mimics the asymmetric differentiation process. In this presentation, we will talk about more details and

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## Portfolio selection using complex network

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The investor does not consider expected return a desirable thing and variance of return an undesirable thing. said Harry Markowitz(1952) in his work. He emphasized the importance of risk diversification mathematically for the first time. Based on his work, many researchers including James Tobin(1958), William Sharpe(1964), Evans and Archer(1968), and Meir Statman(1987) studied the benefit of portfolio diversification. However, these studies assumed that the market is efficient and the investors are rational, i.e. the market follows random walk. In this regard, portfolios risk is calculated from the randomly selected stocks, because they considered each company as an independent object. However, the inefficiencies or inequalities are examined in financial market. Agents in real market are influenced or influence each other by means of the financial relationships, which means that one agent's tragedy is no more one's sole business. As the economic scale is increased and financial relationships are constructed, the market condition has been more unpredictable and more volatile as the inter-connections that have to be considered among domestic or even foreign companies are complicated and become closer through financial relations, which is proved by Domino effect in Sub-prime crisis in 2008. In this constantly varying circumstance, analyzing the market using complex network model is more important than ever. In much of the previous work, [6-11], networks were constructed and analysis was progressed. Portfolio management is an essential problem of financial investment literature. Since Markowitz's portfolio theory introduced, the numerous methods for constructing portfolio set have been proposed in the traditional technology such as the several clustering algorithm and the random matrix theory, while there has been NYSE from 01.03 2000 to 12.31. 2012. To make diverse portfolio sets, we constructed the stock network with winner-takes-all approach. We consider Pearson correlation in order to identify the performance of proposed method and calculated the correlation between the KOSPI and the network-based-portfolio index using in-sample, and out-of-sample return. We find that the correlation value was high enough in overall threshold value in in-sample that can be used as a secondary index. Moreover, we measured portfolio risk using out-of-sample return. We could find that the portfolios risk much lower than Markowitz's random selected portfolio risk with proposed methodology, which leads to better performance of portfolio in terms of Sharpe ratio. This result implies that we can make sound stock portfolio using complex network.

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## Construction of Legendre dualities for generalized entropies via conformal flattening and its applications

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In the research areas of thermostatics or nonextensive sta-

tistical mechanics [2,3], a generalized entropy appears in many aspects and is widely recognized as an important concept. Various deformations of entropies, Massieu functions, divergences or (escort) probabilities without losing the Legendre duality are geometrically equivalent to constructing *dually flat* (or information geometric) structure on the family of probability distributions. The purpose of this presentation is to study the Legendre dualities of generalized entropies in a unified way via this perspective. For this purpose we show that such a deformation can be formulated using concepts called *affine immersion* and *conformal flattening* [1]. For an arbitrarily given generalized logarithmic function we derive explicit representations of deformed quantities, in terms of mutually dual (extensive and intensive) variables or probabilities of each state. In addition to Legendre dualities, obtained structures on the probability simplex satisfy interesting geometric properties, e.g., conformality, projectivity and so on. The results can be regarded as extended ones obtained in [1], which treats the only  $q$ -logarithmic function. As a physical application of such a deformation, we consider several dynamical systems on the simplex, which include what we call the deformed replicator system, and discuss their behaviors from viewpoints of the obtained dually flat structure.

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## Phenomenological vs. Statistical Ideal Gas Entropy

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Phase space statistical mechanics presents in nowadays the cornerstone of the probabilistic description of a thermodynamic systems. By means of the  $3N$ -hypersphere model in which the  $N$  dimensions correspond to the number of particles comprising a system and a hamiltonian consisting solely of kinetic terms, phase space thermostatics determines the density of states  $\Phi$  and herewith the Sackur-Tetrode (ST) entropy  $S_\Phi = k_B \ln(\Phi)$ . The obtained then pressure and temperature expression are the ones of the ideal gases.

It is worth stressing however that the identification of  $S_\Phi$  with the Clausius entropy is far from being a trivial issue. Among others, two drawbacks are presented below. First, the relation determined within the kinetic gas theory between the pressure  $P$ , volume  $V$  and the inner kinetic energy  $U$  is  $PV = \alpha U$ , where  $\alpha$  can be any positive constant [1]. The  $3N$ -hypersphere model on the other hand may reproduce the thermodynamic results only for the specific choice of the constant  $\alpha = 2/3$ . Although the former value can be justified within the kinetic theory it presents a restriction stemming from the phase space statistical model. Second, in the recent study [2] the present author with his collaborator G.B. Bagci deriving the analytical Clausius entropy expression for ideal gases in open systems (with  $\alpha = 2/3$ ) following a rigorous extensivity analysis, demonstrated that the SK-entropy  $S_\Phi$  coincides with the Clausius entropy  $S$  only in the limit  $N \rightarrow \infty$ . Moreover, they succeeded to show that the aforementioned limit is not an inherent thermodynamic property, unveiling herewith that Clausius physical and Sackur-Tetrode statistical entropies are

distinct sharing merely their expressions in the former limit. In this presentation we demonstrate that the phase space equilibrium thermodynamics, as promising as it may initially appear, presents obstacles in rigorously reproducing the phenomenological thermodynamic expressions. For our purpose, we derive the necessary equations for a complete thermodynamic description of an ideal gas, i.e., noninteracting constituent molecules with constant heat capacity for any possible values of the density  $n/V$ . We refer to them as dense ideal gases (DeIG). We show analytically and numerically then, that in this case, the DeIG equilibrium energy deviates from the equipartition energy when  $n/V \rightarrow 0$ . Since the latter energy is a fundamental and very general result of the phase space statistics, our analysis reveals the necessity of revising the former approach [3].

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### Structure of viable clusters in interdependent networks

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An interdependent network is a network of sub-networks, i.e. formed by different kinds of vertices (from different sub-networks) connected by different kinds of edges — links inside each sub-network (connecting the same kind of vertices) and interdependency links (connecting vertices from different sub-networks). In an interdependent network, a viable cluster is defined as the set of vertices (of any sub-network) that are connected by at least one path formed by any kind of edge. In these interdependent networks, we study the system of viable clusters, which, in general, should consist of a giant viable cluster containing a finite fraction of vertices and of numerous finite viable clusters.

Equations for the size of the giant viable cluster in the case of an interdependent network formed by two Erdos-Renyi networks,  $a$  and  $b$ , with multiple interdependencies are obtained. Each vertex in this interdependent network has two degrees: an internal degree  $q_s$  which is the number of neighbours within its network (following a Poisson distribution), and an interdependence degree  $k_s$  (also Poisson distributed) which is the number of interdependency links to the other network (where  $s = a, b$ ). If degrees are uncorrelated, the system is defined, then, by the joint degree distributions of each network. For network  $a$  we have  $P(q_a, k_a)$  while for network  $b$  we have  $P(q_b, k_b)$ , where

$$P(q_a, k_a) = \frac{\exp -\mu_a \mu_a^{q_a}}{q_a!} \frac{\exp -\lambda_a \lambda_a^{k_a}}{k_a!} k_a!$$

and, similarly

$$P(q_b, k_b) = \frac{\exp -\mu_b \mu_b^{q_b}}{q_b!} \frac{\exp -\lambda_b \lambda_b^{k_b}}{k_b!} k_b!$$

We devise and describe an algorithm for obtaining the viable cluster of an interdependent network formed by two generic sub-networks with a general distribution of interdependency links. Implementing this algorithm in numerical simulations and using dedicated analytical techniques we explore the emergence of the giant viable component (hybrid or mixed

phase transition) and obtain the statistics of finite viable clusters. We find the global properties of the interdependent network and the specificities of the critical phenomena at the point of birth of the giant viable cluster.

These analytical results are compared with the numerical ones obtained from the algorithm.

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### From World Time To Event Time: Why Time Matters

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World time is a static time scale that does not pick up price extremes of financial time series. We propose an event-based definition of time, where a price reversal of  $x$  percent from a recent price extreme is a new event on the intrinsic time scale. The use of an intrinsic time scale leads to a radically different approach to model building and paves the way for a relativistic theory of economics.

Researchers in economics and finance have taken the notion of time for granted; when sampling financial time series they use the time grid of world time to sample the data. With intra-day data it is striking, how the physical time grid fails to pick up the price extremes and the information of the extremes is lost in the subsequent analysis. Increased sampling does not solve the issue, because the number of sampled data points, where there is no or very little price action increases much faster than the prices sampled at the tails; this is typical for a bell-shaped distribution of price changes.

We propose an event based definition of time, which ticks, whenever the price has reversed by  $x$  percent from its local extreme. The threshold  $x$  defines the resolution, very small thresholds are the equivalent of one second, larger thresholds represent one minute, one hour or one day scale. The advantage of this approach is that price extremes are well tracked; they are recorded after a rebound of  $x$  percent.

The question arises, what occurs after an intrinsic time event? We observe that the rebound will on average occur after an additional price move of threshold  $x$ . This is true for any threshold  $x$ ; so there exists a scaling law between the grid size of the event threshold and the subsequent price overshoot. The size of the average price overshoot is a valuable input to measure the size of price overshoots. We show how this approach leads to new types of volatility models that can be estimated with much shorter data samples than traditional models.

Agent-based models have received a lot of interest in recent academic literature; typically, the agent models have relied on an abstract notion of time; we depart from this methodology and use an event-based intrinsic time scale. This enables us to configure behavior patterns of agent-based models as a function of their response to price overshoots; this approach is powerful, because it is extremely parsimonious. We explain, how the introduction of an event-based time scale leads to a new approach to model building and paves the way to a

relativistic theory of economics.

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### Analytical calculation of four-point correlations as indicators of collective motion in one-dimensional and two-dimensional systems of repulsive Brownian particles

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Diffusion in a system of Brownian particles with short-range repulsive interaction can become extremely slow. In the cases of two or three dimensions, it occurs if the particles are packed densely, leading to the glass transition (Berthier & Biroli, 2011). Slowdown of diffusion occurs also in one-dimensional systems, referred to as single-file diffusion (SFD) (Barkai & Silbey, 2010), if the repulsion prohibits the particles from overtaking their neighbors. In both cases, since uncooperative motion is forbidden, the particles must move in some collective manner. As indicators of such collective motion in glassy liquids, four-point space-time correlations such as  $\langle \rho(\mathbf{r}_1, 0)\rho(\mathbf{r}_1, t)\rho(\mathbf{r}_2, 0)\rho(\mathbf{r}_2, t) \rangle$  and  $\chi_4 \propto \langle Q \rangle^2 - \langle Q \rangle^2$  have been studied, with  $Q$  denoting the overlap density (Glotzer et al, 2000). However, analytical calculation of four-point correlations has been difficult, because taking four space-time points implies dealing with four-body correlations as long as the plain  $\rho(\mathbf{r}, t)$  is used as the basic field variable.

Here we propose an idea for calculation of four-point correlations, using SFD as an illustrative example (Ooshida et al, 2013). The idea, which may lead to a breakthrough in theories of glassy liquids in general, consists in adoption of the Lagrangian description, borrowed from the statistical theory of fluid turbulence, which allows us to calculate a four-point correlation as a two-body Lagrangian correlation. The displacement correlation is calculated analytically and verified numerically, and as a special case, the mean square displacement (MSD) is also calculated. The linear analysis reproduces the established asymptotic law of anomalous SFD stating that the MSD is proportional to  $\sqrt{t}$  (Ooshida et al, 2011). Nonlinearity is incorporated with a Lagrangian version of the mode-coupling theory (MCT). Since the cages in SFD are eternal, the correlation length grows unlimitedly as the time elapses, and the self part of  $\chi_4$  converges to a finite value which is an increasing function of the density.

In the two-dimensional case, the tensorial character of the displacement correlation must be important. This suggests adopting the correlation of the deformation tensor as a generalization of the density correlation in MCT. A preliminary analysis yields a flow pattern with a pair of swirls, which looks similar to the pattern of correlated motion obtained with particle-based simulations.

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### Radical Uncertainty and Macroprudential policy

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Radical uncertainty describes situations which are uncertain in the sense of Knight (1921), Keynes (1936) and Alchian (1950). At its simplest, a radically uncertain situation is one in which the probability distribution of outcomes is unknown. Systemic financial risk has high dimensional causes against a background of a constantly changing environment in which individual agents have to operate and take decisions. Bayesian learning processes which might converge on at least a local equilibrium are not really feasible in such contexts. Evolution has provided economic agents with human social and sentient faculties, including emotional and narrative capacities. They make it possible to simulate and commit to action for futures that have not yet happened. Such abilities, therefore, are particularly well adapted to decision-making under uncertainty, where the future is necessarily not calculable. It must be imagined.

In this paper, I describe approaches which operationalise the concept of narratives in the macro economy and financial markets.

A great deal of data now exists in digital form in public and private databases and so offers a new opportunity to capture how people think and support their actions. Machine learning algorithms are by now well established, and offer the potential to carry out rigorous and extensive analysis of textual material in order to derive knowledge. Text analysis is also beginning to appear in papers by mainstream economists. Akerlof and Shiller (2009) set out some general principles as to the importance of narrative. However, the selection of words and phrases to be searched for is to a considerable extent ad hoc. Further, none of the papers really illustrates the robustness of the results to variations in the particular terms which are searched, whether sub-sets of the actual list, or extensions to it. The methodology of Directed Algorithmic Text Analysis (DATA), developed by the Centre for Decision Making Uncertainty at UCL, searches texts in a manner directed by the social-psychological theory of conviction narratives. I give empirical examples of the ability of DATA to provide long-range early warnings of shifts in the overall level of economic activity, and to both short and long term early warnings of developments in financial markets (e.g. VIX and Fannie Mae).

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## Majorana fermions: a new computational paradigm

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Majorana fermions are widely expected to be discovered soon in the laboratory. These particles are interesting by themselves due to their exotic statistics. They also offer the means of performing quantum computation in a conceptually new and intrinsically robust way. In this talk I introduce the general properties of Majorana fermions with an emphasis on how they can be employed to encode and manipulate quantum information. I also discuss whether or not Majorana fermions violate Bell-type inequalities. Finally, I present a quantum algorithm for the evaluation of Jones polynomials that emerges from a quantum simulation with Majorana-like particles.

## The Ellipsoidal Nested Sampling and the Expression of the Model Uncertainty in Measurements

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In this paper, we consider the problems of identifying the most appropriate model for a given physical system and of assessing the model contribution to the measurement uncertainty. The above problems are studied in terms of Bayesian model selection and model averaging. As the computational effort of this approach increases exponentially with the dimensionality of the problem it is necessary a numerical strategy to integrate over the parameter space, and to compute and to sample the measurand post-data distribution. When data analysis is carried out by selecting a data model and by proceeding as if it had actually generated the data, the model uncertainty is ignored and the uncertainty could be underestimated. Bayesian methods are a way to include the model uncertainty into the data analysis and the uncertainty budget. Advances in computing technology have allowed to consider different models, each indexed by one or more parameter, where the Bayesian model selection and model averaging provides the probabilistic framework to simultaneously treat both the model and data uncertainties. The evaluation of the integral  $Z = \text{Likelihood} \times \text{Prior}$ , usually called "evidence" becomes impracticable when the dimension of the parameter space has  $20 \div 30$  dimensions. Among the many algorithms drawing random samples from the post-data distributions, we have investigated a nested sampling technique relating the likelihood values to the prior volume. The considered method is based on the study the contour lines of the integrand and on a probabilistic estimate of the volume (parameter space) contained within the iso-likelihood contour. The contour lines are sampled according this pattern: firstly,  $M$  likelihood samples  $L_{\theta_1}, \dots, L_{\theta_M}$  are sampled in the domain of the likelihood. Next, the smallest, indicated a  $L_1$ , is removed, stored in a list and replaced by a new sample,  $L_{\text{new}}$ , subject to the constraint  $L_{\text{new}} > L_1$ . The prior volume enclosed by the iso-likelihood surface  $L = L_1$  is estimated as  $p/(p+1)$ , which is the mean value of the argest of  $p$  uniform samples in  $[0, 1]$ , the total volume of the domain being an overall factor. The discharge of the lowest likelihood  $L_n$ , sampling

of a replacement constrained to  $L_{\text{new}} > L_n$ , and shrinking of the prior volume of the associated iso-likelihood surface to  $V_n = p^n/(p+1)^n$  are repeated until the contribution to  $Z$  of the surviving likelihood samples is less than some pre-defined value. Eventually, by using the rectangle method,  $Z$  can be easily approximated. This paper lays out the essential ideas of this approach.

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## Structural disorder and anomalous diffusion in random packing of spheres

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In the last years, much attention has been devoted to develop techniques for investigating biological tissues in vivo by using non invasive procedures. Diffusion Nuclear Magnetic Resonance has proved to be a successful tool with its ability to provide detailed information about material and tissue properties. The measurement of molecular diffusion is an effective experimental way of probing biological and porous material structures, since the geometrical complexity of heterogeneous systems can be characterized by the way in which small molecules (typically water) diffuse within them. A complex porous structure implies a multi-scale hindering of diffusing molecules and its characterization can reveal the fingerprint of the medium geometry. In conventional dNMR studies, structural complexity in heterogeneous media has been investigated by means of tortuosity indices as the obstruction factor. However, we show here that the such a parameter is not suitable for detecting the most relevant information on global structural complexity (disorder) and structural transitions. In this work [1] we focus on an innovative aspect of dNMR [2] that allows, in the hindered diffusion regime, to describe the disorder properties of a sphere packed medium, to monitor its structural modifications and to reveal structural transitions. In particular, we show that the subdiffusion exponent  $\alpha$ , a measurable dNMR parameter quantifying the anomalous diffusion regime, is related to well known statistical quantities characterizing the disorder in porous media. Notably,  $\alpha$  is able to detect structural transitions and to classify different kinds of disorder. To achieve this goal, we compare the result of a dNMR experiment made on a sample of random packing monodisperse micro-spheres with the outcome of numerical 3D simulations resulting from an original "collage" of validated algorithms to yield essential results in the characterization of complex systems. Specifically, to highlight the physical meaning and relevance of  $\alpha$  we show here a detailed description of structural disorder in random packing of mono-dispersed spheres performing both generative Molecular Dynamics simulations of disordered porous media and extensive 3D Monte Carlo studies of particles diffusion in these kind of systems, eventually connecting the numerical results to experimental dNMR measurements. We explain the results of particle diffusion in the framework

of the continuous time random walk (CTRW), which has been demonstrated to be a general and effective theoretical tool able to quantify anomalous diffusion in both laboratory- and field-scale systems [3].

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## Eliciting Praise and Criticism from Positive and Negative Sentiments

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In this paper, we present a linguistically driven method that focuses on evaluations expressed in text towards specific targets. The concept of evaluation refers to judgment of people and appreciation of objects and is refined in two specific sentiment insights; criticism for negative and praise for positive evaluations. Our primary aim is to detect praise and criticism in text. A second aim is the general sentence-level sentiment classification i.e. binary positive/negative classification of sentences conveying other types of subjective expressions and personal state(ment)s such as, suggestions, reporting problems, and so on. Eliciting subtle types of information such as criticism and praise as a specific subarea of the general positive/negative sentiment landscape can provide multiple advantages for business purposes. For example, restaurants measuring customers satisfaction in terms of the evaluations (e.g. The build-your-own burritos are awesome, Tasty drinks and great price, Staff is always friendly and upbeat) and not only in terms of generic positive/negative sentiment (e.g. I was pleasantly surprised, I would highly recommend, Very horrible experience!, What's even worse is that I'm lactose intolerant!!) could earn insights on how to improve their businesses or about what users care most in their reviews.

To achieve the two aforementioned goals we present a hybrid SA system which integrates the following four types of components: a) Sentiment lexical resources: the EvalLex [2], a lexicon of evaluative language grounded on Appraisal Theory [1] and the Opinion Lexicon [3]. b) EvalGrammar, a set of linguistic rules for modelling phrase-level evaluations. Given an input text, EvalGrammar determines which spotted EvalLex entries express evaluations by performing contextual evaluation detection and polarity disambiguation, c) SVM models that exploit the aforementioned lexica and rules in combination with other types of features (e.g. nigrams, POS tags), and, d) a network formulation and analysis of the output of the previous steps. To train and evaluate our system we developed benchmark datasets in the restaurant reviews domain using subsets of the open dataset from the Yelp Dataset Challenge [4] which contains over 158,000 reviews for about 5000 restaurants. The evaluation has shown that our method outperforms several baselines and is significantly boosted by EvalLex and EvalGrammar. Future work includes exploring other types of sentiment insights (e.g. recommendations, suggestions) as well as different domains, among others.

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## Statistical analysis of time series in anomalous diffusion generated by fractal intermittency

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The interest towards self-organized and cooperative systems has rapidly increased in recent years. This is related to the increasing interest in the evolution of complex networks, such as the web and, more in general, social dynamics (not only associated with web applications), but also biological applications such as neural networks, human brain dynamics and metabolic networks in the cell. This hot research field is sometimes denoted as *complexity science*.

A fully accepted definition of "complex system" does not yet exist, but there's some agreement on some general features. For example, a complex system is typically made of many individual units with strong non-linear interactions. The global dynamics of this multi-component systems is characterized by *emergent properties*, i.e., the emergence of cooperative, self-organized, coherent structures that are hardly explained in terms of microscopic dynamics.

Thus, the typical approach is focused on emergent properties and their dynamical evolution. Along this line, a crucial property that has been observed is given by the meta-stability of the emerging self-organized structures, that is, the dynamics of a complex system is characterized by a birth-death process of cooperation [Allegrini et al., 2009].

In time series analysis, this is mathematically described in the framework of renewal point processes [Cox, 1962] as a sequence of critical short-time events, with abrupt memory decays, thus dividing the time series into separate segments with long-range memory [Paradisi et al., 2013; Allegrini et al., 2013; Fingelkurts et al., 2008]. As a consequence, the inter-event times are statistically distributed according to an inverse power-law, a condition also denoted as *fractal intermittency*. The sequence of renewal events with fractal distribution of inter-event times are associated with anomalous diffusion processes.

Here we will show how simple random walks driven by complex (i.e., fractal intermittent) events generate anomalous diffusion and long-range correlation despite the presence of memory erasing events in the time series. Then, we discuss different approaches for the statistical characterization of diffusion scaling in time series with intermittent events. We discuss the robustness of diffusion scaling generated by event-driven random walks with respect to the presence of

noisy events, here modelled as events with Poisson statistics, as such events are not able to generate anomalous diffusion, but only normal diffusion (i.e., variance linearly increasing in time).

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### A renewal model for the Superconcentration Effect

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Many efforts in the origin of life research focus on the understanding of the very first steps that led to the emergence of organic compounds and metabolic pathways in the pre-biotic solution. A less investigated issue regards those crucial evolutionary steps leading to the formation of membrane compartments, which came into play as hosts for the first forms of cellular metabolism.

Luisi et al. tackled this issue experimentally, studying the behaviour of the solutes in a water solution of ferritine proteins and lipids. In this setting, the lipid molecules, due to hydrophobic forces, tend to organize spontaneously in quasi-spherical structures, called liposomes or vesicles. These structures, while are forming but still open, allow the random passage of ferritine molecules.

Surprisingly, the authors found that, when lipid surfaces close up in a protein-containing solution to form vesicles, the entrapment frequency does not follow the expected Poisson distribution, but tends to assume a power-law behaviour, characterized by many empty vesicles (no or very little entrapped ferritine solute), and a long decreasing tail with extremely crowded vesicles. This is referred to as "*Superconcentration Effect*". Consequently, the experimental finding of Luisi et al. proves that liposomes can spontaneously capture a very high number of macromolecular solutes, even when forming in diluted solutions. This observation overcomes one of the major problems in prebiotic chemistry, i.e., how intravesicular solutes can spontaneously reach the relatively high concentrations needed for the metabolic processes to occur. As a result of this intriguing behavior, these solute-rich vesicles could be able to facilitate and support the emergence of a cellular metabolism, due to the high molecular concentration.

As the Poisson assumption does not apply, in this work we propose a stochastic model based on renewal theory [Cox], describing independent critical events randomly occurring in time. Waiting Times (WTs) among events are then mutually independent and they are only characterized by the WT distribution. We are here interested in describing the Gibbs ensemble of liposomes, with the critical event corresponding to the closure of the liposome membrane, after which no flux of ferritine proteins can occur on the liposome surface.

We show that simple assumptions about the liposome-ferritine interactions, can explain the emergence of a power-law decay in the distribution of ferritine molecules trapped inside the liposomes, thus shedding light on the role of renewal theory in the emergence of self-organized macro-structures in pre-biotic systems.

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### Evolution dynamics of a model for gene duplication under adaptive conflict

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We present and solve the dynamics of a model for gene duplication showing escape from adaptive conflict. We use the Crow-Kimura quasi-species model of evolution where the fitness landscape is a function of Hamming distances from two reference sequences, which are assumed to optimize two different gene functions, to describe the dynamics of a mixed population of individuals with single and double copies of a pleiotropic gene. By mapping the evolutionary dynamics onto the dynamics of a quantum spin chain, we derive the spin coherent state path integral representation and solve the evolutionary dynamics equation under saddle-point semiclassical approximation. The linear fitness landscape with two reference sequences is analyzed to show phase diagrams on the mutation-fitness parameter space. In the long time limit, there is a competition to dominate in a mixed population between individuals with single genes and individuals with duplicated genes. In the  $c \gg \nu$  regime, among individuals with duplicated genes, there is a sharp phase transition between the escape from adaptive conflict, in which each copy of duplicated genes evolves towards sub-functionalization, and the duplicated-loss of function phase, in which one copy maintains its pleiotropic function and the other copy undergoes neutral mutation to lose its original function. In the  $c > \nu$  regime, whether individuals with single genes or individuals with duplicated genes dominate in the population depends on the mutation and selection parameters. We chose the mutation and selection parameters such that individuals with duplicated genes initially have lower fitness than their single-gene parents, but through the accumulation of advantageous mutations can raise their mean fitness higher than those of a single gene population. For these parameters, we showed that there is a sharp change in the composition of the mixed population at time  $t^*$ : before  $t^*$  the population sustains a small, constant fraction of individuals with duplicated genes, after  $t^*$  the population of individuals with duplicated genes starts to exponentially outgrow the population of individuals with single genes and eventually the mixed population consists only of individuals with duplicated genes. We also presented how to estimate  $t^*$  and showed that there is an optimal mutation rate at which  $t^*$  is a minimum.

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### Diversity and noise effects in generalized multi-neuron models of homeostatic regulation of the sleep-wake cycle

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Recent advances in sleep neurobiology have allowed the development of physiologically-based mathematical models of sleep regulation that account for the neuronal dynamics responsible for the regulation of sleep-wake cycles and allow a detailed examination of the underlying mechanisms. Neuronal systems in general, and those involved in sleep regulation in particular, are noisy and heterogeneous by their nature. It has been shown that in various systems certain levels of noise or diversity can significantly improve the signal encoding and the system response, as for example in the phenomena of diversity induced resonance and in that of stochastic resonance [1,2]. However, these specific phenomena and especially the effects of diversity are rarely considered in models of sleep regulation. The present contribution is focused on the study of a neuron-based physiologically motivated model of sleep-wake cycle, in which a mechanism of the homeostatic regulation of sleep based on the dynamics of a wake-promoting neuropeptide orexin is proposed. Initially Orexin used to be thought relevant for some basic phenomena such as appetite and metabolism but the lack of Orexin has been demonstrated in the last years to be responsible also for the appearance of sleep disorders. The effects of diversity and noise affecting the features of the orexin-producing neurons in this sleep-wake cycle model have been studied [3] and shown to lead to a clear diversity-induced resonance. In other words, the best wake-sleep cycle, when suitably defined, corresponds to the presence of an intermediate level of diversity in the synapses of the orexin-producing neurons.

We discuss the main results of the mentioned model and present novel results concerning some generalized versions of the model, in particular those obtained by replacing the single representative glutamate neuron with a set of glutamate neurons interacting between each other and with the set of orexinergic neurons. We consider various cases in which the neurons of the system are connected to each other through different underlying network topologies and discuss the effect of noise and diversity for such network topologies. In fact, the type of the underlying network is known to affect in a crucially different way the global neuronal dynamics [4].

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### Exact Solution of a Loop Model of Critical Dense Polymers in Two Dimensions

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A lattice model of critical dense polymers is solved exactly for arbitrary finite system sizes on the strip, cylinder and torus. The model is related to symplectic fermions and is the first member of a family of  $O(n)$  loop models called logarithmic minimal models. Critical dense polymers is obtained by setting the loop fugacity  $\beta$  to zero. This forbids the formation of closed loops and means that long polymer segments can only terminate on the boundary. Since the polymers enter each face of the square lattice exactly twice, the polymers form space filling Peano curves with fractal dimension 2 in the continuum scaling limit. Exact expressions are obtained for the bulk and boundary free energies but the emphasis in this talk is on the values of the conformal weights and their associated critical exponents. The key to the exact solution is functional equations in the form of inversion identities satisfied by the commuting single and double-row transfer matrices. The form of these equations is similar to those for the Ising model (free fermions). The functional equations are established directly in appropriate locally planar Temperley-Lieb algebras with the transfer matrices acting on suitable vector spaces of link states. The bulk and boundary free energies and finite-size corrections are obtained from Euler-Maclaurin formulas. The eigenvalues of the transfer matrix are classified by the physical combinatorics of the patterns of zeros in the complex spectral-parameter plane. This yields relations with generalized  $q$ -Catalan and  $q$ -Narayana polynomials. It also leads to selection rules for the physically relevant solutions to the inversion identities and explicit finitized characters for the associated conformal representations. In particular, in the scaling limit, we find the central charge  $c = -2$  and conformal weights given by the Kac formula  $\Delta_{r,s} = \frac{(2r-s)^2-1}{8}$  for  $r, s = 1, 2, 3, \dots$  in an infinitely extended Kac table. The partition function on a torus can not be calculated using the usual matrix trace. Instead a modified trace which is a generalization of the Markov trace is required. Finally, we note that critical dense polymers extends to an elliptic lattice model which is exactly solvable off-criticality. An infinite number of generalized order parameters are identified with critical exponents  $\beta_{r,s} = \Delta_{r,s}$  with  $(2r-s)^2 < 8$

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## The Desertification transition in semi-arid ecosystems: early indicators and effect of sprawl phenomena

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Desertification transitions, occurring in arid and semi-arid ecosystems exposed to high external stresses, are important cases of regime shift. The changes associated to desertification can lead to huge and often irreversible consequences, with relevant economic and social effects: the identification of early and reliable indicators are of obvious importance to develop strategies to manage a transition or its effects. In this work we simulated the time evolution of a semi-arid ecosystem by means of a stochastic cellular automaton model (CA) and we analyzed the results in terms of percolation theory, where the percolating phase, associated with empty and degraded regions, is opposed to a vegetation covered phase. By considering the full range of values of the mortality rate, controlling the strength of external stresses, we followed the entire degradation process: from the vegetation fragmentation transition up to the full desertification transition, corresponding to the vegetation extinction and complete soil degradation, occurring for a critical value  $m_c$ . In order to find suitable indicators and precursors, we calculated the spanning probabilities and the percolation thresholds as functions of the mortality rate according to different spanning criteria and we studied the time fluctuations of the sizes of the biggest vegetation cluster and of the biggest non-vegetated cluster for several mortality values. The dependence of the critical mortality on the other parameters of the model is studied as well. Moreover we investigated the effect on the desertification transition of sprawl phenomena. Sprawl phenomena, i.e. a disordered growth of small urban areas, break the connectivity of a vegetation covered area (either natural or rural habitats), thus potentially increasing the fragility of a given ecosystem and the risk of a regime shift. With the aim of detecting smart geometries able to minimize the damage, we introduced in the model ‘forbidden’ cells aggregated in different configurations, simulating the presence of excluded areas, with different topology and sizes, and we analyzed their effect on several features of the desertification transition. We will present here the preliminary results of this study.

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## New clues on the thermodynamic behaviour in small systems

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The concept of stability is essential when describing the behavior and properties of matter. At the microscopic level, the stability of atoms becomes manifest through the existence of chemical elements, from which quantization can be deduced. Macroscopic thermodynamic systems show stability through the existence of distinct phases which can coexist under the same physical conditions. Unlike macroscopic systems, the existence of boundaries in small systems confers on them peculiar characteristics. Finite size induces nonhomogeneities of the interaction energy which may give rise to free-energy barriers separating two different structural configurations or even thermodynamic phases. Transformations between these structural configurations or phases is a matter of thermodynamic transformation theory, a well-understood problem in classical thermodynamics. When a macroscopic system is subjected to destabilizing conditions, it separates into two or more phases that may coexist in equilibrium. This partitioning involves the formation of new free-energy barriers associated with interfaces and finally, from the thermodynamic point of view, to the emergence of new systems with their own free energy determining their physical properties: compressibility, specific heats, etc. However, when the system is finite and small enough, the formation of an interface could imply an excessively high energetic cost and therefore becomes energetically unfavorable. This energetic restriction has been observed, for instance, in the formation of magnetic domains in ferromagnetic materials or in the hysteresis cycles of Li-ion batteries and other energy storage systems. Even more exotic systems such as atomic nanoclusters with magical numbers show peculiar effects on their behavior and properties similar to those of the small systems we will consider here. The energy lost in small systems, as nano devices, can be decisive in order to obtain an optimal functioning of this kind of systems. Under a thermodynamic point of view, the application of the stability theory and classical statistical mechanics to the description of small systems under destabilizing external conditions reveal a constant generation of heat while the system oscillates between two structural configurations or thermodynamic phases

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## Contact and first layer residues prediction in protein dimers using the Gaussian Network model with adjustable number of fast modes

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Protein-protein interactions are still not fully understood. They are more difficult to decipher than protein-DNA or protein-RNA interactions because they are, usually, not based on the attractive electrostatic interactions. I want present my attempt at recognizing protein-protein interactions based on the Gaussian Network Model - GNM (Demirel et al). The method I developed adjusts the number of modes used in the GNMs weighted sum calculation. The number of modes is adjusted on the basis of the ratio of predicted and expected amount of targets (contact and first layer residues). The number of modes gets increased or decreased if the amount of predictions is smaller or larger than expected for a given protein sequence length (number of residues in a chain). The original GNM algorithm, which uses only 5 fastest modes per chain is not able to correctly connect kinetically hot residues to binding patches in heterodimers. My adjustable approach, on the other hand, puts more than 50% of heterodimer chains into a group of good predictions. It applies the adjustable number of modes to dimers with a significant difference in the chain length of their monomer constituents. The algorithm produces usable results when applied to chains with more than 80 residues, which belong to dimers with chain length ratios higher than 2. The prediction gets even better when the initial adjustable approach which spreads the influence of a hot residue to sequential neighbors is extended to a full 3D approach (spatial neighbors).

I tested the adjustable 3D algorithm on the Vakser decoy sets also (41 heterodimer decoy sets). Each set contains 110 decoys, 10 of which are (on average) near native. The 3D adjustable algorithm which spreads the influence of a hot residue to the variable number of neighbors proved to be the most efficient. I also compared the adjustable GNM approach to the statistical potential approach of Lu and Skolnick. The statistical potential produces better overall results, but in a number of cases it is comparable, or even worse than the adjustable GNM approach. It would be interesting to out point that the decoys sets which were badly characterized by the statistical potential also did poorly with the adjustable GNM approach.

The results I present show that kinetically active residues are important in heterodimer interactions. They also suggest that two interacting chains usually exhibit opposite behavior. While the binding surface of one of the chains is usually rigid and stable, its partners is more flexible and adaptable.

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## Evaluating a gamble - A dynamics perspective

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The classic decision-theory problem of evaluating a gamble is treated from a modern perspective using dynamics. Gambles are probability distributions, discrete or continuous, of changes in wealth. Decision theory aims to advise on (normative), or to model (descriptive), the process of choosing between different gambles. In doing so, it maps gambles (i.e. distributions) into scalars through a functional. Early treatments of the problem, in the seventeenth century, chose as a functional the expected net change in wealth. In the eighteenth century it was recognized that this treatment often produces bad advice and does not fit observations. Consequently, the expectation-value functional was generalized into an expectation value of the net change in utility, where utility is a non-linear function of wealth. Utility, as the word suggests, is to reflect the value an individual attaches to owning a given amount of money. This concept is subjective by construction, and arguments over the meaning of the utility function, its measurability, and its permissible form continue to this day. We observe that decision theory has treated the gamble problem, invariably, using expectation values. This often implies an assumption of ergodicity: optimality is ensured for ensemble averages, and optimality over time results (if it does) as a by-product. We approach the problem from the opposite direction, putting time center-stage time irreversibility, after all, is the crucial constraint that makes good decisions important. We ensure optimality over time for the generally non-ergodic wealth process. This does not require the specification of a utility function, but it does require the specification of a dynamic, i.e. a stochastic process that unfolds over time. In our phrasing, optimality of expectation values of ergodic observables results as a by-product, providing a physical interpretation of utility functions as observables whose rate of change is ergodic. For the most natural dynamics, our perspective is at odds with the boundedness requirement for utility functions in the dominant formalism of decision theory. We highlight conceptual and mathematical inconsistencies throughout the development of decision theory, whose correction clarifies that the modern perspective is legitimate and that boundedness of utility functions is not required.

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## Non-stationary covariance functions based on local interaction models

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The study of spatially distributed physical processes usually relies on irregularly sampled data due to cost, time constraints, and instrumental limitations. For example, in the Geosciences it is impossible to have access to every desired sampling location. Similar limitations apply in other

scientific disciplines, even if data sampling is performed in a controlled environment such as a laboratory [1]. Interpolation or prediction of missing values in spatial data sets, especially those involving extreme values, is a mathematical problem of practical interest in signal processing, remote sensing, and environmental risk assessment [2].

Random field theory [3] is a powerful mathematical framework for modeling spatial data and for spatial interpolation methods. In this framework, we assume that measurements of a physical property correspond to a sample path of a multidimensional Gaussian joint probability distribution function (JPDF). The spatial correlations are described by means of covariance functions, which are determined from the data usually under the assumption of statistical stationarity. This is accomplished by fitting permissible covariance function models to the sample covariance, or by maximum likelihood estimation of the JPDF parameters. Spatial interpolation is then performed via the stochastic optimal linear predictor, also known as kriging. Both the covariance model selection and the stochastic optimal linear interpolation are computationally demanding procedures, especially for large data sets.

Spartan spatial random fields (SSRFs) are a relatively new class of Gibbs random fields in which the spatial dependence is derived from local interactions [4] instead of a model covariance. The probability of a given field configuration is determined by an effective local energy functional (ELEF), which involves the integral of the squares of the field values, as well as the gradient and Laplacian of the SSRF. This formulation can be viewed as a Gaussian classical field theory for spatial data. SSRF interpolation has similar prediction accuracy to kriging, but it is computationally more efficient [2], and it provides a conceptually attractive framework for treating non-stationary and spatiotemporal problems.

In modeling localized extreme values, e.g., radioactivity emergencies due to an accidental release, the stationarity assumption no longer holds. In this work, we propose new, non-stationary SSRF covariance functions for the interpolation of such data sets. We treat the extreme values by means of an ELEF perturbation, and we derive approximate but explicit non-stationary covariance functions using the leading-order perturbation expansion [5].

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## Kappa distributions in space plasmas: from origin to consequences

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Non-thermal particle distributions are ubiquitous in space plasmas, their presence having frequently been confirmed by interplanetary missions. The particle velocity distribution functions in space plasmas generally show non Maxwellian suprathermal tails decreasing as a power law of the velocity.

Such distributions are well fitted by the so-called Kappa distribution. Recent advances in space physics show the connection of kappa distributions with statistical mechanics and thermodynamics. Generalizations of thermodynamics based on Tsallis nonextensive entropy formalism have shown that the Kappa distributions result from a new generalized Lorentzian statistical mechanics formulated for a collisionless plasma far from thermal (Maxwell-Boltzmann) equilibrium but containing turbulence in quasistationary equilibrium. In any case, the presence of such distributions in many different space plasmas suggests a universal mechanism for the creation of such suprathermal tails. Turbulence can play a role in the generation of such suprathermal tails, as well as the long-range correlations supplied by the fields and plasma instabilities. Due to the properties of the Coulomb collisions, the energetic particles are non-collisional, even when thermal particles are submitted to collisions.

The enhanced population of energetic particles plays a crucial role in the heating and acceleration of plasma in several important space and astrophysical contexts. They contribute to the heat flux and modify the classical Spitzer-Harm expression. They have important consequences concerning the temperature increase in the solar and planetary atmospheres, as well as concerning the acceleration of the solar wind particles. Such consequences are well evidenced by using Kappa distributions in kinetic models where no closure requires the distributions to be nearly Maxwellians. Such models have been developed for the solar corona, the solar wind, the polar wind of the Earth and other planets like Jupiter and Saturn, the terrestrial auroral regions, the plasmasphere and the radiation belts. Suprathermal electrons generate large ambipolar electric fields along open magnetic flux tubes in stellar coronae and in planetary ionospheres and thus contribute significantly to solar and stellar wind acceleration, outflow from planetary ionospheres and possibly even exoplanetary atmospheric loss.

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## Do classical or quantum transitive preferences always result in indifferent divisions?

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In the series of earlier works [1,2,3,4,5] we considered the effects of the classical and quantum strategies based on the stochastic preferences in a simple two-component issue of sequential choice. These studies concern the problem of the universal property of transitivity of preferences. Thus, the set of possibilities is composed of at least three elements. The aim of the sequential choices is even distribution of results complying with the Laplace's principle of indifference (maximum entropy).

This theoretical problem is vividly illustrated by an example of a younger cat who is offered (by Nature) three types of food, every time in pairs of two types. Cat's optimal preferences leads to complete his diet.

In the work [6] we considered modification of these models, where Nature was replaced with an older cat. At the beginning of each stage of the game the older cat rejects one of the foods. Then the younger cat selects and consumes one food from the other two. The food that was rejected by the younger cat is consumed by the older one. The rules of this game refer to the simple procedure of fair division among two players, known as the “I cut, you choose” mechanism which has been widely discussed in the literature. It turns out that in this case all younger cat’s rational preferences are intransitive.

The principle of the transitivity of preferences is one of the basic assumptions of choice theory (utility theory), however it is known that this principle is not always fulfilled [7]. In the work that is presented in the conference the authors discuss a modification of “I cut, you choose” game. The difference lies in the fact that the older cat may also use food that was rejected at the first stage of the game. The following diagram illustrates the flow of such a game:

	<i>START</i>
<i>CAT<sub>1</sub> 1<sup>st</sup> MOVE :</i>	<i>READ base</i> $\in \{0, 1, 2\}$
	<i>SHOW base</i>
<i>CAT<sub>2</sub> MOVE :</i>	<i>READ cat<sub>2</sub></i> $\in \{0, 1\}$
	$cat_2 + base + 1 \bmod 3 \rightarrow cat_2$
<i>CAT<sub>1</sub> 2<sup>nd</sup> MOVE :</i>	<i>READ cat<sub>1</sub></i> $\in \{0, 1\}$
	$cat_1 + cat_2 + 1 \bmod 3 \rightarrow cat_1$
	<i>SHOW cat<sub>1</sub>, cat<sub>2</sub></i>
	<i>GOTO START</i>

where the strategy (state) *CAT<sub>2</sub> MOVE* can be classical or quantum, the states *CAT<sub>1</sub> 1<sup>st</sup> MOVE* i *CAT<sub>1</sub> 2<sup>nd</sup> MOVE* are classical.

The simplest variant of younger cat strategy will be discussed. This strategy is a mixed state on one beat (classic variant) or one-qubit pure state.

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### Information Geometry Formalism for the Spatially Homogeneous Boltzmann Equation

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Parametric Information Geometry generalizes to infinite dimension by modeling the tangent space of the relevant manifold of probability densities with exponential Orlicz spaces. Precisely, the exponential manifold on a set  $\mathcal{E}$  of mutually absolutely continuous densities is defined by the atlas of charts  $s_p: q \mapsto u = \log(q/p) - E_p(\log(q/p))$ , and inverse charts  $s_p^{-1}: u \mapsto \exp(u - K_p(u)) \cdot p$ , where  $p$  is any density in  $\mathcal{E}$  and  $K_p(u) = \log(E_p(\exp(u)))$ . The tangent space at each  $p \in \mathcal{E}$  is the set of Fisher scores  $\delta p(t) = d \log p(t)/dt = \dot{p}(t)/p(t)$  and it is identified with the  $p$ -centered random variables of the exponential Orlicz space. The exponential manifold  $\mathcal{E}$  is an affine manifold as

the transition maps are affine functions. It has a natural dual vector bundle of the type  $V'_p \subset H_p \subset V_p$ ,  $p \in \mathcal{E}$ , where  $V_p$  is the set of random variable of  $x \log x$  class of  $p$ -centered random variables,  $V'_p$  is the Orlicz space and  $H_p$  is the Hilbert space of  $p$ -centered and square- $p$ -integrable random variables. Vector fields  $F$  are mappings from  $\mathcal{E}$  to random variables such that  $E_p(F(p)) = 0$ ,  $p \in \mathcal{E}$ . Functional differential equations in this setting have the form  $\delta p(t) = F(p(t))$ , where  $F$  is a vector field in the pretangent bundle, that is a mapping  $F: \mathcal{E} \rightarrow V$  such that  $E_p(F(p)) = 0$ . This setting applies to the spatially homogeneous Boltzmann equation  $\dot{f} = Q(f)$  if  $\mathcal{E}$  is the set of positive densities with finite relative entropy with respect to the Gaussian density and the operator  $Q$  is locally presented as a mapping from  $V'$  to  $V$ , that is  $\delta f(t) = Q(f)/f$  with  $E_p(Q(f)/f) = \int Q(f) = 0$ . The Boltzmann operator is analyzed as a composition of elementary operations in the exponential manifold, namely tensor product, conditioning, marginalization. The functional setting allows the proper definition of tangent mapping of real functions on  $\mathcal{E}$ , e.g. Boltzmann entropy or the entropy dissipation functional. We discuss the interpretation of the spatially homogeneous Boltzmann equation as a gradient flow in  $\mathcal{E}$  and then explore the use of Galerkin’s approximation using the natural set of Hermite polynomials as basis

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### New developments in rate theory

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**Presentation:** Orale

We will briefly review the one hundred year history of rate theory, paying special attention to the convergence of the theory as developed in the Physics and Chemistry communities. The different approaches of Wigner and Eyring to rate theory will be discussed. The statistical physics point of view of activated barrier crossing as introduced by Kramers in his seminal paper (*Physica* **7**, 284 (1940)), will be complemented by the chemically motivated transition state theory whose equivalence today is well understood.

Kramers derived expressions for the rate of crossing a barrier in the underdamped limit of weak friction and the moderate to strong friction limit. The challenge of obtaining a uniform expression for the rate, valid for all damping strengths is known as Kramers’ turnover theory. Two different solutions have been presented. Mel’nikov and Meshkov (*J. Chem. Phys.* **85**, 1018 (1986)) (MM) considered the motion of the particle, treating the friction as a perturbation parameter. Pollak, Grabert and Hanggi (*J. Chem. Phys.* **91**, 4073 (1989)) (PGH), considered the motion along the unstable mode which is separable from the bath in the barrier region.

Both theories did not take into account the temperature dependence of the average energy loss to the bath. Increasing the bath temperature will reduce the average energy loss. In this talk, we analyse this effect, using a novel perturbation theory. We find that within the Melnikov-Meshkov approach, the thermal energy gained from the bath diverges, implying

an essential failure of the theory. Within the PGH approach increasing the bath temperature reduces the average energy loss but only by a finite small amount, leading to a reduction of the rate whose magnitude is of the order of the inverse of the reduced barrier height.

In a later paper (Phys. Rev. E, **48**,3271 (1993)), Mel'nikov improved his theory by introducing finite barrier corrections to the rate expression. The challenge of including finite barrier corrections within the PGH formalism will be addressed and answered in this talk. Tests on a cubic potential demonstrate that finite barrier corrections significantly improve the agreement of both MM and PGH theories when compared with numerical simulations.

A central advantage of the PGH formalism, is that it applies to systems with memory friction. The new finite barrier correction derived will be applied to activated barrier crossing in the presence of exponential memory friction.

Finally, recent misunderstandings of the theory in the context of molecular dynamics simulations and recrossing of dividing surfaces will be discussed.

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## Directed walk model of periodic heteropolymer adsorption onto periodic chemically heterogeneous surface

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Adsorption of heteropolymers - polymers composed of monomer units of two and more types, onto chemically heterogeneous surfaces - was intensively studied in the last decades [1]. A particular interest to this problem is motivated to its connection to the question of molecular recognition, playing a crucial role in living organisms and in various biomedical/biotechnological applications. To understand the mechanisms of the polymer-surface recognition, the problem was extensively investigated from different angles by using relatively simple and physically transparent models.

In the present work, adsorption of a periodic heteropolymer onto a periodic heterogeneous surface is studied by using the 2-dimensional partially directed walk model of the polymer. We show how the generating functions approach developed in for the adsorption of a random copolymer onto a homogeneous [2] or random [3] surface in the annealed approximation can be extended to the case of an arbitrary quenched periodic heteropolymer and a chemically heterogeneous surface. The developed approach is then applied to adsorption of a symmetric multiblock copolymer onto a symmetric multiblock two-letter surface. Analysis of the adsorption transition point dependence on polymer and surface block sizes and different sets of polymer-surface interaction parameters reveals interesting "odd-even" and "recognition" effects. Beyond the transition point, temperature dependences of the main conformational and thermodynamic characteristics demonstrate that adsorption can occur as a two-stage process,

where binding of the polymer chain to the surface is followed by "tuning" the chain conformations in order to maximize the number of energetically favourable contacts. The two-stage adsorption is characterized by appearance of the second maximum on the heat capacity curves. The details of the transition essentially depend on the ratio of polymer and surface periods. The sharpest second transition is observed in the special case when polymer and surface periods are equal (ideal matching). On the contrary, adsorption of periodic copolymer onto a homogeneous surface occurs as a conventional one-stage adsorption without additional heat capacity peak; the same happens when the surface period is small (compared to that in the polymer).

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## The pressure of surface-attached polymers and vesicles

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A polymer attached to a wall produces a force because of the loss of entropy on the wall. This can be measured experimentally, and recently has been described theoretically in two dimensions using lattice path models based on self-avoiding walks [1, 2]. There has also been recent work concerning the entropic pressure of a polymer in the bulk [3] using a lattice path model approach.

We shall review the results obtained for several lattice path models [1], looking at the pressure profiles around the point of tether, the effects of finite-size corrections, and the asymptotic decay of the pressure as a function of the distance from the point of tether. We shall also comment on the interplay between surface adsorption and pressure; in the adsorbed phase we find that the pressure decays to a non-zero value that is related to the density of contacts of the polymer with the surface.

We next explore the competition between the bulk internal pressure and the point pressure caused by entropy loss when a vesicle is fixed to a wall in two dimensions. Lattice walks and polygons on two dimensional lattices have in the past been utilised to model simple vesicles, where there can be an internal pressure. Our study therefore involves an exactly solved model of vesicles, namely, area-weighted Dyck paths [4].

In our work [5] we find that the inclusion of osmotic pressure leads to interesting phenomena. The pressure exerted by the vesicle on the wall depends on the osmotic pressure inside, especially its sign. There is a phase transition at zero osmotic pressure between a surface-attached state with minimal area and an inflated state with maximal area. The bulk pressure of the vesicle on the surface vanishes precisely at the transition point. We shall discuss the scaling of this pressure in the different regimes, paying particular attention to the crossover between positive and negative osmotic pressure. In our directed model, there exists an underlying Airy function scaling form, from which we extract the dependence of the bulk pressure on small osmotic pressures. We further find

that the decay to the limiting bulk pressure is governed by different types of finite-size corrections above and below the transition point. For positive osmotic pressure, we provide the precise form of the finite-size corrections using an appealing combinatorial argument.

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### The Determinants of the Distribution of Wealth and Income

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This paper attempts to individuate the determinants of the distribution of wealth and income on the basis of a theoretical model inspired to Fiaschi and Marsili (2012). In the model households own factors of production and rent those to firms producing a homogeneous output. Shocks on production trickle to the households income, affecting the rate of return granted to households for their assets.

With respect to the previous model we try to develop an empirically testable model, extending the setup to the case of heterogeneous households, both in their human capital endowment and portfolio strategies. This seems to be the model as simple as possible but not simpler needed to achieve quantitatively realistic results.

We then moved to develop a procedure to calibrate in a unique way all the parameters of the model from empirical panel data. Our tests will be done on a dataset of Italian households. In this procedure we choose to use an agent based model to avoid making strong assumptions on the joint probability distribution of heterogeneous traits among the population. A large economy will be assumed, together with low network effects, to keep the model solvable and simple.

We will first test the congruence of the simulation with the expected analytical solution for the steady state. Then we will assess the distributions generated by the model against the empirical ones, to check if the mechanism in our model is able to reproduce the real world inequality we are interested in, not only the behavior of the top tail but also of the median part. Different aggregated indexes of inequality will be used to carry on the comparison.

We identify the determinants of the distribution of income in terms of the characteristics of distribution of the sources of income, both from labor and capital. Income appears Paretian distributed in the top tail in all its components, as it is in the empirical data. We investigate the contribution of single sources to overall income inequality and which source appears to exert more influence on the top tail of distribution. Also wealth distribution follows a Pareto distribution in the top tail. We discuss how saving rate, fiscal policy and demographic factors affect wealth distribution, given

the distribution of income. We then analyze under which conditions wealth distribution is less equal and with higher Pareto exponent of income distribution, as observed in the empirical evidence.

In the end we will run some alternative world scenario of our agent based model to assess how the different sources of heterogeneity influence the wealth and income inequality.

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### Fitting of Observed Non-Maxwellian Distribution Function and Interpretation of Lion Roars in Magnetosheath

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Lion roars, the low frequency mode of the Whistler waves having frequencies nearly centered around 100 Hz, are frequently observed in the magnetosheath. Cluster observed the Lion roars and electron velocity distribution function within the magnetosheath during its several crossings. The observed electron velocity distribution functions clearly show non-Maxwellian features such as flat tops at low energies and superthermal behavior at high energies. These observed lion roars were studied and interpreted using bi-Maxwellian distribution function by employing the kinetic theory but could not justify the observations both quantitatively as well as qualitatively [1]. The obvious reason was to employ idealized Maxwellian distribution function in stead of using non-Maxwellian distribution function. We derived the dispersion relation of Whistler waves by using a non-Maxwellian distribution function such as the generalized (r,q) distribution function which is the generalized form of kappa and Maxwellian distribution functions. In limiting cases, when  $r=0$  and  $q=k+1$  generalized (r,q) distribution function reduces to the kappa distribution and when  $r = 0$  and  $\rightarrow \infty$  it reduces to the Maxwellian distribution [2,3]. It is supposed to be the best distribution when the real distribution contains flat tops or shoulders at the low energies especially. We employed kinetic theory to obtain the modified expressions for real frequency and established the necessary and sufficient condition to achieve growth/damping rates based on the generalized (r,q) distribution function. We then compare our numerical values of real and damping/growth rates with the Cluster observations, a good quantitative and qualitative agreement between the observed and theoretically obtained values have been found. At times when lion roars are observed (not observed) in the data and bi-Maxwellian could not provide the sufficient growth (damping) rates, we showed that when generalized (r,q) distribution function is employed, the resulted growth (damping) rates exactly match the observations. Moreover, Maxwellian theory predicted in some cases that the Lion roars are observed remotely as the theoretically obtained damping rates were too low to predict the local generation of Lion roars. However, when we use generalized (r,q) distribution function, the growth rates become sufficiently high so that we can predict that Lion roars are generated locally not remotely as predicted by Maxwellian theory.

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### Credit-driven business cycles in agent-based macro models

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The study addresses the crucial issue of the interplay between credit and business cycles in an economy by means of the agent-based Eurace model and simulator. Eurace is a fully-specified agent-based model of a complete economy that includes different types of agents and integrates different types of markets (Cincotti et al, 2010, Raberto et al. 2012, Teglio et al. 2012). Agents include households which act as consumers, workers and financial investors, consumption goods producers as well as capital goods producers, banks, a government and a central bank. Agents interact in different types of markets, namely markets for consumption goods and capital goods, a housing market, a labor market, a credit market and a financial market for stocks and government bonds. Except for the financial market, all markets are characterized by decentralized exchange with price setting behavior on the supply side. Agents decision processes are characterized by bounded rationality and limited information gathering and computational capabilities; thus, agents behavior follows adaptive rules derived from the management literature about firms and banks, and from experimental and behavioral economics of consumers and financial investors. The dynamics of credit money is endogenous and depends on the supply side by the banking system, which is constrained by Basel capital adequacy regulatory provisions, while on the demand side depends on firms financing production activity and households indebtedness for housing needs and speculation. Results point out a non-trivial dependence of real economic variables such as gross domestic product, unemployment rate and aggregate capital stock on banks' capital adequacy ratios; this dependence is in place due to the credit channel and varies significantly according to the chosen evaluation horizon. In general, regulations allowing for a high leverage of the banking tend to inflate asset bubbles and boost the economy in the short run, while result in bubble bursts and economic depression in the medium and long run. Results also point out that the stock of money is driven by the demand for loans, therefore supporting the theory of endogenous nature of credit money. The study then also clarify the nature of endogenous money, giving a contribution to a debate that has grown stronger over the last two decades.

### The Higher Dimensions of Stock Market

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SAV Integrated Systems

It seems to be very unlikely that all relevant information

in the stock market could be fully encoded in a geometrical shape. Still, the present paper will reveal the geometry behind the stock market transactions. The prices of market index (DJIA) stock components are arranged in ascending order from the smallest one in the left to the highest in the right. In such arrangement, as stock prices changes due to daily market quotations, it could be noticed that the price of a certain stock get over /under the price of a neighbor stock. These stocks are crossing. Arranged this way, the diagram of successive stock crossings is nothing else than a permutation diagram. From this point on the financial and combinatorial concepts are netted together to build a bridge connecting the stock market to a beautiful geometrical object that will be called stock market polytope. The stock market polytope is associated with the remarkable structure of positive Grassmannian. This procedure makes all the relevant information about the stock market encoded in the geometrical shape of the stock market polytope more readable.

A general stock market index is a method to measure the state of a market evaluating the prices of a selection of stocks owned by the most representative companies. These selected stocks are the components of the index. The market index is computed typically by a weighted average of the selected stocks prices and the performance of stock market can be asses. Such a global view of market performances, although extremely valuable, is not offering an image of what happen with the stock components in the evolution of the market quotations. Viewing the relations between stock components of a market index on a daily basis gave valuable complementary information that help in the investment decision making process.

Prices of the stock index components can be arranged in a table in ascending order starting from smaller stock prices, in the left, to companies having higher stock prices, to the right. Section 2 is devoted to explain the concept in detail, here is suffice to underline the main reason for such stock order. It could be noticed that along the time some stocks surpass or come under the price of neighbors stocks in the parallel series. Other way to see this is to imagine the stocks are crossing their price series.

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### Diffusion Process Peculiarities in the Liquid-Crystalline Cell Fabricated through the Contact of Two Cholesterics with Different Pitches

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We have considered, both experimentally and theoretically, some properties of chiral photonic crystals with a pitch gradient. The liquid-crystalline cell was prepared through the contact of two cholesteric liquid crystals (CLCs) with different pitches. One of the substrates was coated with the green CLC (1), and the other substrate was coated with the red CLC (2). Such CLCs were chosen that the diffusion could take place very slowly (for a few months) so that detailed investigation of the reflection spectrum for different moments

of the diffusion process were possible (and not only that at the beginning or at the end of the process). Using diffusion, we created a pitch gradient of the helix. The pitch gradient was stable for a few months. Thus, using the sensitivity of the helical pitch, we created a new structure. It was assumed that due to the freeze effect, during the diffusion process we could fix intermediate states of the pitch gradient, which would lead to new applications such as: band mirrors, band filters, notch filters and so on. Our studies provide important insights into self-organizing materials photonic investigations to go into details of the diffusion mechanism between the individual films inside the cell. A theoretical model of the pitch change during the diffusion of the CLCs was developed. The simulation of calculations was made on the base of *Ambartsumians layer addition modified theory adjusted to solutions of such problems*. The experiment showed that theoretical reflection spectra at different moments of the diffusion process coincides with the experimental results with a high accuracy, which confirms the validity of the theoretical model of the diffusion process. Let us also note that recently the chirped photonic crystals (PCs) and cholesteric pitch gradient films have been widely applied as lasers with spatially tunable laser emission [1, 2]; as wavelength splitting devices based on the PC super prism effect [3]; as a dispersion controlled optical group delay device [4]; as a magneto-optically operated multichannel adddrop multiplexer [5]; as omnidirectional mirrors and as optical diodes. There is also papers, where it was shown that the precise control of longitudinal localization of surface plasmons in plasmonic nanoparticle chains is possible by simply imposing a gradient in the refractive indices of the surrounding host medium, which can behave like a plasmonic switcher. So the investigation of optical properties of PCs with variable parameters of modulation is very important.

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### Instabilities in the topological structure of financial simplicial networks

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Modeling complex systems and in particular complex networks with simplicial complexes proved to be useful for acquiring insight into qualitative features and structural organization which is unattainable with standard methods [1] - [3]. The framework of our approach consists in constructing different types of simplicial complexes, either directly from the available data or from an existing complex network. We use a term *simplicial network* for simplicial complexes obtained from graphs or networks. Since simplicial complexes are intrinsically multidimensional they are adaptable to multiresolution analysis based on dimensional partition. The phenomena of major concern for financial modeling and networks are predictability and prevention of financial crisis.

We study the network obtained from the correlation matrix of stocks traded in the US equity market in the period 1996-2009 (data courtesy of Tomaso Aste). Using a PMFG filtering algorithm [4], we have formed a neighborhood complex [2] in which an arbitrary simplex  $\sigma(i)$  is associated to the firm  $i$ , and  $\sigma(i)$  is defined by firms to which firm  $i$  is correlated. The simplicial communities [3] were detected as connectivity classes in multiresolution analysis. The organization of firms according to similar or related industrial sectors is evident, and interesting information is obtained from merging simplicial communities (and hence industrial sectors) at different resolution levels. Applying time windows of various length (3 months, 6 months, 1 year) of the same period, an "obstruction vector" is evaluated, whose entries enumerate the number of structural gaps between simplicial communities at a particular resolution level. We define stability (or instability) at a particular resolution level if the corresponding component of the obstruction vector remains the same (changes). At each resolution level and for the temporal period of interest we define the *characteristic instability dimension* as the one at which maximal change happens. We evaluate the *simplicial structural complexity* for each time period and discuss how temporal dynamics of both quantities indicate the possible coarse-graining due to the change of time-scale. We show how these properties of this particular simplicial network shed new light on the analysis presented in [4].

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### Nonextensivity as a measure of self-organization and the criterion for the optimal wavelet representation

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In the most general sense, the term self-organization refers to the process that develops over time and which causes the emergence of structures and organized behavior without external influence. A necessary condition for self-organization is an increase in statistical complexity implying that optimal prediction of such a process requires more information. So far, the only methodology that satisfies rigorous requirements for discovering, describing, quantifying and predicting patterns is the so called  $\epsilon$ -machine. It represents the minimal model for the process that is optimally predictive. Recently, we have developed a  $w$ -machine ( $w$  stands for wavelet) in analogy with the  $\epsilon$ -machine, which is a versatile method for quantifying complexity that at the same time determines the optimal wavelet for capturing and quantifying self-organization of the dynamic system and which also performs superior denoising. It also shares the same properties which used to be unique to the  $\epsilon$ -machine. The framework is founded on the wavelet-domain decomposition and the properties of the

wavelet tree (the graph of wavelet coefficients) and statistical properties of the wavelet coefficients. A parametric model for a wavelet tree distribution attributes hidden variable to each node forming a hidden Markov model of the tree. The wavelet decomposition is sparse so that most of the energy is packed into a small number of large coefficients, while the remaining large number of small coefficients, carry remaining energy. The tree is considered as a self-organizing system by identifying hidden states with local causal states. The local statistical complexity of a spatio-temporal process is defined as the Shannon entropy of the local causal state, while the global complexity is the corresponding entropy of the whole tree. Instead of the Shannon entropy, we now introduce the Tsallis entropy of the local causal state which we calculate recursively for the wavelet tree using pseudoadditivity due to Abe and Suyari. We present expressions for the entropy of the wavelet tree in the Abe and Suyari representation and discuss their properties. The quantification of self-organization is illustrated on several dynamic systems and also in detecting forgeries of artistic paintings.

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## Random Walks in Random Media

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A general description of random walks on a regular, infinite lattice in  $d$ -dimensional Euclidean space is presented. In the continuous limit, generating function for the probabilities is the analog of the massive propagator in particle physics with mass  $m$  and momentum  $k$ . To propose a transition probability  $P_p$  for a random walk in a random media is necessary to consider the geometric characteristics in percolating systems, i.e., given two sites  $x, y$  in the lattice generated with a probability  $p$ , the walker can only arrive from  $x$  to  $y$  if they are connected through the same finite or infinite (only if  $p \geq p_c$ , where  $p_c$  is the percolation threshold) cluster. Connectivity through a finite clusters is given by the connection function while in the infinite case is given by the quantity  $P_\infty$ . Introducing this last fact and doing a time standardizing (to prevent the new transition probability has a time-dependent spatial normalization for a given time  $t'$ ) we are able to establish a bridge with the scalar quantum field theory. With this proposal, it is found that  $P_p$  satisfy the discrete diffusion equation with initial condition  $P_p(x, t; y, t) = \delta_{x,y}$ . In particular, the motion in a finite cluster is also a diffusion process. On the other hand, Green function is calculated as an average over all possible values of the time and in the continuous limit, the probability generating function is proportional to the Klein-Gordon propagator, in the quantum field theory context for a massive particle. This suggests that the propagator found corresponds to a free particle with different mass and momentum that the particle that belongs to limit case

$p = 1$ . We find that the mass ( $m + m_p$ ) and momentum  $k_p$  depend on the geometrical characteristics  $P_\infty$  and the time standardizing, distinguishing two limiting cases: i) if  $p \sim p_c$ ,  $m_p/m \rightarrow 1$ , while  $k_p/k \rightarrow 0$ ; ii) if  $p \sim 1$ ,  $m_b/m \rightarrow 0$  and  $k_p/k \rightarrow 1$ . It means that the propagator of a random walk in a percolating system correspond a particle with a greater mass and less momentum compared with the limit case  $p = 1$ . If we impose that the walker to be a Markov process, it is possible obtain a path integral representation for Green function. In the context for a random walk in a random media we obtain that the corresponding classical action is that of a free particle with mass ( $m + m_p$ ), in agreement with the Klein-Gordon propagator in the quantum field theory context.

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## Simulations of plasma obeying Coulomb's law and the formation of suprathermal ion tails in the solar wind

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A pair plasma was simulated with a self-consistent force given by Coulomb's law. A summary of the results are as follows: 1) the VDF quickly forms a power law tail of index 5 and evolution to a  $\kappa$ -type distribution (Livadiotis and McComas 2009) is observed at slightly later times, 2) for a fixed initial state (set by a seed for a random number generator) and varying the density,  $n$ , and thermal speed,  $v_{th}$  of the initial Maxwellian, we find an exact scaling of the time evolution of the simulation that is given by any function of  $n^{2/3}v_{th}^{-1}$ , 3) the probability distribution of the modulus of the electric field vector at the location of each particle (the EDF) has a shape that is also a  $\kappa$ -type distribution, 4) for different random realizations of the same initial set of plasma parameters, the parameters of the fitting distributions for the initial EDF correlate very well with those of the VDF at the end of a simulation, and 5) the shape of the EDF is given by the power law index of the force law used, and for Coulomb's law, the fitted power law index of the EDF is 5.

We have provided a self-consistent theory based on first principles (Coulomb's law) that can generate a spectrum that looks like the solar wind suprathermal tail (Gloeckler et al. 2012) in many respects. First, a power law index of 5 occurs regardless of parameters, with very small statistical variation. Secondly, a broken tail is observed at small times when a Maxwellian is used to initialize the simulation. When an interstellar pick-up ion shell distribution is used, very steep spectra are seen at early times, reminiscent of quasi-linear diffusion, but at slightly later times the same form of a power law with an index of 5 is recovered. The final VDF is well described by a self-similar  $\kappa$  distribution, which has been used to fit solar wind electron distributions (Kiran et al. 2006). We also found that the EDF not only follows the same self-similar  $\kappa$  distribution, but that the parameters of that distribution determine the parameters of the VDF.

Although actual 1 AU solar wind bulk parameters were not simulated due to limited computer resources, the same

behavior is reproduced for all bulk parameters that were simulated, with a single free parameter (dependent on the initial conditions) which is essentially the time it takes to form the ST tail. For bulk parameters typical of the solar wind at 1 AU, it is estimated that the ST would form in a time of order 1 s, which is much shorter than the transit time from the Sun to 1 AU ( $\sim 4$  days).

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### Micro and Macro Benefits of Random Investments in Financial Markets

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Noise and randomness are very familiar to physicists. In experiments one usually tries to avoid the effect of both of them, since they can perturb the phenomenon under investigation and mask the laws under scrutiny. However, it is well known that quite often, in particular if present in a limited amount, they can have an important and constructive role which have been realized since long time. But physical phenomena are not the only ones that can benefit from noise and randomness: in fact, the noise produced by the random action of the many elementary constituents or by the environment, has also a great influence in the complex dynamics of living cells, of neurons and of many other biological systems. It is then very likely that many other dynamical systems, including socio-economic organizations, share a similar situation. Along this direction, the role of stochastic noise and, more specifically, that of random strategies, have been investigated to try to understand their eventual positive and constructive features in several socio-economic systems. For example, we have found how random strategies can help to improve the efficiency of a hierarchical group or of a public institution like a Parliament. Interesting experimental results come also from the investigation of the role of random strategies in financial markets. For these reasons more recently we have been investigating in detail the efficiency of random trading with respect to standard technical strategies, both from an individual point of view and from a collective perspective. In this talk we will present recent results about the positive role of random strategies in financial markets. From a microeconomic perspective, we will show that, if investors chose a completely random approach to decide about their investments, instead of costly and arbitrary technical strategies, they would end up with, in average, the same wealth but with minor risks. On the other hand, from a macroeconomic point of view, we will present new results which show that, the mere existence of a few random investors in a 2D small-world lattice, representing a trading community of interacting agents characterized by a self-organized critical dynamics, is able to reduce significantly the magnitude of financial avalanches. Finally, we will show how different network structures do not change our general

findings and, more important, that just a very small number of hyper-connected investors (i.e. hubs in a scale-free network of traders) are required in order to obtain a stabilizing effect and dampen financial bubbles and crashes.

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### A Non Equilibrium Thermodynamic Model for Ferroelectric Hysteresis

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Over the last decades ferroelectric materials have drawn considerable attention due to their unique properties and applications and have given rise to various models attempting to describe such characteristics. Many ferroelectric materials exhibit hysteretic behaviour, particularly in the spontaneous polarization-applied electric field relationship; this is associated with the irreversible processes taking place in the material, which tend to align the polarization of the domains with the direction of the electric field. The macroscopic theory of non-equilibrium Thermodynamics is well-suited to deal with a number of problems concerning irreversible processes, by means of the laws of conservation of mass, momentum and energy, applied to systems where the properties of the materials may be assumed to be continuous functions of the space coordinates and time. In the first place, this work obtains a model for the polarization vs electric field hysteresis curve, using the concept of pseudoparticle and by invoking an entropy maximum principle [1]. Though this models applicability in calculations is limited, it will give insight on the various phenomena involved in ferroelectric hysteresis, such as domain switching [2-4]. Secondly, we will derive a phenomenological macroscopic model by close analogy to plasticity theory, introducing ideas like yield surface, flow rule and hardening law [5]. The main advantage of this methodology is that it makes the model ready for its implementation in finite element codes, with a relatively few internal variables, such as backfield and electric field hardness. With the aid of the first model we may interpret the yield surface in terms of microscopic processes taking place. The derivation of the model is based in the entropy functional, which is to be obtained from Gibbs equation and the conservation laws. In so doing, we obtain the entropy source term (associated with intrinsic losses), containing the various contributions of heat conduction, dielectric relaxation, straining of the material. By invoking the first models entropy maximization, we will get the flow rule for the system, yielding that the remanent polarizations increment is perpendicular to the switching surface. Thus, a relatively easy to implement phenomenological model for a ferroelectric material is obtained, which proves to be thermodynamically consistent and that can be explained in terms of a more complex micromechanical model.

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### Kinetics of subdiffusion limited precipitation: Fractional Ham approach

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Precipitation of atoms and defects in solids can substantially change their mechanical, electrical and optical properties. Investigation of precipitation kinetics is important for predicting undesirable degradation, and for new technologies aimed at creating materials with adjusted properties. Experimental kinetics of precipitation is well described by the empirical law:  $\xi = \xi_0 \exp(-Kt^n)$ , where  $\xi$  is the fraction of non-precipitated particles. In the classic Ham approach [1], the precipitation is considered as a process limited by Gaussian diffusion of atoms, and for spherical precipitates the theory gives  $n = 3/2$ . But parameter  $n$  determined from the experiment often takes values less than  $3/2$ . In [2], authors considered subdiffusion limited coagulation and annihilation in one dimension and obtained that anomalous diffusion substantially influence on their dynamics.

In present work, we calculate the kinetics of subdiffusion limited growth of precipitates in disordered solids. Considering thermo-activated hopping of atoms in disordered potential we obtain the following integro-differential equation for the concentration of unprecipitated particles:

$$\frac{\partial \rho(\mathbf{r}, t)}{\partial t} = C \int_0^t Q(t - \tau) \nabla^2 \rho(\mathbf{r}, \tau) d\tau.$$

The kernel  $Q(t)$  is expressed through the localization time distribution, and its time dependence is determined by density of states [3]. In case of the exponential density of states, trapping times are distributed according to the asymptotic power law, and the equation transforms into the fractional diffusion equation:

$${}_0D_t^\alpha = C_\alpha \nabla^2 \rho(\mathbf{r}, t).$$

Here, the fractional operator is the Riemann-Liouville derivative.

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### Fractional stable statistics in bioinformatics

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This work provides a short review of applications of fractional stable distributions in bioinformatics and molecular biology. The set of distributions  $q(x; \alpha, \beta, \theta)$  ( $\alpha \in (0, 2]$ ,  $\beta \in (0, 1]$ ,  $\theta \in [-\theta_\alpha, \theta_\alpha]$ ) represents a new class of probability distributions

appeared in [1] (univariate distributions) and [2] (multivariate distributions) and investigated in [3] where they were named fractional stable distributions. The fractional stable random variables is equal in distribution to the ratio of two independent stable random variables:

$$X(\alpha, \beta, \theta) \stackrel{d}{=} Y(\alpha, \theta) / [S(\beta)]^{\beta/\alpha}.$$

Here  $\alpha$  is the characteristic exponent and  $\theta$  is skewness parameter and  $S(\beta)$  is the subordinator. In terms of probability density functions, one can write

$$q(x; \alpha, \beta, \theta) = \int_0^\infty g(xy^{\beta/\alpha}; \alpha, \theta) g_\beta(y) y^{\beta/\alpha} dy.$$

These distributions were obtained as limit distributions for Continuous Time Random Walk model.

First application considered here is associated with a statistical analysis of DNA microarray data to filter the differentially expressed genes and assess the correlation between the expression profiles. Fractional stable pdf approximates well the distribution of microarray probe intensities. We have applied fractional stable statistics to analyze the radiation-induced dynamics of gene expression in cells K562, Me45 and HCT116. Transcriptome is analyzed after 1, 12 and 24 hours after irradiation using the Affymetrix microarray HGU133A series. It was found that the probability characteristics of expression differences depend strongly on the intensities of the reference level, and this dependence is nonlinear in general case. We take this fact into account using the "noise envelope" algorithm in filtering, clustering and grouping. The effectiveness of the procedures can be estimated from the results of hierarchical clustering and using the method of group averages. For filtered genes, dendrograms are constructed, and comparison of gene dynamics in key signaling pathways associated with programmed cell death and DNA repair is provided. Another applications are related to the description of anomalous diffusion of signaling molecules along the dendrites of neurons and the searching process of a binding protein for a specific site on DNA.

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### Conformal invariant stochastic processes

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Unlike KPZ where the dynamical critical exponent  $z = 3/2$ , conformal invariant stochastic models have  $z = 1$ . As a consequence of the space-time symmetry of the models, one can make clear predictions about the correlation functions. In the present talk we show several conformal stochastic models which are derived using different Temperley-Lieb algebras and various presentations of the same algebras. The Raise and Peel model is an example of a one-dimensional stochastic process which is conformal invariant. It describes the interface of fluid deposited on a surface in the presence of a rarefied gas. The adsorption is local but the desorption

is not. The interface is peeled. The average height increases logarithmically with the size of the system. The stationary state has magical combinatorial properties and various correlation functions have been obtained. They reflect the conformal invariance of the system. The Hamiltonian which gives the time evolution of the system is expressed as a linear combination of generators of the Temperley-Lieb algebra at the percolation point. The model is derived by mapping the vector space in which the generators of the Temperley-Lieb act, into the configuration space of Dyck paths which represents an interface with fixed end-points. To obtain the spectrum of the Hamiltonian, one uses a different representation of the algebra in terms of Pauli matrices. One obtains in this way the XXZ integrable quantum chain which has a known spectrum. The periodic Temperley-Lieb  $PTL_L(x)$  algebra with  $L$  generators is infinitely dimensional. Taking  $x = 1$  (the percolation point) and different quotients one obtains different stochastic models on a ring. NASEP (nonlocal ASEP) model is an example. An expression of the current for any system size was conjectured and verified by computer simulations. Recently an  $N2^L$  dimensional representation of the periodic Temperley-Lieb algebra was studied. It is also a representation of the cyclic group  $Z_N$ . We define a Hamiltonian as a sum of the generators of the algebra acting in this representation. In the finite-size scaling limit, the spectrum of the Hamiltonian contains representations of the Virasoro algebra with complex highest weights. The partition functions are known analytically. As a consequence of the existence of complex highest weights, the correlation functions have an oscillatory behavior for large system sizes. The two-boundaries Temperley-Lieb algebra can be used to study the open NASEP model. One adds particles at one end of the system and take them out at the other end. The expression of the current was derived in this case too.

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## Weierstrass random walks and complexity

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We make use of the so-called Weierstrass Random Walk Model [1], a forerunner of complexity science developed by E.W. Montroll and M.F. Schlesinger, to discuss various current theoretical and application issues in statistical-mechanical and complex systems. Amongst the former are discrete scale invariance, central limit theorems and renormalization group (RG) concepts. Amongst the latter are fluid structure and criticality, anomalous diffusion and complex systems. The three themes, extremal entropy, self-similarity, and RG theory were highlighted in Ref. [2] where one of the examples chosen as an illustration was the Levy flight and its lattice random walk analog [1]. Here we consider a generalization of this walk, in its symmetric one-dimensional version, to an infinite family of walks such that an RG transformation can be introduced in the discussion [3]. From our analysis it is clear that the Weierstrass walk and the simple nearest-neighbor

step walk are both RG fixed points. The first one is nontrivial in the sense that it is associated with an infinite-ranged step distribution that can be reached via the RG transformation only from other such infinite-ranged distributions required to approach asymptotically the Weierstrass walk. The single-step distributions (SSD) span the critical hypersurface, and we identify the quantities that appear as irrelevant variables that vanish as the RG transformation is repeatedly applied. The other fixed point is trivial since it is generated by the application of the RG transformation to any noncritical finite-ranged SSD. Subsequently, we demonstrate the connection that exists between the extremal properties of entropy expressions and the renormalization group (RG) approach when applied to systems with scaling symmetry such as the walks studied. The particular entropy expressions considered relate to the SSDs.

Then, we make use of the same lattice walk to describe the critical phenomena in a lattice gas or Ising model, a connection that can be exhibited through the existing analogy [4] between a random walk and the Ornstein-Zernike relation for the pair correlation functions in a fluid or magnet. The relationship between the SSD and the direct correlation function of the statistical-mechanical model is used to associate the random walk properties with those of the pair correlations at criticality. The anomalous dimension at criticality is identified with the index of the Levy distribution, therefore relating criticality below the borderline dimension to anomalous diffusion. Finally, we sketch applications to helix coil transitions in a DNA model and to small world properties of a complex network model.

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## Boltzmann distribution with power law tails: new multi-parametric analytical approximations for new cars sells and for known distributions of income and wealth

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There are two problems in a description of experimentally obtained Boltzmann exponential distributions with power-law asymptotes (tails). At first, experimental values of probability distribution function in zero is usually not determined. Then, the transition between exponential part and power law asymptote part is distributed, and the first point of tail part is not determined strictly. Thus the approximation included gap in zero and smooth transition between Boltzmann and power law parts is necessary.

We introduced 4-parameters approximation in form

$$\begin{aligned}
 W_{T\beta\theta}(R - R_{min}) &= \frac{1}{\sqrt{\pi T}} \int_0^\infty \cos(x\sqrt{R - R_{min}}) \\
 &\times \left( \frac{2}{\Gamma(\beta - 3/2)} \left( (\beta - 3/2) \frac{xT}{4\theta} \right)^{\theta/2 - 1/4} \right. \\
 &\times K_{\beta - 1/2} \left( \sqrt{(\beta - 3/2) \frac{xT}{4\theta}} \right)^\theta dx
 \end{aligned} \tag{1}$$

for  $R \geq R_{min}$ , and

$$W_{T\beta\theta}(R - R_{min}) = 0 \quad (2)$$

for  $R < R_{min}$ , respectively. Here  $R_{min}$  is the minimal value of random value  $R$ ,  $T$  is the normalization coefficient (temperature) in Boltzmann part,  $\beta$  is the power law exponent,  $\theta$  characterizes the transition between Boltzmann and power law parts,  $\Gamma$  is Euler function,  $\Gamma(\beta - 3/2)$  is the Bessel function of the second kind. This approximation looks like

$$W_T(R - R_{min}) \simeq \frac{1}{T} \exp\left(-\frac{R - R_{min}}{T}\right) \quad (3)$$

for comparably small  $R$ , and  $W_T \sim R^{-\beta}$  for large  $R$ , i.e. satisfies introduced requirements.

We applied obtained formula to describe known distributions of incomes and wealth [1] as well as recently measured distributions of new car sales [2]. Our analysis shown the function well-describes experimental distributions [1,2]. Moreover, we could define the power law exponents in all experimental distribution more accurately by value  $0,5 - 1$  units.

Since new car sales distributions are found to be corresponding to known distribution of incomes in countries with the advance tax system, it may be used to find the incomes distribution by new car sales distribution in countries with not enough advanced tax systems.

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### Diffusion constants to reaction-spreading on random clusters

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Reaction diffusion processes have been extensively studied in the past years as systems able to shed some light on various problems of different disciplines. These phenomena occur in nature in many ways, for example: epidemic evolution or forest-fires, the growth of populations, the activity of catalyzers and the formation of stars and galaxies. In this context, studies on reaction dynamics on percolating clusters appear very interesting for their physical relevance and their applications in many different scientific and technological fields. Studies of reaction and diffusion processes are oriented to solve a differential equation like in the FKPP model. In the same meaning, there is a quantum field theory approach, such case is formulated by a Langevin equation in the Ito sense or using a diffusion equation where the population density evolves according with a multiplicative noise. Another alternative is developed in this work, using the central limit theorem we can do the study of reaction-diffusion processes on random media, specifically dynamics by radius of the cluster growth on square lattices at nearest-neighbor (SQ2N) and next-to-nearest neighbor (SQ3N), triangular (TR) and honeycomb (HC) lattices around their respective percolation threshold. We present a definition of the radius  $R(t)$  of the

process as time function. In this way, we define the displacement of radius at time  $t$  as follows:  $\Delta R(t) = R(t) - R(t-1)$ , so we can set up the random variable  $\Delta X(t) = \Delta R(t)$ , which we can interpreting as the displacement of a brownian particle in one dimension. Now, the random variable  $X(t) = \sum_{s=1}^t \Delta R(s)$  represents the total displacement of the brownian particle until instant time  $t$ . Now we study the mean squared displacement using Einstein's equation  $\sum_{s=1}^t \Delta R(s)^2 = 2D_x t$ , with this, we can define a diffusion constant, on the other hand, we can defined the diffusion constant of the reaction-spreading process as  $D_R = D_X + \sum_{s=1}^t \Delta R(s)^2 / 2t$ . We are interested in study the behavior of  $D_X$  and  $D_R$  as a function of the probability  $p$ . We may note that both constants present a phenomenon with phase transitions around the percolation threshold  $p_c$  which are characterizable by power-laws with order parameter  $\epsilon = p - p_c$ , in the following way:  $D_X \sim \epsilon^{-\alpha}$ ;  $D_R \sim \epsilon^{1/\delta}$ . The critical exponents obtained by computer simulation of the same order as those reported for other systems.

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### Spintronics Of A Bipolar Semiconductor With Fermi-Dirac Statistics

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Spintronics is an emerging technology exploiting both the intrinsic spin of the electron and its associated magnetic moment, in addition to its fundamental electronic charge, in solid-state devices.

For years, spintronics has searched for a material whose properties can be tuned by electrostatic gates that can also transport spin at room temperature. This property was finally realized in graphene which is composed of a single atomic layer of carbon atoms arranged in a honeycomb lattice. The carbon material can be both electron-doped and hole-doped, something that is important for advancing real-world graphene electronics. It has been found that each pair of boron dopants contributes a hole into the graphene sheet.

The ensemble of electrons can be described by the Boltzmann spinor equation, which recently has given rise the interest of mathematicians [2]. In particular, asymptotic expansion techniques have been utilized for the construction of macroscopic equations (see [2] and references therein). In these models the phonons are considered as a fixed background at a given temperature. Moreover an isotropic parabolic dispersion relation is assumed for electrons. Most of the work is performed under the assumption of Maxwellian distribution function for electrons. However, since spin and statistics are linked, as soon as the spin is introduced, at the mean time the exclusion principle (Fermi-Dirac correction) should be considered. Moreover, in general, the presence of holes should be taken into account.

In the present paper we propose a spintronics for bipolar

semiconductors i.e. carrier spintronics, where the correct Fermi-Dirac statistics of carriers is accounted for. We start from the (spinor) Bloch-Boltzmann-Peierls (BBP) coupled equations for the distribution functions of carriers and phonons.

After that, by means of an expansion of both the unknowns and the interaction kernels with respect to a small parameter which accounts for the umklapp processes (with no momentum conservation), the lowest order equations show that the displaced Fermi-Dirac approximation is justified [3,4].

A closed set of equations for the chemical potential of carriers, the temperatures, and the drift velocities can be constructed, which recalls the extended thermodynamics model [1]. Moreover, equations for the time evolution of the spin densities are added, based on a new model in which the spin flip is induced by phonons.

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## Population dynamics and surname distribution

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The frequency distribution of surnames turns out to be a relevant issue not only historical demography but also in population biology, and especially in genetics, since surnames tend to behave like neutral genes and propagate like Y chromosomes. A brief review of the century-long studies on surname distributions and on the related issues of isonymy, consanguinity and migration dynamics is presented.

The stochastic dynamics leading to the observed scale-invariant distributions has been studied as a Yule process, as a branching phenomenon and also by field-theoretical Renormalization Group (RG) techniques, always obtaining the same prediction for the dependence of the power-law exponents from mutation and migration coefficients.

In special cases (absence of mutations) the theoretical models are in good agreement with empirical evidence; however in many concrete instances there is a still unexplained discrepancy between the theoretical and the empirically measured exponents.

Hints for the possible origin of such discrepancy are presented and discussed, with some emphasis on the difference between the asymptotic frequency distribution of a full population and the frequency distributions observed in its samples. We show how to construct large sets of measurable quantities whose expectation value is independent of the sample size.

It is also theoretically promising to explore the connection between the above mentioned results and the statistical properties of genealogical trees, which may be investigated theoretically by RG techniques thanks to the obvious self-similarity of the trees, but can also be studied empirically by exploiting the very large online genealogical databases presently available, especially concerning many families of the European nobility, for which records of birth and marriage referring to several generations have been consistently preserved.

Some statistical results are presented and discussed, based on the study of the family links of more than 12,500 individuals belonging to the rather compact and closed social group formed by the German *Hochadel* in the Modern Age (1500-1800).

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## Dynamics of entanglement between two harmonic modes in stable and unstable regimes

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The investigation of entanglement dynamics and growth in different physical systems is of great current interest. Quantum entanglement is well known to be an essential resource for quantum teleportation and pure state based quantum computation, where its increase with system size is necessary to achieve an exponential speedup over classical computation. And a large entanglement growth with time after starting from a separable state indicates that the system dynamics cannot be simulated efficiently by classical means, turning it suitable for quantum simulations. In this work [1] we examine the dynamics of the entanglement between two harmonic modes generated by an angular momentum coupling, and its ability to reproduce typical regimes of entanglement growth in more complex many body systems, when starting from an initial separable gaussian state. Such system arises when considering a particle in a rotating anisotropic harmonic trap or a charged particle in a fixed harmonic potential in a magnetic field, and exhibits a rich dynamical structure [2], with stable, unstable and critical regimes according to the values of the rotational frequency or field and trap parameters. Consequently, it is shown that the entanglement generated from an initially separable gaussian state can exhibit quite distinct evolutions, ranging from quasiperiodic behavior in stable sectors to different types of unbounded increase in critical and unstable regions. The latter lead respectively to a logarithmic and linear growth of the von Neumann entanglement entropy with time. It is also shown that entanglement can be controlled by tuning the frequency, such that it can be increased, kept constant or returned to a vanishing value just with stepwise frequency variations. Exact analytic expressions for the entanglement entropy in the different dynamical regimes are provided. The ground state and thermal entanglement properties of the model are also discussed in detail [3], through the analysis of the entanglement entropy and thermal negativity. It is shown that they exhibit as well critical behavior at the borders of stability. A full analytic treatment of these quantities is provided.

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### Parallel and Cross-field streaming instabilities in a dusty Lorentzian Plasma

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Dust kinetic Alfvén wave (DKAW) instability with perpendicularly streaming kappa distributed ions have been examined rigorously in a uniform dusty magnetoplasma. A dispersion relation of low-frequency KAW/DKAW instability on the Whistler and dust acoustic velocity branch is obtained in a low beta Lorentzian plasma. The solutions are compared to various scenarios in dusty and dustless plasma. Under certain limits, the dispersion relation of Modified Two stream Instability (MTSI) is obtained. It is shown that the presence of dust particles and superthermal streaming ions sensibly modify the dispersion characteristics of KAW/DKAW. Numerical calculations for the growth rates are carried out by taking different dust parameters into account. The dust number density plays a significant role to study the above phenomenon. A sufficiently large amount of dust grains and for the beam velocity larger than the Alfvén velocity, we get two unstable solutions. One is the low frequency electromagnetic mode kinetic Alfvén waves coupled with Lorentzian and Doppler-shifted ion plasma oscillations in the beam resulting in the unstable branch and the other is near lower hybrid frequency; therefore, two separate unstable frequency bands may arise. The dispersion branches of KAW instability It has been observed that the lower instability stabilizes due to the high concentration of the negatively charged dust grains, while the upper instability will increase for increased dust number density when  $V_0=10$ . For fixed plasma parameters, lower instability extends to short wavelength and large amplitude as compared to the upper instability.

It is shown that the dispersion relation of DKAW is modified with perpendicular streaming ions as the charged dust grains in a magnetized plasma are highly coupled to the waves due to cyclotron resonances. The dust parameters are found to have significant effect on the instability, for instance, the dynamics of dust particles introduce a cutoff in the Alfvén wave known as dust lower hybrid frequency.

The dispersion relation for the DKAW instability has been derived and found to be dependent on the Lorentzian index. The ubiquitous presence of dust grains and high energy tail distribution, plasma is able to support a number of unstable branches.

It is also found that nonthermality is more effective for kinetic Alfvén waves in the perpendicular component as compared to parallel one. The results obtained have been analyzed and presented graphically which can be applied to various low-beta regions of space and astrophysical plasma.

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### Actin flows mediate a universal coupling between cell speed and cell persistence

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In the first part of this talk, I will present a recent joint experimental and theoretical work that revealed a universal coupling between cell speed and cell persistence. Eukaryotic cell migration is essential for a large set of biological processes, including morphogenesis, wound healing, tumor spreading or immune responses]. Assessing quantitatively the exploratory efficiency of cell trajectories is therefore crucial. In the absence of external guidance, cell movement can be described as a random motion, and proposed models have ranged from simple Brownian motion to persistent random walks, Levy walks or composite processes such as intermittent random walks. Such models essentially differ in the cell persistence, which quantifies the ability of a cell to maintain its direction of motion. The variety of behaviors, observed even along a single cell trajectory, stems from the fact that, as opposed to a passive tracer in a medium at thermal equilibrium, which generically performs a classical Brownian motion, a cell is self-propelled, and as such belongs to the class of active Brownian particles. Here, we show on the basis of extensive experimental data for various cell types and migration contexts in vitro and in vivo that cell persistence, which quantifies the straightness of trajectories, is robustly coupled to migration speed. We suggest that this universal coupling between speed and persistence (UCSP) constitutes a generic law of cell migration, which originates in the advection of polarity cues by a dynamic actin cytoskeleton undergoing flows at the cellular scale. Our analysis relies on a theoretical model that we experimentally validate by measuring the directional persistence of dendritic cells upon gradual modulation of actin flow speeds. Beyond the quantitative prediction of the UCSP, the model yields a generic phase diagram of cellular trajectories that reproduces migration patterns of most cell types. This opens the way to the control of cell migration patterns, and therefore of the capacity of cells to explore tissues.

In the second part of this talk, is dedicated to a theoretical study the first-passage properties of the Pearson random walk. We calculate exactly in 1D and, using an approximated scheme, in 2D and 3D the mean first-passage time (MFPT) to a single target located at the center of the domain. We find that persistent random walks with exponentially distributed excursion lengths make possible the minimization of the MFPT as a function of the persistence length. We show that the existence of such a minimum is robust to changes in the geometry in the specular or diffuse nature of the boundary conditions.

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## Power-law statistics from nonlinear stochastic differential equations driven by Lévy stable noise

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Complex dynamical systems that are governed by anomalous diffusion often can be described by Langevin equations driven by Lévy stable noise. The Lévy stable processes constitute the most general class of stable processes while the Gaussian process is their special case. The physical reason behind the Lévy stable processes is the inhomogeneous structure of the environment, in particular, fractal or multi-fractal [1]. Nonlinear stochastic differential equations with additive Lévy stable noise have been explored quite extensively for past 15 years [2]. In this contribution we investigate equations with multiplicative Lévy stable noise. We have obtained nonlinear stochastic differential equations driven by Lévy stable noise and giving a power-law steady state distributions of the signal intensity. To do this we have considered the corresponding fractional Fokker-Planck equations and assumed that the drift and diffusion coefficients should have a power-law form. We have investigated two cases when the signal is positive and when the signal can also be negative. In contrast to the equation with the Gaussian noise, the constant in the drift term is different in those two cases. Our equations with Lévy stable noise appear to be generalizations of earlier proposed [3] nonlinear stochastic differential equations with Gaussian noise yielding power-law behavior of spectrum. Similarly to equations with Gaussian noise, the signal generated by our generalized equations also has a power-law behavior of the spectrum, including  $1/f$  noise, in a wide range of frequencies. By considering the scaling properties of the equations we have obtained analytical expression for the power-law exponent of the spectrum and confirmed it by comparing with numerical solution of stochastic differential equations. This power-law exponent depends not only on the parameters of the equations but also on the index of stability  $\alpha$  of the Lévy stable noise. For numerical solution of the stochastic differential equations we used Euler's method and employed a variable step of integration. We expect that our equations may be useful for describing  $1/f$  fluctuations in inhomogeneous systems subjected to Lévy stable noise and to investigate Lévy flights in non-equilibrium and inhomogeneous environments.

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## Scaling behavior in the $3d O(n) \phi^4$ model in half-space and slab geometries with free boundary conditions at $n = \infty$

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In 1977, Bray & Moore [1] solved exactly the  $d$ -dimensional  $n$ -component  $\phi^4$  model in the half-space  $z > 0$  with free boundary condition in the limit  $n = \infty$  directly at the bulk critical point  $T_c$ . We have obtained the exact solution of the

same model in the half-space geometry at  $d = 3$  in the whole critical region  $T \geq T_c$  at zero magnetic field. The logarithmic singularity of the surface free energy  $f_s(t)$  as the function of the temperature parameter  $t \sim (T - T_c)/T_c$  was found, and the explicit expression for the surface spin-spin correlation function in the whole critical region was obtained. The presence of the free boundary at  $z = 0$  in the  $\phi^4$  model breaks explicitly its the translation symmetry in  $z$ -direction. As the result, the generalized spherical model, to which the  $O(n) \phi^4$  model reduces at  $n \rightarrow \infty$ , contains the non-uniform effective potential  $V(z, t)$ . At a given temperature  $t$ , the latter should be determined from the  $z$ -dependent self-consistency equation [1,2]. Since it turns out impossible to solve this equation analytically with respect to  $V(z, t)$  at  $t \neq 0$ , a different strategy has been used, which applied the methods of the inverse scattering theory. We reformulated the self-consistency equation in terms of the scattering data of the associated one-dimensional Schrödinger operator  $-\partial_z^2 + V(z, t)$  in the half-line  $0 < z < \infty$ , and then determined these scattering data in the explicit form at all temperatures  $t \geq 0$  in the scaling region. Afterwards, the desired physical quantities were calculated in the scaling region directly from the scattering data without explicit knowledge of the self-consistent potential  $V(z, t)$ .

In the slab geometry, the finite-size scaling arguments imply the following expansion for the free energy (normalized by the unit area of the boundary surface) of the slab of width  $L$  in the scaling region:  $f(t, L) = Lf_b(t) + 2f_s(t) + \Theta(x)/L^2 + \dots$ , where  $f_b(t)$  is the bulk free energy density,  $f_s(t)$  is the surface free energy, and  $\Theta(x)$  is the universal Casimir free energy scaling function, which depends on the scaling parameter  $x = tL$ . Based on the results of the inverse scattering solution of the half-space problem, we have calculated the asymptotics of the scaling function  $\Theta(x)$  at  $x \rightarrow \pm\infty$ , and described its singular behavior at  $x \rightarrow \pm 0$ . Obtained analytical results were confirmed by recent precise numerical calculations of the Casimir force and Casimir free energy scaling functions [3].

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## Baryon masses in the three-state Potts field theory in a weak magnetic field

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Confinement of the kink topological excitations typically takes place in one-dimensional quantum systems having a discrete vacuum degeneracy, if the latter is explicitly broken by an external field. The ferromagnetic Ising field theory provides the simplest and most studied example, in which application of the weak external field induces confinement of kinks into the two-particle bound states - the mesons [1,2].

The similar mechanism leads to confinement in the ferromagnetic 3-state Potts Filed Theory (PFT), which describes the scaling limit of the 3-state Potts model in the  $2d$  lattice in the ferromagnetic phase near the phase transition point. There are three degenerate vacua  $|\alpha\rangle$ ,  $\alpha = 1, 2, 3$  in the 3-state PFT in the ferromagnetic phase at zero magnetic field  $h = 0$ . Elementary excitations consist of six kinks  $K_{\alpha,\beta}$ ,

$\alpha, \beta = 1, 2, 3$ , which interpolate between different vacua  $|\alpha\rangle, |\beta\rangle$ . If a positive magnetic field  $h$  that couples to the spin component  $\sigma_3$  is applied, the energy of the vacuum state  $|3\rangle$  decreases, while the vacua  $|1\rangle$  and  $|2\rangle$  remain degenerate and become metastable. This leads to the long range linear attractive potential between kinks, which causes their confinement [3]. Besides the meson (two-kink) states, there appear also two series of the baryon (three-kink) bound states  $p_n^{(\kappa)} = K_{3,1}K_{1,2}K_{2,3} + (-1)^\kappa K_{3,2}K_{2,1}K_{1,3}$ , which differ in their parity  $\kappa = 0, 1$ . In the leading order in  $h \rightarrow +0$ , the masses of lightest baryons  $M_n^{(\kappa)}$ ,  $n = 1, 2, 3 \dots$ ,  $\kappa = 0, 1$  in the 3-state PFT are determined by the stationary Schrödinger equation, which describes relative one-dimensional motion of three kinks with coordinates  $0 < x_1 < x_2 < x_3 < \infty$  interacting via the linear confining potential  $(x_3 - x_1)f_0$ . Here  $f_0 \sim h$  is the string tension, and the wave function  $\Psi(x_1, x_2, x_3)$  should vanish at  $x_1 = x_2$ , at  $x_2 = x_3$ , and at  $x_3 - x_1 \rightarrow \infty$ . By the scaling reasons, the masses of the lightest baryons  $M_n^{(\kappa)}(f_0)$  depend on the string tension  $f_0$  and the kink mass  $m$  as:  $M_n^{(\kappa)}(f_0) = 3m + (f_0/m)^{2/3}Z_n^{(\kappa)} + \dots$ , in the leading order in  $f_0 \rightarrow +0$ . Through numerical solution of the described above boundary problem, I obtained approximate values for the several initial constants  $Z_n^{(\kappa)}$ :  $Z_1^{(0)} \approx 4,602$ ,  $Z_2^{(0)} \approx 5,912$ ,  $Z_3^{(0)} \approx 7,095$ ,  $Z_1^{(1)} \approx 6,649$ ,  $Z_2^{(1)} \approx 7,735$ ,  $Z_3^{(1)} \approx 8,753$ .

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## Thermal environments in quantum games

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A two player quantum game is considered in the presence of thermal decoherence. It is shown how the thermal environment modeled in terms of the rigorous Davies approach affect pay-offs of the players. The general definition of a quantum game would be involved. Here by a quantum game we understand a quantum system that can be manipulated by at least one party and for which utilities of moves can be reasonably defined [1]. The only fully natural source of decoherence affecting quantum systems is due to their environment causing both energy and information dissipation. For our model considerations we assume that at least one of two qubits in a state shared by Alice and Bob just before applying their strategies  $\mathbf{S}_{A,B}$  interacts with its own environments  $E_{A,B}$ . As Alice and Bob can be separated from each other we neglect any direct interaction both between their qubits (via proper Hamiltonian term) and the environments  $E_{A,B}$ . In other words, Hamiltonian of the total system is simplified to the form:

$$H = H_A + H_B + H_{AE_A}^{int} + H_{BE_B}^{int}$$

We also assume that qubits  $A$  and  $B$  are identical:

$$H_A = H_B = \frac{\omega}{2}(|1\rangle\langle 1| - |0\rangle\langle 0|)$$

Instead of using the full power of Davies semigroups, we consider certain elements of Davies dynamical semigroups: Davies maps [2]. The appropriate Davies map  $D = D_{A,B}(p, A, G, \omega, t)$

reads as follows [2,3]:

$$\begin{aligned} D|1\rangle\langle 1| &= [1 - (1-p)(1 - e^{-At})]|1\rangle\langle 1| \\ &+ (1-p)(1 - e^{-At})|0\rangle\langle 0| \\ D|1\rangle\langle 0| &= e^{i\omega t - Gt}|1\rangle\langle 0| \\ D|0\rangle\langle 1| &= e^{-i\omega t - Gt}|0\rangle\langle 1| \\ D|0\rangle\langle 0| &= p(1 - e^{-At})|1\rangle\langle 1| \\ &+ [1 - (1 - e^{-At})p]|0\rangle\langle 0| \end{aligned}$$

where  $p \in [0, 1/2]$  is related to the temperature (here we set  $k_B = 1$ ) via:

$$p = \exp(-\omega/2T) / [\exp(-\omega/2T) + \exp(\omega/2T)].$$

Our analysis shows that the dependence is strongly affected by thermal environment. Moreover, the presented analysis stress that the payoffs can vary dramatically in cases when the environment is attached either to Bob's or Alice's qubit. The results will be presented graphically for various special cases of payoff differences. It would be of great interest to adapt this approach to quantum games on networks of agents.

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## Quantum auctions and quantum mechanism design

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We would like to discuss the concept of a quantum auction, its advantages and drawbacks. Quantum auction are quantum games designed for goods allocations [1]. There is hope that due to the quantum computation speed up resulting from unitary evolution and entanglement they might be some day an alternative for "classical" auctions in cases where combinatorial and computational problems hinder the designers in their work. Quantum game theory, whatever opinions may be held due to its abstract physical formalism, have already found various applications even outside the orthodox physics domain. We envisage that the implementation might not be an easy task. At least a restricted model e.g such as the one advocated by the Hewlett-Packard group [2] seems to be a feasible idea. Quantum auctions, if ever implemented, would be designed for very specific and limited business circles: the volume must be huge and items combined. The presentation will be organized as follows. We will begin by presenting the general idea of a quantum auction and methods of gaining an advantage over "classical opponent". Then we will proceed to the quantum mechanism design problems, that is the theory of construction of quantum games with equilibria implement a given social choice rules [3]. Finally we will try to show some problems that should be addressed in the near future. In discussion we will use quantum auction theory as a formal theoretical tool but the broadcasted message would be that it would probably be used for massive combinatorial auctions in the future [4] or in compound securities trading. On the other hand quantum algorithm may be used for solving computational problem in the classical setting that is without

actual implementation of a quantum action. Quantum auctions are certainly an interesting theoretical alternative for complex and massive auctions but are they feasible? Encoding bids in quantum states is a challenge to (quantum) game theory: quantum auctions would almost always be probabilistic and may provide us with specific incentive mechanisms etc. As the outcome may depend on amplitudes of quantum strategies sophisticated apparatus and specialist may be necessary. Therefore, we envisage some changes in the law and habits. Despite the promising quantum-like experiments commercial implementation of quantum auctions is a demanding challenge that would hardly be accomplished without a major technological breakthrough in mastering quantum devices. Extreme security and privacy are other strong points of quantum auctions. Currently, it is difficult to find out if this is a feasible task.

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### Complex-valued Time Parameter for the Dynamics of Time-dependent Random Hamiltonian

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We investigate the dynamics of a quantum system generated by a random Hamiltonian of the form  $\mathbf{H}_\omega(t) = \mathbf{H}_0 + \mathbf{V}_\omega(t)$ , where  $\omega$  denotes a configuration in the probability space. Here the deterministic Hamiltonian operator  $\mathbf{H}_0$  is assumed solvable and time-independent, while  $\mathbf{V}_\omega(t)$  is a smooth time-dependent random potential operator. By extending the time-parameter to be complex valued and defining the Hamiltonian off the real line from its evolution history as  $\mathbf{H}_\omega(t + i\tau) := \mathbf{H}_\omega(t - \tau)$ , we can formulate the evolution of the system's state under the random Hamiltonian, keeping the potential's history in the imaginary time. The average of interested operators over the state  $\rho$  at time  $t$  and the two-epoch time correlations are consequently performed in similar fashion to the Keldysh-contour formalism. After applying the mean-field approximation and perturbation from a bath to the dynamics, when the potential  $\mathbf{V}_\omega(t)$  is assumed periodic in  $t$  and its  $\omega$ -average is time-independent, we find that the Kubo's formula, which connects the correlation  $\text{Tr}(\sigma_{\beta'} [\mathbf{A}(t), \mathbf{B}(t')]) - \sigma_{\beta'}$  is a whole equilibrium state with respect to inverse temperature  $\beta'$ - and response functions  $\chi_{\mathbf{AB}}(t, t')$ , has to be modified by adding a term arising from the correlation between the random part of Hamiltonian and the bath as the following: for  $t > t' > 0$

$$\chi_{\mathbf{AB}}(t, t') = 2i\theta(t-t') (\text{Tr}(\sigma_{\beta'} [\mathbf{A}(t), \mathbf{B}(t')]) + \Delta_{\mathbf{AB}}(t, t')),$$

where

$$\Delta_{\mathbf{AB}}(t, t') := \frac{1}{2} \text{Tr} \left[ \left( \int_0^{t'} e^{-i(t-t')\mathcal{L}} \right. \right. \\ \left. \left. [\mathbf{B}, e^{-i(t'-s)\mathcal{L}} [\mathcal{C}(\mathbf{H}, \rho(s)), \sigma_{\beta'}] ds] \mathbf{A} \right) \right];$$

and

$$\mathcal{C}(\mathbf{X}, \mathbf{Y}) := \text{Cov}(\mathbf{X}, \mathbf{Y}) - \text{Cov}(\mathbf{Y}, \mathbf{X})$$

is a difference of  $\omega$ -covariances between  $\mathbf{X}$  and  $\mathbf{Y}$ . This suggests that, under the given random potential, the statistical properties of the system can be appropriately modelled and predicted by the deterministic Hamiltonian  $\mathbf{H}_0$  and the history of the time evolution of the random potential.

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### Phase locked solution of the Kuramoto model on trees

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Synchronization is one the most important dynamical processes which is vastly studied in many areas of science and engineering. The Kuramoto model is a well known model for investigating synchronization on complex networks. In this model some phase oscillators (which are considered as nodes of a complex network) are coupled through the sine of phase differences. We use the corresponding 'line-graph' of the network to find the exact solution for phase-locked states on networks with tree topologies. Line graph of a graph is a graph in which nodes correspond to the links of the original graph and nodes are neighbor if the corresponding links in the original graph have one common end. Using the properties of the line graph, we find some analytic conditions for the existence of such solutions. The results are confirmed by numerical simulations for some graphs with tree topologies including random trees.

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### Optimization of Traffic Efficiency under Bounded Information accessibility and structural effect

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It is crucial for many socioeconomic systems to find out an optimum for use of limited information, especially traffic systems are impossible to facilitate whole range of resources. Based on bounded information accessibility such as high cost or technical limitation, we find a new optimization strategy to improve traffic efficiency in the traffic system with lights and intersections. There are various researches to achieve the maximum vehicle speed or the minimum waiting time on the given traffic condition, including the study fulfilled by Chowdery and Schadschneider (ChSch). In this paper, we introduce the modified version of ChSch with independently functioning decentralized control system. With the new model, we figure out the optimization strategy under the bounded information accessibility, which proves the existence of optimal point to cause the phase transition in the system. The paper also gives to us a clue, with which an architect make the traffic system

efficient using the analysis of local site area and symmetry. Moreover, we support to our results with the statistical analysis of the empirical traffic data in Seoul, Korea. Traffic flow represents various and intriguing complex phenomena. There are abundant studies of traffic system problems which have motivated great researches from physicists, in particular for their statistical and dynamical features. With the inherent property of traffic system, Agent-based model (ABM) has been used for study on traffic dynamics. We can classify two categories of study for traffic system with ABM, pedestrian dynamics and vehicle dynamics. The pedestrian problems have been investigated with microscopic models, including cellular automata (CA) models, the social force model, the magnetic force model and the centrifugal force model. In contrast to the pedestrian problems, the vehicle problems focus on not the individual behavior, but the structural effect of the important variables. Especially, traffic lights is the one of the most important variable to describe the phase transition such as transition from jam to free flow phase. Although there are plentiful references on traffic jams, chaotic traffic flows, pedestrian flows, sequence of traffic lights, and etc., there is lack of studies on the system efficiency with traffic lights to make traffic condition more efficient in urban traffic system. The establishment for regulation of efficient traffic lights is important to improve mobility and lessen waiting time of vehicles on the road because there are intersections to be the greatest reason of speed reduction in the city like gigantic metropolis.

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### Random geometry and the KPZ universality class

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We consider random metrics on  $R^2$  which are flat on average and only present short-range correlations. Specifically, we focus on the statistical properties of balls and geodesics, i.e., circles and straight lines. Despite the geometric setting, we can reformulate this problem as one in kinetics, by realizing that it is a continuous analogue of a first passage percolation (FPP) problem. The key insight is to assume an underlying Euclidean metric on our random manifold, and consider distances as times required to travel between different points. Balls correspond to the spread of walkers following geodesics starting from a given point [1].

The roughness of a ball of radius  $R$  is shown to grow as  $R^\chi$ , with  $\chi = 1/3$ , while the lateral spread of the minimizing geodesic between two points at a distance  $L$  grows as  $L^\xi$  with  $\xi = 2/3$ . These results allow us to postulate that balls in these random metrics pertain to the Kardar-Parisi-Zhang (KPZ) universality class, with  $1/\xi$  relating to the dynamical exponent ( $z$ ) and  $\chi$  to the growth exponent ( $\beta$ ) within the

Family-Vicsek scaling Ansatz for kinetic roughening.

The random metrics are generated point-wise, assuming a random distribution for the eigenvalues of the metric tensor and the directions of the eigenvectors. The balls are studied via numerical simulations. We have devised a discretization scheme which relies upon the Huygens principle, which is stable and very natural geometrically. In order to study the lateral fluctuations of the geodesics, we have employed the dynamical programming principle, and studied the fluctuations in the intersection of pairs of balls starting from different points. Indeed, the minimizing geodesic between two points will pass through the intersection point between the two balls of equal radius emerging from each of them.

The KPZ universality class is believed to convey universal critical amplitudes, in our case universality in the radial fluctuations. Indeed, in many known cases of circular geometry, the radial fluctuations follow the Tracy-Widom distribution from random matrix theory for the Gaussian Unitary Ensemble (TW-GUE) [2,3]. Unexpectedly, we find that the radial fluctuations do not follow Tracy-Widom statistics. Nonetheless, by exploiting the analogy between our random metric model and first passage percolation, we find that TW statistics do show up in a different observable: the fluctuations of arrival times.

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### The role of topology on the Kardar-Parisi-Zhang universality class

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The kinetic roughening properties of 2D stochastic interfacial growth models fall into a small number of universality classes. Among them, the KPZ (Kardar-Parisi-Zhang) class has received a lot of attention recently. The reason is that systems in this class not only share the values of the critical exponents, but also those of universal amplitudes, and even the full probability distribution of the fluctuations. Moreover, these properties depend on the topology of the configuration space, i.e., whether it is a cylinder or a plane. In a cylinder, which is the case also known as band geometry, the height fluctuations follow the Tracy-Widom distribution for the Gaussian Orthogonal Ensemble (TW-GOE). Meanwhile, for a plane, the case also known as circular geometry, the radial fluctuations follow the Tracy-Widom variant for the Gaussian Unitary Ensemble (TW-GUE). This behavior has been proved analytically for a number of cases, and has been checked numerically for others [1,2].

In 2011 we proposed a covariant version of the KPZ equation [3], which provides its natural extension to an arbitrary Riemannian metric, and is devoid of the usual small-slopes and

no-overhangs approximations:

$$\partial_t \vec{r} = (A_0 + A_1 K(\vec{r}) + A_n \eta(\vec{r}, t)) \vec{u}_n.$$

Here,  $\vec{r}$  is the position vector of any point in the interface;  $A_0$  is the average growth rate,  $A_1$  is the surface tension with  $K(\vec{r})$  being the local extrinsic curvature,  $\eta$  is a white noise with amplitude  $A_n$ , and  $\vec{u}_n$  is the outwards pointing normal. Along with the equation, we provide a natural discretization scheme which makes simulations efficient and stable. These simulations indicate that the short-time behavior corresponds to different universality classes depending on the topology: While for the cylinder they correspond to the Edwards-Wilkinson class [3], for the plane they correspond to a self-avoiding walk [4]. Moreover, it was shown that the presence of noise renormalizes the remaining terms in the equation.

In this contribution we present a general framework to understand the relation between topology and the KPZ class, by studying the behavior of the above equation on a series of manifolds which interpolate smoothly between the cylinder and the plane, namely, cones of increasing aperture. The Gauss-Bonnet theorem provides a conserved quantity with a topological origin, which helps explain the different behaviors. Moreover, we study the KPZ class in homogeneous spherical and hyperbolic geometries.

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## Reconstructing countries diversification by a simple competitive model

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Economic Complexity is a brand new framework for the Macro-Economic analysis: using tools from complex systems, it provides non monetary metrics to measure the competitiveness of countries. In this frame, trade data can be rearranged in a total exports binary matrix,  $M_{cp}$  and, using Fitness and Complexity algorithm (a sort of two dimensional PageRank for the bipartite network of countries/products, [1]), it is possible to give a valuation of the ability of countries of exporting exclusive products. Rearranging the M-matrix by Fitness and Complexity reveals that developed (and high fitness) countries tend to diversify their productions, from highly exclusive commodities to common ones, while poor countries focus their exports on a fixed finite subset of fitter countries ones; in this way the  $M_{cp}$  matrix shows a peculiar triangular shape. Such a behaviour has been observed in completely different biological system, as the evolution of insects/flowers populations in a isolated environment, [2] and it has been measured by the “nestedness”, a global property for bipartite network adjacency matrix. The  $M_{cp}$  and its dynamics are completely different from the biological case, but tools can be borrowed for different aims. The model by

[3] is able to give the right values of nestedness and other proper signatures of  $M_{cp}$ , using few parameters and the Economic Complexity concepts of capability, i.e. the building blocks for producing commodities. The authors build a hierarchy of products, based on the capabilities contents and assign capabilities directly to countries, thus providing the export basket to countries. The algorithm is able to give a snapshot of the M-matrix and give some hints about its constitutive properties, but it does not provide a dynamical evolution. Basing our analysis on [3], we propose two different evolutionary algorithm that produce an  $M_{cp}$  with the observed properties: the first approach makes countries move on the product networks proposed by [3], while the second one makes the product network evolve together with its occupation by countries. Both the two approaches avoid an explicit use of the capabilities concept, thus not entering in the debate about their nature. The results of our simulation are compatible with the observation and shed new light in the evolutionary processes in Economic Complexity.

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## An effect of natural disasters to socio-economic systems and a procedure to estimate physical exposures

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Our social and economic activities are strongly influenced by natural or man-made disasters. After social infrastructures and human resources were damaged by disasters, socio-economic performance had gone down and human behavior had changed drastically. In this talk, I will show several case studies on influence of natural disasters to our society and propose methods to estimate risks of our society from both social and economic points of view. As the first example I will show changes in the Tokyo stock exchange market. This study comprises a comprehensive analysis of time series segmentation on the Japanese stock prices listed on the first section of the Tokyo Stock Exchange during the period from 4 January 2000 to 30 January 2012. A recursive segmentation procedure is used under the assumption of a Gaussian mixture. The number of each quintile of variance for all the segments is used as an indicator of macroeconomic situations and is investigated empirically. The results show that from June 2004 to June 2007, a large majority of stocks were stable and that from 2008 several stocks were unstable. In March 2011, the number of unstable stocks increased dramatically due to societal turmoil after the Great East Japan Earthquake. It is concluded that the number of stocks included in each quintile of volatility provides useful information about the Japanese macroeconomic situation. Next, I will show an example from Japanese tourism industry. This study investigates the impact of the Great Japan Earthquake (and subsequent tsunami turmoil) on socio-economic activities by using data on hotel opportunities collected from an electronic hotel booking service. A method to estimate both primary and secondary regional effects of a natural

disaster on human behavior is proposed. It is confirmed that temporal variation in the regional share of available hotels before and after a natural disaster may be an indicator to measure the socio-economic impact at each district. I further propose a procedure to estimate regional risks of tsunami run-up events based on 1 km square grid statistics of data on Japanese landscape and population census with the generalized Pareto distribution. This method computes physical exposures defined by both exposed value and hazard frequency. I estimate regional risks of tsunami turmoils in Japan.

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### Memory-induced sign reversals of the spatial cross-correlation for particles in viscoelastic shear flows

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Recently, several studies have focused on the dynamics of underdamped Brownian particles trapped by harmonic potentials and exposed to shear flows [1-4]. However, in most papers it is assumed that the interaction of Brownian particles with shear flow is characterized by Stokes friction [1-3]. The latter is irrelevant for shear flow in viscoelastic media, where anomalous diffusion occurs (e.g., in the cytoplasm of living cells, in colloidal suspension and dusty plasmas) [4]. Moreover, the previous calculations are based on models without using multiplicative noise. It is important to note that multiplicative noise arises in a natural way in quantitative measurements with laser-optical tweezers, where the stiffness of the effective trapping potential may fluctuate. Motivated by the experimental results of Ref. [1] and by the theoretical results in [4], we study the behavior of shear-induced cross-correlation functions between particle fluctuations along orthogonal directions in the shear plane for harmonically trapped Brownian particles in a viscoelastic shear flow. A generalized Langevin equation with a power-law-type memory kernel is used to model the system dynamics. The interaction with fluctuations of environmental parameters is modeled by a multiplicative white noise (fluctuations of the stiffness of the trapping potential), by an internal fractional Gaussian noise, and by an additive external white noise. It is shown that the presence of a memory has a profound effect on the behavior of the cross-correlation functions. Particularly, memory-induced reentrant sign reversals of the spatial cross-moment between the orthogonal random displacements of a particle are established, i.e., an increase of the memory exponent can cause a sign reversal from positive to negative, but by further increase of the memory exponent a reentrant transition from negative to positive values of the cross-correlation appears. Let us note that in the case of Stokes friction the corresponding cross-moment is always positive. Moreover, the dependence of the cross-correlation functions on the lag time is analyzed. It is shown that additive external and internal noises cause qualitatively different dependences of the cross-correlation functions on the time lag. The occurrence of energetic instability due multiplicative noise is also discussed.

We believe that our results suggest some possibilities for interpreting experimental data in applications where the issues of viscoelasticity and multiplicative noise can be crucial.

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### The Gonihedric Paradigm. Extensions of the Ising Model

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We suggested a model of random surfaces demanding that geometrically nearby configurations of the triangulated surfaces must have close probabilities. This principle leads us to a new energy functional, which is called Gonihedric functional [1]. The model can be considered as a natural generalization of the Feynman integral over the random walks to an integral over the random surfaces because these probabilities, for a random walk and a random surface, coincide when a surface degenerates into a single path. The partition function is defined as an integral over all random surfaces of a given triangulation and a summation over all topologically different triangulations. We analyze the properties of the partition function and its convergence. We prove that the contribution of a given triangulation to the partition function is finite and have found the explicit form for the upper bound [2]. The original model of gonihedric random surfaces was formulated as embedding of random surfaces into the Euclidean space. It can also be formulated as a model of random surfaces embedded into the hypercubic lattice [3]. The advantage of the lattice formulation is that one can construct a spin system, which is a generalization of the Ising model with ferromagnetic and antiferromagnetic interactions, such that its interface energy is equal to the gonihedric energy. These spin systems have very high symmetry, one can flip the spins on any flat hypersurface without changing the energy of the system. The lattice formulation the partition function of the random surfaces is exactly equal to the partition function of the corresponding spin system and can be studied by powerful analytical methods, as well as by Monte-Carlo simulations. It is well known that the two-dimensional Ising model is self-dual system and that three-dimensional Ising model is dual to the gauge spin system. This duality was an important fact allowing to find the exact solution of the Ising model in two-dimensions. We were able to construct dual systems for the gonihedric spin systems in three and four dimensions [4]. These results allows to construct the corresponding transfer matrix, to prove that is describe the propagation of closed loops and to find its spectrum [5]. This is a unique exact solution of the tree-dimensional statistical spin system [6]. Because gonihedric spin systems have exponentially degenerated vacuum states, which are separated by the potential barriers, one can suggest to use such systems as storages of the binary information. There is no interface energy proportional to the area in these systems therefore one can store a one bit of information in a very small region of the lattice [7].

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## $\kappa$ -Deformed Fourier Integrals Transforms

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As known, Fourier integral transform is a very useful mathematical tool widely used in statistics and physics. It is a linear operator transforming a function  $f(x)$  with a real argument  $x$  into a function  $\mathcal{F}(\omega)$  with a complex argument  $\omega$ . Formally, this transformation belongs to the family of integral transform defined as

$$\mathcal{H}[f(x)](\omega) = \int_{-\infty}^{+\infty} f(x) h(\omega, x) dx,$$

in which the kernel of the transform  $h(\omega, x)$  is of exponential type.

In application to statistics, Fourier transform of the density distribution gives the characteristic function that encodes all information restored in the probability distribution. In fact, the momenta of a random variable can be obtained by means of derivative of any order of the characteristic function. More importantly, Fourier transform is employed to proof several limiting theorems including the important central limit theorem [1].

Generalization of the above integral transforms has been proposed in different mathematical framework. In this contribute, we explore several possible definitions of deformed Fourier transforms corresponding to the following kernels

$$\begin{aligned} h_{\kappa}^{(1)}(\omega, x) &= \left( \exp_{\kappa}(-\omega \otimes x) \right)^i, \\ h_{\kappa}^{(2)}(\omega, x) &= \left( \exp_{\kappa}(-x) \right)^{i\omega}, \\ h_{\kappa}^{(3)}(\omega, x) &= \left( \exp_{\kappa}(-\omega) \right)^{ix}, \\ h_{\kappa}^{(4)}(\omega, x) &= \left( \exp_{\kappa}(-\omega x) \right)^i. \end{aligned}$$

which follow by employing the  $\kappa$ -exponential

$$\exp_{\kappa}(x) = \left( \kappa x + \sqrt{1 + \kappa^2 x^2} \right)^{1/\kappa}, \quad (4)$$

a function derived from the mathematical formalism underlying to the  $\kappa$ -deformed statistical mechanics [2,3,4,5], a theory developed with the purpose to study non Gibbsian statistical systems characterized by power-law distribution functions.

In all the cases discussed the usual transform is recovered in the  $\kappa \rightarrow 0$  limit, since in the same limit all the kernels reduce to the standard exponential one  $h_{\kappa}^{(l)}(\omega, x) \rightarrow \exp(-i\omega x)$ .

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## Intermittency, multifractal vector fields, Lie cascades and stochastic Clifford algebra

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Intermittency is a key issue in statistical physics. A major breakthrough occurred with multifractals in the 1980s: intermittency became understood as resulting from an infinite hierarchy of fractal supports of the field singularities. However this was mostly done for scalar fields, whereas the fields of interest, e.g. the velocity for turbulence, are generally vector fields. This gap has prevented many developments.

Some time ago, "Lie cascades" were introduced (Schertzer and Lovejoy, 1993, 2011; Schertzer et al., 1997) to deal with multifractal vector fields. This was done by first considering a scalar multiplicative cascade as a non trivial limit for a vanishing inner scale of the exponentiation of a generator, defined as an additive stochastic process with a logarithmic divergence with the inner scale. Although non trivial, this limit is by noway limited to scalar processes. Indeed, the latter are only the simplest case of an exponentiation from a stochastic element of a Lie algebra to its Lie group of transformations. The concerned transformation corresponds to the fine graining/downscaling of the field to higher and higher resolution (Schertzer et al., 2012).

However, the theoretical efforts were mostly concentrated on the Levi decomposition of this algebra into its radical and a semi-simple algebra. Developments were again paused, this time due to the possible large number of degrees of freedom of the latter, in particular with respect to the information that can be easily extracted from a d-dimensional vector field. In short, some physics was missing.

Although Clifford algebra  $Cl_{p,q}$  have been mentioned at once for rather straightforward generalisations of complex cascades, they were no more than mentioned. This is in sharp contrast with the rather frequent use of the "pseudo-quaternions"  $l(2, R) = Cl_{2,0} = Cl_{1,1}$ , back to Schertzer and Lovejoy (1985), to generate generalised scales to analyse and simulate anisotropic scaling (scalar) fields. This was not done by chance:  $Cl_{p,q}$  has a basis whose vectors anti-commute and square to the identity or its opposite, and is generated by a quadratic form  $Q$  of signature  $(p, q)$ . These properties being physically meaningful are convenient to understand, analyse and simulate intermittent vector fields. The applications of the corresponding Lie cascades will be illustrated with the help of simulations of geophysical processes.

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## Speed Gradient and MaxEnt principle for Shannon and Tsallis entropies

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The notion of entropy is widely used in modern statistical physics, thermodynamics, information theory, engineering etc. In 1948, Claude Shannon introduced his information entropy for an absolutely continuous random variable  $x$  having probability density function (pdf)  $p$ . In 1988, Constantino Tsallis introduced a generalized Shannon entropy. Tsallis entropy have found applications in various scientific fields such as chemistry, biology, medicine, economics, geophysics, etc.

A phenomenon when system tends to its state of maximum entropy is known as the maximum entropy (MaxEnt) principle. Since seminal works of E.T. Jaynes (1957) and until recent years the MaxEnt principle attracts a strong interest of researchers. The MaxEnt principle defines the asymptotic behavior of the system, but does not say anything about how the system moves to an asymptotic behavior. Despite a large number of publications studying the maximum entropy states, the dynamics of evolution and transient behavior of the systems are still not well investigated.

In this paper we consider dynamics of non-stationary processes that follow the MaxEnt principle. We have derived a set of equations describing dynamics of pdf for Shannon and Tsallis entropies. Systems with discrete probability distribution and continuous pdfs are considered under mass conservation and energy conservation constraints. The uniqueness of the limit pdf and asymptotic convergence of pdf are examined. Convergence of pdfs does not lead to the convergence of the corresponding differential entropies. Based on sufficient conditions the nontrivial convergence of differential entropies is proved.

We use the speed-gradient (SG) principle originated in control theory [1]. Applicability of the SG principle has already been experimentally tested for the systems of finite number of particles simulated with the molecular dynamics method [2,3]. We apply similar approach for the systems with discrete and continuous probability distributions considering Shannon and Tsallis entropies.

SG principle generates equations for the transient (non-stationary) states of the system operation, i.e. it gives an answer to the question of "How the system will evolve?" This fact distinguishes the SG principle from MaxEnt principle, the principle of maximum Fisher information and others characterizing the steady-state processes and providing an answer to the questions of "To where?" and "How far?"

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## Asymmetrical free diffusion with orientation-dependence of molecules in finite timescales

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Diffusion is one of the most fundamental matter transport

phenomena. Free diffusion of particles from the molecular to the macroscopic scale suspended in liquids or gases has been extensively studied. Tracking of particle diffusion is always considered as a result of the random walk of particles, and thus the displacements of the particle are handled as isotropic. In recent years, it has become well-recognized that extensive biological, chemical and even physical processes including conformation changes usually occur in nano-, pico- and even femto-seconds, and very small spaces. For example, signaling molecules in cells typically diffuse within several nanometers, where the solvents are discrete water molecules and the structure of these molecules should not be ignored. Thus the dynamic behaviors of diffusion at nanoscale could not be random any longer and the correlation time of thermal motion may also not be zero any more. Until recently, conventional theories are still widely applied in studying the behavior in various systems with finite timescales and length scales, which may have led to misunderstanding. Using molecular dynamics simulations, we present the spontaneous asymmetric diffusion, which is orientation-dependent diffusion of asymmetric molecules in finite timescales. Here, we use molecules including methanol, glycine to illustrate this idea, and we find the asymmetric part of the diffusion reaches about 10% of the total diffusion when the diffusion distances of the particle reach about 1 nm. Interestingly, the asymmetric part is saturated when the time is sufficiently large, which means this asymmetry is negligibly small at the macroscopic-level but comparable with diffusing distance in picoseconds. We find that the orientation-dependent diffusion results from the orientation-dependent damping force of the asymmetric nanoparticle together with a finite time required to regulate the particle orientation from the initial orientation. This finding extends the work of diffusion to the nanoscale beyond random Brownian motion since most of the particles have asymmetric structures. We expect that the orientation-dependent diffusion may have a critical role in the dynamics of bio-molecules in living cells and various nanoscale devices, such as chemical separation, sensing and drug delivery.

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## Discrete time quantum walk is quantum dynamical simulator

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Cellular automata, discrete models that follow a set of rules, have been analyzed in various dynamical systems in physics, computational models, and theoretical biology; well-known examples include crystal growth and the Belousov-Zhabotinsky reaction. To simulate quantum mechanical phenomena, Feynman proposed a quantum cellular automaton (the Feynman checkerboard), and the model, defined generally by Meyer, is known as the discrete-time quantum walk (DTQW). Since the DTQW on the graph is a model of a universal quantum computation, it is of particular use, especially in quantum information. Furthermore, the DTQW has been

demonstrated experimentally in various physical systems.

As the cellular automaton can be mapped to various differential equations by taking the contentious limit, some DTQW models can be mapped to the Dirac equation, the spatially discretized Schrödinger equation, or the Klein-Gordon equation. These equations have ballistic transport properties, which are reflected mathematically in the one-dimensional (1D) DTQW with a time- and spatial-independent coin operator, i.e., a 1D *homogeneous* DTQW. Therefore, these can be taken as the 1D quantum dynamical simulator. We consider here the 1D DTQW model. Physically speaking, the standard deviation of the homogeneous DTQW is  $\sigma(t) \sim t$ , whereas the unbiased classical random walk has a standard deviation of  $\sigma(t) \sim \sqrt{t}$ . In this Letter, we analyze a specific nonlinear DTQW model that incorporates a nearest-neighbor interaction with the coin operator. We show numerically that the probability distributions of the nonlinear DTQW model have anomalous diffusion characterized by  $\sigma_{q=0.5}(t) \sim t^{0.4}$ , where  $\sigma_q(t)$  is the  $q$ -generalized standard deviation defined later. These dynamics are consistent with the time evolution of the self-similar solution of the porous medium equation (PME), which is known to describe well the anomalous diffusion of an isotropic gas through a porous medium. Furthermore, we show analytically that these dynamics do not originate from the interference terms in our model whereas the ballistic transport property of the homogeneous DTQW is derived from its interference terms.

In this talk, we discuss the concepts of the quantum dynamical simulator, the relationship to the discrete time quantum walk, and our recent results.

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## A novel transition in robustness of open systems

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The stability/robustness of complex systems has attracted broad interest, along with essential theoretical findings such as the general instability of large and densely interacting systems [1], the self-organized criticality (SOC) [2], and the relation between the robustness and the network structure of the systems [3]. In this study, we approach to this classical problem with putting emphasis on a key and universal feature of various real complex systems such as ecosystems, reaction networks in living organisms, and social communities: its openness [4].

In open evolving systems, their complexity emerges as a result of (or at least persist) successive introductions of new elements. Therefore it is natural to ask how can a community or system, which consists of lots of elements interacting each other, grow to more complex structure by adding new elements to it. To tackle this general question, let us consider the following Ising model of open evolving systems:

0) Prepare a system that consists of certain number of nodes (elements, species), connected with weighted and directed links (interactions).

1) At each time step  $t$ , a new node is added to the system.

The newly introduced node has  $m$  new links. The interacting nodes are chosen randomly from the resident nodes and the direction of the link is also determined randomly with equal probability. The weights of the links are drawn from the standard normal distribution.

2) Calculate the fitness for each node, which is simply

$$\text{defined as: } f_i = \sum_j^{\text{incoming}} a_{ij}$$

3) If the minimum fitness is non-positive, delete that node and then re-evaluate the stability of the system (go back to the step 2)

4) If the fitness of the node are all positive, proceed the time to  $t + 1$  (go to the step 3).

Because both the introduction process and the survival condition are neutral, it is not easy to predict whether model yields continuous growth or not. Fascinatingly, it turns out that both can happen depending on the unique parameter of the model,  $m$ . The system grows infinitely if the number of the interactions is in a moderate range.

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## Experimental study of fractal pattern observed in gravitational instability

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Gravitational instability, that a higher-density solution (HDS) sinks vertically, occurs, when HDS is placed on the top of a lower-density solution (LDS). In a sinking of HDS, a fractal pattern drawn by HDS is observed on a surface of LDS. It has been discussed that the fractal pattern is formed in accordance with a similar behavior to viscous fingering, because it is observed that LDS with a lower viscosity penetrates into HDS with a higher viscosity. It, however, has not been confirmed quantitatively. Thus, we compare the experimental results with those obtained from miscible viscous fingering, and verify above discussion.

Magnetic fluid with 1.4 g/ml and PEG solution with 1.0 g/ml are used as HDS and LDS, and these solutions are miscible. The viscosities are 25 Pa·s and 1.75 Pa·s, respectively. A volume of HDS is controlled using a micro pipette from 20  $\mu$ l to 80  $\mu$ l.

A dimension of fractal pattern found in our experiments is  $1.88 \pm 0.03$ . The value agrees with 1.88 of ordinary studies, and is almost constant with a time. Before the fractal pattern is formed, a stick-like pattern with a periodic structure appears at a region surrounding a surface pattern due to interfacial instability. The sticks repeat to grow and confluent toward a center of the pattern from a region surrounding a surface pattern, and the fractal pattern is formed. The experimental result shows that the stick is a source to form the fractal pattern. Thus, we focus on an appearance of the sticks. The periodic wavelength  $\lambda$  of the stick-like pattern increases with a volume  $V$  of HDS, and is independent of the viscosities of

the two solutions. In increasing of  $V$ , a radius of the stick-like pattern is almost constant, whose value is  $25.5 \pm 1.2$  mm. A vertical sinking of HDS, derived from gravitational instability, is not observed in a formation of the stick-like pattern. It means that HDS volume is kept at the surface. Thus, a thickness  $b$  of the HDS at the surface is estimated using values of the pattern radius and a HDS volume. The results provide that a relationship between  $b$  and  $\lambda$  obeys  $\lambda = 7.6b$ .

We propose a model of miscible viscous fingering without an upper wall, which is similar to our experimental condition with a free boundary, in order to understand the appearance of the sticks. The model provides  $\lambda' = 8.0b'$ , where  $\lambda'$  and  $b'$  mean a wavelength due to interfacial instability between two solutions and a thickness of a higher-viscosity solution. The coefficient value 8.0 is close to 7.6 obtained from our experiments. The result implies that sticks are formed by a similar behavior to miscible viscous fingering.

We also consider a formation of the fractal pattern after an appearance of the sticks. When sticks grow towards the center from the edge of the pattern, a distance between sticks becomes short. A decreasing of the distance must make sticks difficult to exist independently, and lead a confluent of sticks. A model to contain the growth and the confluent of sticks is simulated. In a comparison between a fractal dimension obtained from the computer simulation and 1.88 of our experiments, the formation of the fractal pattern is discussed.

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### Dimension changing phase transition in a lattice model: a new universality class?

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There are lots of natural processes that can be described as dimension-changing phase transitions, characterized by a change of dimension, with the onset of a low-dimensional ordered phase below a critical point. Most of them are order-disorder transitions, where a disordered system is quenched in an ordered phase that has a lower dimensionality. Examples can be given, ranging from liquid crystals to liquid-solid transitions and even solid-solid transitions.

Notably, some years ago, a simple lattice model has been proposed[1-3], describing a continuous transition that mimics the orientational ordering of a covalent crystal, where a small coordination number gives rise to frustration and eventually to the low-dimensional structures that can be observed in some phases of polymers and molecular crystals like iodine and hydrogen halides.

We point out that the model might describe a general universality class of solid-solid dimension-changing transitions, with peculiar exponents that make the model different from the other known classic lattice models.

While in two dimensions the model is exactly equivalent to the Ising model, in higher dimensions the frustration gives rise to low-dimensional ordering, with correlated chains (polymers) that point toward a common direction inside each two-dimensional plane of the lattice. Such planes are weakly

correlated in the low-temperature phase, and the system acquires a two-dimensional behavior.

In three dimensions The model has been studied by analytical and numerical methods, and a critical exponent  $\nu = 0.44 \pm 0.02$  has been found by finite size scaling. That value seems to be peculiar of the dimension-change transition, and cannot be found in other classic lattice models like two-dimensional two-state (Ising) and three-state Potts models, whose exponents are  $\nu = 1$  and  $\nu = 0.83$ , respectively, or three-dimensional Ising model ( $\nu = 0.64$ ), and the three-state three-dimensional Potts model which is known to undergo a first-order transition.

As it is well known that the critical properties do not depend on the microscopic details of the model, we expect that the same universality class should describe several orientational transitions that occur in complex molecular systems, especially under pressure. New experiments are called for in order to test such ideas and explore the broad universality class of the model.

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### Exact Expression for Number of Energy States in the Two-Dimensional Ising Model

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The number of energy states  $g(E)$  gives the number of microstates of energy  $E$  in the canonical ensemble. In classical canonical systems, one can derive all energy-related thermodynamic properties of the system from the known  $g(E)$ . We present the exact formula for the number of energy of states for the two-dimensional Ising model, as the coefficients in the low-temperature series expansion of the partition function  $Z(x) = \sum_{N=0}^{\infty} g(\varepsilon N)x^N$ , where  $\beta = 1/(k_b T)$  is the temperature factor,  $x = e^{-\beta \varepsilon}$  is the low-temperature expansion's variable, and  $\varepsilon = 4J$  is the energy portion, with  $J$  being the coupling energy in the Ising model. The coefficient was first found by Beale [Beale, 1996], who used Wolfram Mathematica for simplification the famous Kauffman's solution of the two-dimensional Ising model, and obtained the exact formulae for the  $g(\varepsilon N)$ . Beales result had a huge impact on the numerical studies in the statistical physic. It was due to the fact that the number of energy states is strictly related to the probability distribution that the studied system has energy equal to  $E$ :  $P(E) \propto g(E)e^{-\beta E}$ . Therefore, Beale's results was often used as a benchmark for new numerical methods and as a tool for improving the known methods. Despite importance of Beale's result it has one important drawback. It is not general approach: it works well for the two-dimensional Ising model and it is not clear how generalize this approach for other models of statistical physics. Our result has much more general. We have used a new combinatorial technique introduced in Ref. [Fronczak, Fronczak, 2014]. The technique benefits from the dual, combinatorial and analytical, nature of the Bell polynomials. It is not limited for the two-dimensional Ising model and allows to calculate the number of states for a wide class of lattice models which are described with the

canonical ensemble. This approach based on expansion of the Free Energy  $F(x)$  of the system into a power series of the variable  $x$ . Due to Bell Polynomials of derivatives of the Free Energy  $\partial^n F(x)/\partial x^n \Big|_{x=0}$  one can obtain coefficients of the low-temperature expansion of the partition function i.e. number of energy states  $g(E)$ . We present this approach in the case of the two-dimensional Ising model, for which we expand Kauffman's [Kaufman, 1949] Free Energy and use described approach.

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### Transition from the mechanics of material points to the mechanics of structured particle

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The conventional explanation of the irreversibility mechanism is based on the property of exponential instability of Hamiltonian systems and the hypothesis of the fluctuations existence. But this hypothesis is alien to the classical mechanics. Nevertheless, it appears that in the frame of the Newton's laws the explanation of irreversibility exists [1-3]. To come to this explanation, necessary to take into account that all natural bodies are systems, have the internal energy and that the internal energy of systems can be changed by their interaction. To do this, the mechanics should be based on the systems motion equations rather than the motion equation for material points (MP) or any other nonstructural elements. Moreover, in deriving the systems motion equation necessary to exclude the hypothesis of holonomic constraints. This hypothesis is used in the obtaining of the canonical of Lagrange's equation [4]. You can satisfy all of these requirements if the mechanics of systems will be built under the following conditions: 1. All the bodies are consisting from a set of interacting of structured particles (SP) (SP is equilibrium system consisting from a large enough number of potentially interacting MP); 2. The SP energy must be represented as a sum of the motion energy and internal energy with the help of transitions to macro and micro-variables respectively; 3. The motion equation for SP should be obtained from the energy equation provided that the Newton's laws for MP have a place. We explain how the SP mechanics can be built within the laws of classical mechanics based on the expression for the energy. We show how in the mechanics of SP the symmetry breaking in time occurs and how into the frame of classical mechanics can introduce the concept of entropy. We consider how the mechanics of nonequilibrium systems can be constructed in the thermodynamic approach of the local equilibrium. We are also explain why the hypothesis of holonomic constraints used in the derivation of canonical Lagrange equation, eliminates the possibility of describing of irreversible dynamics. Difference of SP mechanics from the mechanics of MP will be discussed. Why the system may arise only from those elements that have a structure, will be shown. How the paradoxes of classical mechanics about irreversibility can be explain based on the equations of motion SP will be discussed. The main advantage of the SP mechanics consists in accounting in the SP motion equation of transformation of the system motion energy into its internal energy. This

transformation is the key reason of symmetry breaking of time.

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### Noise-induced Effects in Nonlinear Relaxation of Condensed Matter Systems

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Nonlinear relaxation phenomena in three different systems of condensed matter are investigated. (i) First, the phase dynamics in ballistic graphene-based Josephson junctions (JJ) is analyzed. The model ruling the JJ phase dynamics is called the Resistively and Capacitively Shunted Junction (RCSJ) model. A superconductor-graphene-superconductor (SGS) system exhibits quantum metastable states analogous to those observed in normal current biased JJs. The average escape time from these metastable states in the presence of Gaussian and correlated fluctuations is calculated, varying the noise source intensity and the bias frequency. Noise induced phenomena are observed, such as the noise enhanced stability (NES) and the resonant activation (RA). (ii) Second, the transient dynamics of a long-overlap Josephson junction (JJ) subject to thermal fluctuations and non-Gaussian noise sources, Lvy type, is investigated. The focus is on the switching events from the superconducting metastable state, and in particular on the mean switching time (MST). The dynamics of this system is ruled by a nonlinear partial differential equation, the perturbed stochastic sine-Gordon (SG) equation. Nonmonotonic behaviors of the MST versus noise intensity, frequency of the external driving, and length of the junction are obtained. A characteristic length, after which the MST reaches asymptotically a constant value, exists. Exceeding this characteristic value, the formation of SG travelling wave solutions, that is solitons, is allowed. The analysis of the time evolution of the order parameter highlights the influence of the noise induced solitons on the MST behavior. Moreover, in the presence of Lvy flights, another localized SG solutions, the breathers, can be observed. (iii) Third, the electron spin relaxation process in n-type GaAs crystals driven by a fluctuating electric field is investigated. Two different sources of fluctuations are considered: (a) a symmetric dichotomous noise and (b) a Gaussian correlated noise. Monte Carlo numerical simulations show, in both cases, an enhancement of the spin relaxation time by increasing the amplitude of the external noise. Moreover, we find that the electron spin lifetime vs. the noise correlation time increases up to a plateau in the case of dichotomous random fluctuations, and shows a nonmonotonic behaviour with a maximum in the case of bulks subjected to a Gaussian correlated noise.

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### Fast dynamics of mesoscopic systems and fluctuation dissipation theorem

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This contribution will deal with perspectives of description of full non-equilibrium quantum many body system dynamics within the Non-equilibrium Greens Functions (NGF) [1, 2]. The basic aim of this approach is to describe time development of the many-body system out of equilibrium from its initial state over its transient dynamics to its very long time (if e.g. steady state exists), dynamics. A transient response of electrons in a mesoscopic system depends on the joint effect of the initial state with correlations and of the driving external disturbances abruptly setting on at the initial time. The early period of the transient requires the full NGF for its description. After the disturbances cease acting the system may enter, under favorable conditions, the non-equilibrium quasi-particle mode. The loss of initial correlations due to interactions and related renormalization processes permits reduced description by a quantum transport equation.

The fluctuation-dissipation (FD) theorems are known to be a constitutive aspect of thermal equilibrium. Here, we address the FD theorem, which permits to write all components of the equilibrium one particle equilibrium Green's function in terms of the spectral density and a thermal factor. The fluctuation-dissipation relation between the correlation and the retarded components of the NGF depends on the existence of temperature and is lost out of equilibrium. Still, the components of the NGF retain some inner interconnection which may be termed the fluctuation-dissipation structure out of equilibrium. Historically, this was grasped in an approximate manner by the Kadanoff-Baym Ansatz, but there is an exact formulation in terms of reconstruction equations generating the correlation components in terms of the propagators and the non-equilibrium distribution function either of particles, or, under restrictive conditions, of the non-equilibrium quasi-particles. These equations can lead to a Generalized Master equation (GME) or to a Markovian master equation (ME) [1, 2, 3].

This approach will be related to the dynamics of open quantum systems, when finite time initial conditions, quantum interferences and their attenuation play important roles. To study all these phenomena in conditions of general non-equilibrium, open systems are represented by a simple structure of a molecular bridge. Three stages of non-equilibrium evolution of the molecular bridge model, the first described by the full NGF description, the second, ruled by the asymptotically exact GME, and the third, governed by a ME, will be related to each other [3].

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### Comparative studies of the statistical and field theoretical descriptions of vorticity self-organization in two-dimensional fluids

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The asymptotic states of two-dimensional fluids, magnetized plasmas and of the planetary atmosphere consist, in certain conditions, of highly ordered flows with the shape of coherent vortices. For low dissipation these vortical structures are stationary and the motion takes place on only the streamlines. The process that underlies this relaxation to highly ordered flows is separation of opposite-sign small-scale vortices together with concentration of like-sign vortices. It is the *self-organization of the vorticity*, a process specific to the 2D fluids, which in the turbulent state is manifested as inverse cascade. The evolution is spontaneous and does not involve heat exchange or other thermodynamic processes.

The usual description is based on the statistical analysis of a discrete set of point-like vortices interacting in plane by a potential generated by themselves. When the interaction is long-range, for any positive energy the statistical temperature of the system is *negative*. The asymptotic states are identified as maximum entropy states and the equation verified by the streamfunction of the flow is *sinh*-Poisson. There is no clear success of the statistical approach for plasma/atmosphere (where the interaction is short range).

A different approach can be developed consistently, in terms of a classical field theory (FT). The fields are: *matter* (connected with the density of point-like vortices), *gauge field* (connected with the potential of interaction in plane). In the Lagrangian the Lorentz-type motion of elementary vortices is represented by the Chern-Simons term. The model must be non-Abelian ( $sl(2, \mathbf{C})$ ) since the objects are vortices.

We examine comparatively the two approaches: statistics of point-like vortices and respectively the field theoretical model of the continuum limit. We show that both identify the asymptotic states (deriving the *sinh*-Poisson eq.). The FT reveals the property of self-duality for the Euler fluid and its absence for plasma and atmosphere. The solutions however (for the latter case) compares favorably with experiments and observations. The Abelian version reinstates the self-duality and derives the equation  $\Delta\psi = \exp(\psi)[\exp(\psi) - 1]$  with solutions ring-type vortices. These are not accessible to the statistical analysis. It results that the statistics is equally efficient as the FT approach only for absolute minimum, but cannot cover all situations. We discuss the possible inclusion of the dissipation in these models.

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## Early-warning signals of topological collapse in interbank networks

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The financial crisis clearly illustrated the importance of characterizing the level of systemic risk associated with an entire credit network, rather than with single institutions. However, the interplay between financial distress and topological changes is still poorly understood. Here we analyze the quarterly interbank exposures among Dutch banks over the period 1998-2008, ending with the global crisis. We find that, after controlling for the link density, many dyadic and triadic structural properties display an abrupt change in 2008, providing a clear (but unpredictable) signature of the crisis. By contrast, if the heterogeneity of banks connectivity is controlled for, the same properties show a gradual transition to the crisis, highlighting a slow build-up phase starting three years in advance and representing an early-warning signal of the upcoming collapse. These results show a dramatic consequence of the heterogeneity of the interbank system. Moreover, we find that the pre-crisis phase is preceded by an earlier period characterized by anomalous unreciprocated debt loops among triples of banks. Given the Over-The-Counter (OTC) nature of interbank transactions, debt loops could have led to the systematic underestimation of counter-party risk. Our results also show that, during the build-up of crises, interbank networks can keep moving away from the expectations derived only from the knowledge of bank-specific properties. In this out-of-equilibrium regime, the local connectivities of banks become less and less informative about the network as a whole. This loss of topological predictability speaks against the use of maximum-entropy techniques aimed at reconstructing the most likely configuration of an (unobserved) interbank network when only local information about the total assets and liabilities of each bank is available. Our results suggest that this technique might yield a realistic guess of the real network only in tranquil times. When the network is under stress, maximum-entropy techniques would instead provide a greatly distorted picture of it. So, by construction, the early-warning signals identified by our approach are completely undetectable if the network is reconstructed from partial bank-specific data, as routinely done. Supervision based only on bank-specific information, and not on the knowledge of the entire network, is thus likely to remain oblivious to warning signals of structural changes in the run-up to the crisis. These considerations show that OTC transactions have the potential to create unintentional but emergent and destabilizing patterns, and feed into the debate on how OTC markets can be monitored and regulated.

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## Scaling analysis of time series of stock market indices of transitional economies in the Western Balkans

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In this paper we have analyzed scaling properties of time series of stock market indices (SMIs) of developing economies of Western Balkans, and have compared the results we have obtained with the results from more developed economies. We have used three different techniques of data analysis to obtain and verify our findings: Detrended Fluctuation Analysis (DFA) method, Detrended Moving Average (DMA) method, and Wavelet Transformation (WT) analysis.

Following extensive research in the area of econophysics of national and international stock markets, we were interested to contribute to this body of knowledge by analyzing the dynamics of market behavior of transitional economies in the Western Balkans, and to compare data from these emerging economies with data from more economically developed countries. Analyses of stock market behavior of the emerging economies of South America, or the developing Asian or African markets have shown that the values of scaling exponents, calculated from the time series of stock market indices, could be used to estimate the efficiency of markets in question. With that in mind, by applying the theoretical approach of statistical physics, we aim to offer a new perspective on stock market dynamics in the Western Balkans and contribute to better understanding of the development process in the region's economies.

We have found scaling behavior in all SMI data sets that we have analyzed. Scaling of SMI series changes from long-range correlated to slightly anti-correlated behavior, i.e. the appropriate scaling exponents decrease in value with the increase in growth and/or maturity of the economy the stock market is embedded in. Scaling exponents  $\alpha$ ,  $H$ , and  $\beta$ , corresponding to the DFA, DMA, and WT technique, all cross the 0.5 (and zero) line, marking this alteration.

We also report the presence of effects of potential periodic-like influences on the SMI data that we have analyzed. One such influence is visible in all our SMI series, and appears at a period  $T_p \approx 90$  days. We propose that the existence of various periodic-like influences on SMI data may partially explain the observed difference in types of correlated behavior of corresponding scaling functions. The application of time-dependent scaling analysis (tdDMA) proved that these influences are of a complex type, that is, they can not be easily distinguished from a local correlations profile.

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## Combined generation and quantum walks of biphotons for reconfigurable entanglement on a nonlinear chip

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Quantum walks of photon pairs have been realized in various optical structures, including integrated arrays of evanescently coupled waveguides [1]. Such correlated walks involving quantum interference of several walkers can provide a speed-up of quantum algorithms delivering an exponential acceleration with the number of correlated walkers [2]. However, in all schemes to date, the correlated photon pairs were generated externally to the array by using bulk photonic elements. Such bulk elements may introduce quantum decoherence and impose stringent requirements on the losses associated with the connection of the array to the photon sources.

We propose and develop a novel scheme of quantum walks based on nonlinear coupled waveguides, involving simultaneous generation of correlated photon pairs and their quantum walks inside a single photonic element [3]. Specifically, we find that spontaneous parametric down-conversion (SPDC) can provide an on-chip source of entangled photons in structures with quadratic nonlinearity, so that there is no need to generate entanglement externally to the quantum circuit. Moreover, we show that biphotons with any desired complex spatial entanglement can be generated in nonlinear waveguide arrays (WGA) with specially optimized characteristics. This photon source can be integrated on a chip for processing of the generated path-encoded qudits.

The propagation of biphotons in a waveguide array is governed by a linear Schrödinger-type equation representing a continuous-time quantum walk with an additional source term accounting for the pump. Then, the driving of all three waveguides simultaneously will produce a linear superposition of the output states that would be produced if the waveguides were driven individually. Hence altering the amplitude and phase of the driving lasers allows switching between the states spanned by the set of single waveguide outputs.

In order to manipulate the biphoton generation within the WGA, we engineer the sign of the effective quadratic susceptibility along the length of each waveguide. This can be achieved in practice by altering the domain structure through electric poling, representing a feasible route towards an on-chip reconfigurable source of spatially entangled photons.

We also show that by employing adiabatic waveguide coupling, we can generate spatially entangled photon pairs in a required Bell state while simultaneously providing spatial pump filtering and keeping photon-pair states pure [4]. We estimate the performance of the pump filtering to be of the order of 72 dB. Moreover the proposed method is highly tolerant to fabrication inaccuracies.

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## Multiscale analysis of many-body transport

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The discrete, generalized Frenkel-Kontorova (FK) model consists of a chain of nodes sitting in a periodic lattice potential, interacting through an arbitrarily non-local harmonic force, typically thermalized through a Langevin equation of motion. As the model supports a kink mechanism and has analytical limits it has become a prototypical many-body system, finding wide application in condensed matter physics and elsewhere since Prandtl in the late 1920s (O Braun and Y Kivshar, Springer, Berlin, 2004).

Despite the model's popularity, there are very few rigorous results on the transport properties of the FK chain, often the main quantities of interest for the condensed matter community. In my talk, I will outline a way to derive exact bounds on the transport properties of the FK chain through the novel application of rigorous multiscale analysis (T D Swinburne, PRE July 2013).

Multiscale analysis has been used since Hilbert to isolate coarse grained quantities of interest, rescaling space and time to generate an asymptotically independent hierarchy of evolution equations which may be systematically solved to investigate thermodynamic limits (G Pavliotis and A Stuart, Springer, Berlin, 2008). Whilst previous results have been concerned with the transport properties of point particles, by transforming to a coordinate system which clearly distinguishes bound and unbound variables, I show how multiscale analysis can be used to rigorously investigate many-body Fokker-Planck equations, with specific application to the FK chain.

Significantly, it is shown that the free energy barrier is a lower bound to the true finite temperature migration barrier for this general and popular system, in both equilibrium and driven environments. Numerical simulation confirms these conclusions, whilst limiting cases provide a connection the results of transition state theory and other non-equilibrium approaches. We find that the low-temperature upper bound exactly reproduces the results of Kramer's transition state theory, whilst at high temperature thermal fluctuations exponentially suppress any migration barriers in close analogy to Debye-Waller broadening.

The generality of these techniques and their application to problems in crystal plasticity will be discussed (T D Swinburne et al., PRB February 2013).

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## Dynamics of Opinion Spread in Social Networks

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Human behavior is profoundly affected by the influenceability of individuals and their social networks. We discuss the dynamics of spread of opinions using two fundamental models for Social Contagion: the binary agreement model (BAM) to study the role of committed minorities, and threshold model, to investigate cascades of opinions. Both models, for certain setting of their parameters, undergo significant change in their dynamics.

In BAM model, all individuals initially adopt either opinion A or B. Then, repeatedly, randomly selected node acts as a speaker and chooses randomly one of his acquaintances as a listener. Speaker selects one of its opinions and sends it to listener who either adds this opinion as its own, if they are different, or both speaker and listener keep only opinion sent by the speaker. When a small fraction of all individuals commits to their opinion, those individuals are immune to influence from speakers, but otherwise follow the prescribed rules for opinion change. We will show that the prevailing majority opinion in a population can be rapidly reversed by a small fraction of randomly distributed committed individuals. When committed individuals exist for both opinions, the difference between larger and smaller fractions of them needed for rapid majority conversion decreases as the smaller minority increases. We will discuss how these results can be used to understand and to influence the social perceptions of ideas and policies as well as applied to reputation protection. In threshold model, all nodes start in the initial state-zero, except for initial spreaders that are in state one. The node whose fraction of neighbors in state one is higher than the threshold, changes its state to one. We will discuss how the threshold model can be used to find efficient spreaders, to develop fast heuristic for spreader selection strategies, and to measure the impact of clustering on system dynamics. We will show that even for arbitrarily high value of threshold, a critical initiator fraction of spreaders exists beyond which the cascade caused by them becomes global. Network structure, in particular clustering, plays a significant role in this scenario. Similarly to the case of single-node or single-clique initiators studied previously, we will demonstrate that community structure within the network facilitates opinion spread to a larger extent than a homogeneous random network. Finally, we will discuss the efficacy of different initiator selection strategies on the size of the cascade and the cascade window.

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## The complex nested structure of the Countries-Products matrix

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In contrast with the Ricardian paradigm of specialized industrial production at a national level, data about actual production shows that the most successful countries are those who diversified their industries, while the poorest are those with the lower diversification [1]. Interestingly the products that are produced by poorly diversified countries are a small set, which almost any country is able to produce, whereas the more complex products are produced only by a very limited number of highly diversified countries. In other words the adjacency matrix of the bipartite network connecting countries with the products they produce can be arranged to have a triangular-like shape. This implies that there is a certain amount of nestedness in countries production, i.e. products of poorly diversified countries tend to be a subset of those of the more diversified ones. Here we focus on possible micro-founded explanations of how this nested structure built up. A very useful tool to discriminate the outputs of statistical models that produce such countries-products matrices are the metrics introduced in [1]. In particular, by using these metrics, we show that binomial approaches such as the one introduced in [2] as well as more sophisticated ones are not able to generate matrices with statistical features even close to those of the real one. We identify the key aspect in a zipf-like distribution of the capabilities needed to produce a given product: in other words there exist a small set of capabilities that are needed to produce almost any product. This framework shows very interesting results that span from the nestedness in bipartite networks to concepts like "poverty traps" and "malthusian traps", in surprising agreement with the data. While this assumption might seem "ad-hoc", we show [3] that by introducing a tree-like hierarchical relation among products (i.e. being able to produce a leaf product means being able to produce all the simpler ones until the root) the zipf-law comes out as a consequence. This leads us to conclude that such hierarchical relation between products is likely to exist and sets the basis to explore methodologies to extract such information from the data, supported by benchmarks of reverse engineering of our model. Moreover this framework can be used to build more realistic dynamical models of development.

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## Understanding Dynamics of Emotions and Emergent Collective Phenomena in Online Social Networks

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Social interactions rely on information and emotion exchange. These processes integrate in the brain function of each interacting person, i.e. by involving the cortical circuitry that are associated with empathy, mirror-neuron system, or other brain areas; these brain processes are currently investigated in social neurology [1]. On the other hand, a pattern of such pairwise interactions often drives to aggregated group behavior; based on the empirical data from various online social networks, the aggregated behavior of users can be studied as a complex system in the physics laboratory. To investigate these empirical data, methods of statistical physics are joined with machine-learning techniques by which facts can be deduced from the text of every communicated message. Among others, our recent study of the data from Blogs, Chats and MySpace network [2,3] provided strong evidences that, in these collective phenomena, emotions expressed via written messages play a significant role. Here, we use agent-based modeling strategy firmly related with these empirical systems [3,4] to investigate the underlying stochastic processes of emotional interactions. We present in detail the model where the structure of social links, rules and parameters are matching the ones in MySpace social network. In Blogs and Chats, the network co-evolves with user interactions, thus enhancing the collective dynamics. On the other hand, the mostly fixed structure of links in the social networks like MySpace and Facebook tends to constrain the flow of emotions. Our aim is to describe the collective dynamics arising on fixed social networks and to compare its key features with the ones which characterize the co-evolving social structures. Explicitly, by simulation of emotional communications between agents in a large empirical network, we show that the external inputs—pace of agent's stepping into the virtual world and emotions that they receive from the outside world—are critical for developing the observed collective behavior. By varying the level of extrinsic noise, we analyze the simulated message streams to determine temporal correlations, returns and clustering of events which carry a specified emotional substance. The concerned quantitative measures are compatible with non-extensive statistics with Tsallis  $q$ -entropy. We discuss the likely cause of non-extensivity in the emotion dynamics at a large scale and examine how the parameter  $q$  varies with the noise level and the emotion contents.

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## Proposing a project of Financial Observatory of Real-time Market fluctuations

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Due to development of information technology and game strategies the present financial markets are highly complicated computer systems. They are new targets of scientific study which cannot be analyzed without collaboration of researchers from wide variety of fields, such as econophysics, mathematical statistics, information sciences, and economics. We plan to build the world's first Financial Observatory of Real-time Market fluctuations (FORM). Thousands of real time market data will be analyzed in parallel and check the stability of the markets scientifically. The system will give alarms for abnormal signals and malfunction of markets. The FORM will provide scientifically reliable information about stability and malfunction of markets, it is not aiming for profit of investors. The system will become the base for the nation to make a calm decision about economy, contributing to avoid financial turmoil.

The observatory is planned to be consisted of the following 5 divisions. The main observatory: Real-time market data are analyzed immediately in parallel and the results are announced by the homepage of the observatory. In order to show the whole world-wide market conditions, a new way of display will be introduced. In the case of detecting a dangerous symptom, some possible measures to avoid the risk will be simulated and presented. The time-series analysis group: The methods of change-point detection and characterization of abnormal fluctuations will be developed. The cause-result analysis group: The methods of causality detection from thousands of variables by eliminating spurious correlations will be developed. The simulation group: The computational simulation methods of markets will be developed especially to reproduce bubbles and crashes. Also measures for avoiding catastrophic situations will be pursued by repeating numerical simulations with different conditions. The data archive center: All the data will be conserved in this center for the purpose of future scientific study for general scientists. Some data will be provided to researchers for academic purposes.

The methods of detecting change points and causality will be invite contributions from the general researchers. The key technology at present includes the particle filter PUCK analysis [1] and the change-point detection using Fisher-exact [2]. Causality detection methods include the big-data analysis method that has been developed in semiconductor factories to estimate the cause of troubles.

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## Similarity between colloidal random walks and financial random walks

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By analyzing comprehensive market data of order book of foreign exchange markets, we clarify the correlations between motion of the market price and the changes of order book [1]. The order book is separated into the inner layer with short memory and the outer layer with long memory. The Langevin equation is naturally derived from the dynamics of order book, and the resistance coefficient is estimated from the data. It is demonstrated that the motion of market price can be directly compared with the colloidal Brownian particle in water molecules.

By observing the mean free path of this financial Brownian particle, we can define the Knudsen number that characterizes the validity of continuum description of this motion. It is found that the continuum description is not automatically fulfilled in the real markets.

The background: Financial market price motions have been often described by continuum stochastic processes including a single derivative term of time a priori, however, the real markets sometimes show dynamical behaviors that cannot be described in such formulation. In order to describe the dynamical motions we need to introduce a kind of market potential force [2]. By the effect of such potential force the equation of motion of the market price is known to follow a Langevin type equation, which includes the acceleration term with second derivative of time and a resistivity term with a single derivative of time [3]. Like the colloidal particles' motions we show that the market price motion is also described by a Langevin equation, not by a simple integration of white noise.

The merit of order book analysis is that we can observe the detail motion of surrounding molecules, which are generally non-observable for materials. By carefully analyzing the motion of financial Brownian particle and the surrounding molecules, we can directly check the validity of fluctuation-dissipation relation. It is shown that in the case of markets the random force term in the Langevin equation is not white, but we can observe that the response function is given by an exponential function that is consistently derived from the data analysis. It is very interesting that the markets are generally not in equilibrium, but the F-D relation and Langevin equation generally holds approximately.

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## Stochastic models of blocking in particulate flows

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If the carrying capacity of a channel conveying a particulate

stream is limited, blockage or clogging may result. This phenomenon occurs in a wide variety of applications including particle filtration, vehicular and pedestrian traffic flow, and the transit of macromolecules through artificial or biological channels. Depending on the application, there may be a single stream of particles moving in more or less the same direction, or two opposing streams. An example of the former is provided by an experimental study of the filtration of glass beads through a wire mesh [1]. A significant fraction of the beads is retained even when the individual beads are small enough to pass through a mesh hole. Opposing vehicular traffic flow on single track roads in remote areas is an example of a process involving counterflowing streams. If two opposing cars enter the passage with an insufficient time delay they block the passage and prevent each other, and following vehicles, from passing. Similar situations may occur in urban areas where the capacity of the road is reduced by parked vehicles and also in dense pedestrian flows in narrow passages. The Asymmetric Simple Exclusion Process (ASEP) is a well-known lattice model for stochastic transport phenomena [2]. It has been applied to model traffic flow and transmembrane macromolecular transport. Particles can hop to vacant adjacent sites, but blockage is explicitly avoided. We have introduced, and obtained exact solutions for, stochastic models in which particle flow is instantaneously interrupted as soon as the carrying capacity of the channel is exceeded for both concurrent and countercurrent flow situations [3]. These models are complementary to the lattice gas models in that they are continuous in both space and time and are most appropriate for low density flows. In the concurrent flow model particles arrive at the entrance of a channel according to a Poisson process of constant intensity. A single particle exits the channel after a transit time  $\tau$ . If, however, two particles are simultaneously present in the channel, blockage occurs instantaneously and the flow ceases. More abstractly, the channel acts as a filter for the Poisson point process. The quantities of interest are the probability of blockage (failure) as a function of time and the final outcome, i.e. the number and type of particles that exit the channel before blockage occurs. We also consider generalizations of the model where more than two particles must be present in the channel to cause blocking. A reversible version of the concurrent flow model can also be solved analytically [4].

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## Is percolation of long rods possible?

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Percolation [1] and jamming [2] phenomena are investigated for relaxation random sequential adsorption (RRSA) of linear  $k$ -mers (particles occupying  $k$  adjacent adsorption sites along a line; also denoted in publications as rigid rods, needles, sticks)

on two-dimensional square lattices  $L \times L$  with periodic boundary conditions. Numerical simulations by means of Monte Carlo method and finite-size scaling analysis have been performed.

Effect of anisotropic placement of the objects on the jamming concentration,  $p_j$ , and percolation threshold,  $p_c$ , has been investigated. In anisotropic problem, the vertical and horizontal orientations of  $k$ -mers occur with different probabilities and degree of anisotropy can be characterized by the order parameter  $s$  defined as

$$s = \left| \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}} \right|,$$

where  $N_{\uparrow}$  and  $N_{\downarrow}$  are the numbers of  $k$ -mers oriented in the vertical and horizontal directions, respectively.

To place an object, the lattice site is randomly selected and an attempt of deposition of a  $k$ -mer with orientation defined by order parameter  $s$  is done. The unsuccessful attempt is not rejected and a new lattice site is randomly selected until the object will be deposited. In contrast to the known random sequential adsorption model with diffusion, the RRSA model does not restrict the movement of species by the nearest sites only. The species may move all over the substrate searching for a sufficiently large empty space. The deposition terminates when a jamming state is reached along one of direction.

It was observed that partially oriented and disordered systems consist of the blocks of vertically and horizontally oriented  $k$ -mers and voids between them.

A nonmonotonic size dependence for the percolation threshold has been confirmed in isotropic case. We propose a fitting formula for percolation threshold

$$p_c = a/k^{\alpha} + b \log_{10} k + c,$$

where  $a$ ,  $b$ ,  $c$ ,  $\alpha$  are the fitting parameters depending on anisotropy. For isotropic system, parameters are  $a \approx 0.36$ ,  $\alpha \approx 0.81$ ,  $b \approx 0.08$ ,  $c \approx 0.33$ .

The jamming concentration follows the power law

$$p_j(k) = p_j(\infty) + d/k^{\beta}.$$

For isotropic system,  $p_j(\infty) \approx 0.65$ ,  $d \approx 0.42$  and  $\beta \approx 0.72$ . For isotropic systems, the ratio  $p_c/p_j$  increases proportionally to  $\log_{10} k$  for large  $k$ :

$$p_c/p_j = f \log_{10} k + g,$$

where  $f \approx 0.12$  and  $g \approx 0.51$  are the constants.

We predict that for large  $k$ -mers ( $k \approx 10^4$ ) isotropically placed at the lattice, percolation cannot occur even at jamming concentration.

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### Universal scaling in the statistics and thermodynamics of the Bose-Einstein condensation of an ideal gas in different mesoscopic traps

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We present a complete analytical theory of the critical phenomena in the Bose-Einstein condensation (BEC) of an ideal gas in the mesoscopic traps. We find that in the whole critical region the BEC statistics and thermodynamics have a universal self-similar behavior for arbitrary parameters of the system – number of atoms, volume, temperature, etc. We prove that there are only two universality classes, Gaussian and anomalous, which have essentially different scaling, and analytically describe their properties.

That constitutes a full analytical solution to the problem on how BEC critical fluctuations and thermodynamic quantities (such as the Gibbs free energy, the average energy, and the specific heat) depend on the confining potential, shape, and boundary conditions of a trap. We give transparent illustrations of this solution by considering boxes with periodic and zero Dirichlet boundary conditions as well as various power-law and harmonic traps. In particular, we find analytically, for the first time, the universal fine structure of the famous discontinuity in the specific heat at the  $\lambda$ -point.

The theory of the BEC critical phenomena cannot be done within the standard grand-canonical-ensemble approach which employs only an approximate condition of a fixed expectation value of number of atoms and leads to the incorrect results. Our study of the mesoscopic Bose-systems is based on the canonical ensemble description that takes care on the exact conservation of the number of atoms in the trap.

We calculate the probability distribution of the number of noncondensed atoms in the trap and the partition function by linking the system with exactly  $N$  atoms with an auxiliary ‘unconstrained’ system with unlimited number of independent excitations and implementing the appropriate reduction of the many-body Hilbert space. Statistics derived in that way is not an approximation or a kind of a model, but an exact and rigorous solution which obeys the fundamental Gibbs distribution.

We show that the statistics is determined by the one-particle energy spectrum of the trap and especially by its lowest-energy modes, which are sensitive to the confining potential, and, hence, the universal self-similar structure of the BEC thermodynamics pronouncedly depends on the shape of the trap (in particular, on the trap’s boundary conditions). This dependence manifests itself even in the bulk limit of a macroscopically large system, and we propose experiments for its direct measurement.

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### Virtual Currencies. A Risk Analysis

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Information and communication technologies are rapidly transforming our financial system, and our society in general, to something very different from what we were used to think

about, during the last decades. The financial industry is becoming more heterogeneous in its business proposition because technology standards open to new products, new services and new business models. Those familiar with technology often refer to the exponential growth of the Moores Law so named for the Intel co-founder Gordon Moore who postulated it in 1965. He predicted that transistors on processors (used for processing power) would double every two years. Many predicted this would be a short-term law and could not continue for long. Yet nearly 50 years later, it is continuing. Every few years the world of technology witnesses a revolution, the next big development with the potential to change how we function and to spawn a highly profitable new industry. There has been the explosion of personal computers, followed by the Internet, smartphones, tablets, cloud computing, and other large and small disruptive technologies. The latest such game-changer seems to be the arrival of virtual currencies. A virtual currency, among its various names (electronic currency, crypto currency, digital currency) is a digital medium of exchange that acts as an alternative unit of accounting, i.e., money. Technically, a virtual currency relies on cryptography and it is created on the proof-of-concept principle that transactions can be securely processed on a decentralized P2P network without the need for a central clearinghouse. Currently, virtual currencies are not produced by government-endorsed central banks nor necessarily backed by national currencies. The ECB (2012) report issued in October 2012 on virtual currency schemes concluded saying: Given that the current assessment of risks is highly dependent on relatively small-sized virtual currency schemes, the assumption that virtual currency schemes will continue to grow means that a periodical examination of the developments is needed in order to reassess the risks. From that time on, the virtual currencies market capitalization has rapidly increased from a value 130 Millions USD to 15 Bilions USD. Equivalent to an expansion larger than 11.500% in only 15 months. Therefore, this paper attempts to picture the ongoing innovation of virtual currencies from a market risk perspective. We will go into a comprehensive level of details about the impact of virtual currency scheme on the following central banks areas: price stability, financial stability and payment system stability.

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### Reduced dynamics of dimer subsystem in a molecular chain

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We study a group of  $N$  oscillators each located at a fixed site. These oscillators are coupled to their nearest neighbor, and at the same time each of them is coupled linearly to a phonon field common to all of them. The coupling is modulated by the population quantum number or site energy of the corresponding site. This model was used to describe the transfer of excitation energy, or exciton, in photosynthetic light-harvesting complex through dipole-dipole interaction between chlorophyll molecules (May). It was also used to study the transfer of vibrational energy, or vibron, along molecular chain, for example, between the peptide groups in

alpha-helix protein chain (Christiansen).

In this model the transfer of energy or vibrational mode is affected by the molecular displacement of the underlying protein matrix or molecular chain. We will apply a few changes of basis, through a series of unitary transformations on the phonon basis and then on the dimer subsystem basis, to bring the Hamiltonian of the system, and in particular a dimer subsystem among the oscillators, to a familiar form.

Under a weak coupling assumption in the interaction between the sites and the phonon, the original interaction term can be eliminated in the transformed basis. A new interaction term involving the dimer and the phonon then emerges. Under a further rotating-wave approximation, the Hamiltonian of the dimer subsystem acquires a form similar to the trilinear boson model, in which the dimer interacts as a collective unit with the phonon. The trilinear boson model was used to describe the processes of parametric amplification and frequency conversion in quantum optical systems (Louisell). The transformed Hamiltonian also has the same formal structure as the Lee model for bosonic system (Lee), and the one-particle sector of its oscillator subsystem is equivalent to the Friedrichs model (Friedrichs).

In the Markovian approximation, the reduced dynamics of the dimer subsystem in the trilinear boson model, where the dimer is regarded as a collective system, has the form of Kossakowski-Lindblad equation. This equation can be solved analytically in terms of the bosonic representation of the  $SU(2)$ . The total occupation quantum number of the individual oscillators is a constant of motion under the reduced dynamics. Consequently, the underlying space of the dimer separates into an infinite number of subspaces disconnected from one another. Within each subspace, the reduced dynamics behaves like a finite-level system under a cascade process. As a result, there is a set of non-degenerate metastable states in the reduced dynamics.

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### Hierarchies in tagged data

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Folksonomies - classifications based on tags applied by users - are becoming an increasingly important way of categorizing information, contrasting traditional hierarchies. It can be found in blog posts, online stores, journals, biological datasets, movie databases, photo sharing sites, scientific publications, social bookmarking sites, etc. giving short "description" of the items. In traditional hierarchies, categorization is done by a few experts in a top-down approach, and the items typically belong to a single category. In folksonomies, users can attach tags to items, speeding up the process significantly,

thus allowing the categorization of very large datasets, possibly containing tens of thousands of items or more, in reasonable time. Tags can be arbitrary words or expressions deemed applicable by the users, and an item can have several tags, resulting in a bottom-up, self-organized categorization. The arising dataset is a tri-partite item-tag-user network, however, it has no evident organization in general. Although there are no prescribed relations between tags, hierarchical relationships are expected to appear between them, reflecting the hierarchical structures like special-general relationships, e.g. "orange" is a special case of "fruit". Therefore, a directed acyclic graph (DAG) of tags is expected to exist. Reconstructing such directed acyclic graphs solely from the co-appearance of the tags is an interesting challenge with a potential for practical applications, like more fluid navigation in the original dataset, providing recommendations, or aiding the categorization of newly arriving items. Here we present a new reconstruction method, along with a benchmark system, including similarity measures for directed acyclic graphs. Tuning the parameters of the benchmark system allows investigating the behavior of the reconstruction methods in a controlled environment, while similarity measures can be exploited to compare the reconstructed directed acyclic graph to the original one. We compare our method to existing ones (the ones of P. Heymann and H. Garcia-Molina [1], and of P. Schmitz [2]) on our benchmark, resulting in very promising scores, significantly better compared to the existing methods. We also apply it to various real-world examples, like the Flickr photo sharing website and the Internet Movie Database (IMDb).

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## A Reciprocity Based Model of Social Networks

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Modeling and understanding social networks has been the subject of intense research as social networks enable and drive many technological advances and fundamentally shape the economic and political landscape. A set of questions emerging from these studies refer to the inherent distinguishing features of social networks: What makes social networks "social"? How the distinguishing features of social networks emerge from human behavioral mechanisms? A fundamental empirical observation is that structurally, social networks are characterized by assortative mixing by degree (simply referred to as "assortativity") and transitivity. While the transitivity property, expressing the higher-than-random prevalence of closed triples has extensively been studied; there is less understanding on the behavioral mechanisms responsible for the observed assortativity of social networks. Assortativity describes the preponderance of connections between nodes of similar degrees, indicated by a positive Pearson correlation coefficient  $r$  between the degrees of connected nodes. It is a signature that appears to consistently differentiate social networks from other natural and technological complex systems, in the latter of which low-connectivity nodes tend to attach preferentially to high connectivity nodes, resulting in dis-assortatively mixed structures ( $r \leq 0$ ).

The fact that social networks are consistently assortative, suggests that this property is generated by universal behavioral features that are independent of the particular context associated with the actor and its social relationships. Here we propose that the observed assortativity has its roots in fundamental physical and cognitive constraints accompanying human communications both at the agent and dyad (agent-interaction) level. Incorporating these constraints, we present a parsimonious agent-based model of the emergence of assortativity in social networks, based on Jaynes' Maximum Entropy Principle. The presented model offers an explanation of positive degree assortativity in social networks in terms of the simple mechanism of preference for reciprocity in communication flows: actors tend to drop relationships in which they invest more than their partners and attempt to search for relationships in which they invest as much or less. Our model explains how degree-assortative social networks characterized by system-wide connectivity can emerge from random initial conditions through a process of decentralized decision-making constrained only by the local neighborhood topology.

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## Evaluating Advertisement On Social Network By Extended Sir Model

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Many companies are starting to rely on marketing activities over social networks. There are two main categories of marketing on social network: (1) advertisement of word of mouth (WOM) by customer behaviour such as like and share, and (2) direct access (DA) marketing such as banners displayed on the window of customers.

It is known that large-degree nodes can become the bottleneck of spreading information on complex networks. In this research we will analyse the spread process of WOM advertisement on social network by using extended heterogeneous SIR (Susceptible-Infectious-Recovered) model. We compare the effectiveness of WOM with direct marketing. Further, we will discuss the possible improvement of WOM advertisement.

Our SIR model on the network of size  $n$  can have different heterogeneity of emitting and receiving of WOM. Let  $k_i$  and  $w_i$  be the emission level and receptivity of individual  $i$ . The emission level  $k_i$  corresponds to the number of WOM emitted to the network (typically, the number of  $i$ 's friends or the degree of  $i$ ). Also  $w_i$  corresponds to the probability that one WOM floating in the network is absorbed by  $i$  (typically, ratio of the degree of  $i$  to the sum of all degree). The state of individual  $X_i(t)$  starts from  $S$  (susceptible: no WOMs have reached yet). When  $i$  absorb one WOM, the state is changed to  $I$  (Infectious: emit secondary WOMs to the network) for some time, and eventually  $i$  become silent and the state is changed to  $R$  (Recovered: emit no WOMs). Let  $S_i(t) = PX_i(t) = S$ ,  $I_i(t) = PX_i(t) = I$ ,  $R_i(t) = PX_i(t) = S$ , then the system of PDEs for describing WOM spread is the

following:

$$\frac{dS_i(t)}{dt} = -\lambda \left( \sum_{j=1}^n k_j I_j(t) \right) w_i S_i(t), \quad (1)$$

$$\frac{dI_i(t)}{dt} = \lambda \left( \sum_{j=1}^n k_j I_j(t) \right) w_i S_i(t) - \mu I_i(t), \quad (2)$$

$$\frac{dR_i(t)}{dt} = \mu I_i(t), \quad (3)$$

where  $\lambda$  is the emitting rate and  $\mu$  is the recovering rate. On the other hand, DA model, which has no infectious state, can be described by:

$$\frac{dS_i(t)}{dt} = -\frac{\sigma}{n} S_i(t), \quad (4)$$

$$\frac{dR_i(t)}{dt} = \frac{\sigma}{n} S_i(t), \quad (5)$$

where  $\sigma$  is the emitting rate of direct advertisement. We solve these extended SIR models of network and discuss the efficiency of advertisement of WOM and DA. by the power of network, WOM can have better spreading performance than DA. However, as observed in SIR on complex networks, when  $k_i$  and  $w_i$  is proportional to the degree, large-degree nodes can be bottleneck to the spread of WOM.

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## Exploiting networks of concepts in learning paths

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Almost everyone met in his life a great teacher. Beside the individual nature of those mentors, a question arises whether it is possible to highlight some general directions an efficient learning (teaching) strategy should follow. A line of research focuses on optimal strategies to present concepts in order to maximize scholars learning abilities. Think for instance you have to learn a language or in a simplified picture a list of words. You should find a trade-off between look at new material and reviewing what you have already seen. I will review a simple mathematical model that captures this issue in an idealized form, including the "spacing effect" famous in cognitive science, according to which, loosely speaking, review is useful only if it is neither too soon nor too late. In this basic model the words to be learned are deprived of their semantic value and considered as a list of uncorrelated items. However, semantic relations play undoubtedly a fundamental role in learning processes. We investigate this aspect by considering different strategies for exploring concepts to be learned when those are represented as nodes in a network of semantic relations. We highlight both semantic structures and ways to exploit them which are optimal with respect to learning efficiency. Both results on synthetic graphs and results on real world structures, such as the collaborative on-line encyclopedia Wikipedia and the graph emerging from a words-association game (Human Brain Cloud) will be discussed.

## The dynamics of correlated novelties

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Innovations drive, at different scales, the evolution of biological systems, human society, technology, and spice our daily lives in the form of little novelties: we continuously meet new people, adopt new words, listen to new songs, use a new technology. From this perspective, novelties and innovations can be viewed as first time events at the individual or collective level, respectively. Such new experiences are often triggered by some earlier new experience, thus providing a correlation between them. The scientific significance of such concept has been summarized by the notion of *adjacent possible* introduced by Stuart Kauffman and popularized by Steven Johnson: "The adjacent possible consists of all those things (depending on the context, these could be ideas, molecules, genomes, technological products, etc.) that are one step away from what actually exists, and hence can arise from incremental modifications and recombinations of existing material. .. The strange and beautiful truth about the adjacent possible is that its boundaries grow as you explore those boundaries." In this picture, the advance into the adjacent possible is the driving force for correlating innovative events, and novelties are produced through an exploration of a space – physical, conceptual, technological, or biological – that enlarges itself whenever one reaches a point of the space never visited before. Though the creative power of this idea is widely appreciated at an anecdotal level, it remains poorly understood theoretically and unquantified empirically. I will propose a first mathematical model for the dynamics of novelties correlated via the adjacent possible. The model is based on Polya's urn, suitably generalized, and can be mapped into a random walk process on a growing graph. I will derive testable, quantitative predictions from this model: the statistical laws for the rate at which novelties happen (Heaps' law) and for the probability distribution on the space explored (Zipf's law), as well as signatures of the process by which one novelty sets the stage for another, measured by means of suitably defined entropic measures. I will show that these predictions are borne out in four data sets capturing diverse aspects of novelty and innovation in social and technological systems: the edit events of Wikipedia pages, the emergence of tags in annotation systems, the sequence of words in texts, and listening to new songs in online music catalogues, suggesting that our hypotheses capture the essential features of real systems.

## Karhunen-Loève expansion of Spartan spatial random fields

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Flexible and efficient representations of random fields (RFs) are important in the analysis and modeling of environmental data. The Karhunen-Loève (KL) expansion allows representing RFs in terms of an orthogonal basis derived by the eigen-decomposition of its covariance function and a denumerable set of uncorrelated random variables. The method can handle both Gaussian and non-Gaussian RFs depending on the probability distribution that the chosen

set of random variables follows [1]. The truncated basis of functions ensures smaller energy loss than representations by any other basis of the same dimension and the corresponding eigenvalues measure the contribution of each basis function to the total energy. As a means of extracting dominant features and trends the KL expansion has applications in turbulent flows [2] and nonlinear threshold systems [3]. The so-called Spartan spatial random fields (SSRFs) are Gaussian RFs with additional parameters [4]. The additional non-trivial parameters provide the ability to vary the RFs' smoothness and the dependence between the correlation length, the integral range and the characteristic length of the covariance model. This implies more flexibility in capturing the local variability than existing covariance models. In this work we derive the KL expansion of the Fluctuation-Gradient-Curvator(FGC) SSRF. The FGC-SSRF covariance is a family of three classes of covariance functions each one accounting for a different range of values of the shape parameter  $\eta_1$ . The model includes oscillating covariance functions for  $-2 < \eta_1 < 0$  [4]. The eigenfunctions and eigenvectors of the KL expansion are obtained by the solution of a homogeneous Fredholm equation of the second kind. For the FGC-SSRF covariance function the integral equation corresponds to a fourth order linear ODE, in contrast with the second order ODE of the well-studied exponential covariance case. This property is also present in the associated FGC-SSRF fourth order Langevin equation. The Fredholm equation is solved analytically and the eigenvalues are obtained by the numerical solution of two transcendental equations for each class of covariance functions. For  $\eta_1 = 2$  the derived results are similar with the ones presented in [5] for the modified exponential kernel. Simulations of the FGC-SSRF using their KL expansion will be presented and discussed.

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## Extreme waves in linear and nonlinear systems

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Ocean rogue waves (RW) huge solitary waves have for long triggered the interest of scientists. Rogue waves emerge in a complex environment and it is still unclear if their appearance is due to linear or nonlinear processes. Recent works have demonstrated that RWs appear in various other physical systems such as microwaves, nonlinear crystals, cold atoms, etc. We will present two types of investigations focusing on discrete nonlinear lattices that are models for weakly couple materials or metamaterials as well as continuous optical systems with specific variation of the index of refraction.

In the lattice models we first perform statistical analysis on discrete nonlinear waves generated through modulational instability in the context of the Salerno model that interpolates between the integrable Ablowitz-Ladik (AL) equation and the nonintegrable discrete nonlinear Schrödinger (DNLS) equation. We focus on extreme events in the form of discrete rogue or freak waves that may arise as a result of

rapid coalescence of discrete breathers or other nonlinear interaction processes. We find power law dependence in the wave amplitude distribution accompanied by an enhanced probability for freak events close to the integrable limit of the equation.

Furthermore, we induce spatiotemporal complexity in a two dimensional nonlinear disordered lattice through the modulational instability of an initially weakly perturbed excitation. In the course of evolution we observe the formation of transient as well as persistent localized structures, some of which have extreme magnitude. The probability of creation and dynamical properties of these two types of the extreme events are correlated with the competition of two distinct localization mechanisms dating from the quenched disorder and nonlinearity. We analyze the statistics of occurrence and the return time probability of these extreme collective events and find that the appearance of transient extreme events is more likely in the weakly nonlinear regime, while in the opposite case trapped highly pinned structures dominate. These findings that help to find optimal regimes for the EE generation are compatible with earlier work on the role of integrability in the generation of extreme waves.

In the optical work we investigate optical wave propagation in strongly scattering random lattices embedded in the bulk of transparent glasses. In the linear regime we observe the appearance of RWs that depend solely on the scattering properties of the medium. Interestingly, the addition of nonlinearity does not modify the RW statistics, while as the nonlinearities are increased multiple-filamentation and intensity clamping destroy the RW statistics. Numerical simulations agree nicely with the experimental findings and altogether prove that optical rogue waves are generated through the linear strong scattering in such complex environments.

## Nonlinear Lattices embedded in nonzero Heat baths

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A nonlinear lattice of anharmonic ( $\phi - 4$ ) oscillators supports nonlinear excitations in the form of intrinsic localized modes or discrete breathers, i.e., spatially localized and time-periodic oscillations [1], which crucially affect its thermodynamic properties. By means of the thermal shock technique we study various statistical properties of the one-dimensional chain of anharmonic oscillators with free-end boundary conditions. The isolated system is thermalized initially at a given temperature  $T_0$ . Subsequently, at time zero, our system is put into contact with a heat bath of nonzero temperature  $T_1$  city. As expected, we record velocity distributions which deviate from the Gaussian distribution (Boltzmann equilibrium). Moreover, the higher the energetic level of the metastability is, the more the distribution deviates from the former one, yet in a consistent manner so that a quantitative description is possible. Further analysis is related to the energy relaxation time. After initially thermalizing the lattice at temperature  $T_0$ , we put, at time zero, a large amount of energy on one site. This corresponds to the creation of a breather, following the reverse procedure than previously. We

then study the relaxation time of the breather with respect to the energy, by varying in each set of experiments the coupling constant  $k$  between neighboring oscillators. We observe that the former relaxation time  $\Delta\tau$  follows a power-law behavior with respect  $k$  for small values, i.e.,  $\Delta\tau(k) \sim k^{-\alpha}$

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### Liquid-liquid demixing and droplet nucleation in electric field gradients

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When a mixture of two simple liquids, or a pure liquid in coexistence with its vapor, are under the influence of spatially uniform electric field, the critical temperature may change by a small amount, typically in the mK regime. A different scenario occurs when the external fields has gradients. In this case the change to the coexistence temperature is 2-100 larger than the change in uniform fields. The phase separation is reversible: when field gradients are turned off, the mixture becomes homogeneous again. We will show theoretical and experimental evidence for this new electro prewetting transition and will discuss its implications.

Our theoretical approach relies on a coarse-grained mean-field free energy. It allowed us to recently study (i) liquid-vapour coexistence in pure and dielectric materials, (ii) interactions between colloids immersed in a binary mixture containing salt, and (iii) the dynamics of phase separation in a cylindrical capacitor.

Preliminary experiments have revealed several interfacial instabilities, occurring as a competition between surface tension and electrostatic forces. In addition, as the size of the electrodes is reduced, surface tension becomes more dominant; at the scale of 50 m we are able to achieve a large array of small drops of one liquid embedded in a matrix of the second liquid.

The phase transition may have various applications in the nanotechnological world, since it benefits from field gradients near small conducting objects. Some of the promising directions studied by us are demixing in microfluidics channels, MEMS, and electro lubrication.

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### Spectral Renormalization Group for a scalar $\psi^4$ theory on the Cayley tree

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We use a deterministic lattice, the Cayley tree, to illustrate how the analogue of a “momentum shell” renormalization group can be constructed on a network which is not embedded

in a metric space. The spectral renormalization group we have recently proposed [1] utilizes the eigenvectors of the graph Laplacian as a basis in which to expand the fields living on the vertices of a graph. The field components corresponding to eigenvectors with higher eigenvalues are then integrated out, and the remaining terms in the Hamiltonian rescaled in order to obtain a renormalized Hamiltonian. The scaling behaviour of the spectral density of the graph Laplacian near the lower end of the spectrum plays an important role in determining the critical behaviour of the fluctuating field living on the network, notwithstanding claims to the contrary [4].

The spectral renormalization group is applied to a scalar  $\psi^4$  model on the Cayley tree, which has an effective graph spectral dimension  $\bar{d} = 2$ , with the discrete spectral density  $\rho(\omega) \sim \omega^\beta$  being consistent with a scaling exponent  $\beta = 0$ , although the discrete  $\delta$ -function peaks have an envelope with a slope of unity [2]. The nearest neighbour interaction is diagonalized by using the eigenvectors of the graph Laplacian, giving rise to a Gaussian theory for continuous fields. Explicit solutions for the eigenvectors involve harmonic functions and display discrete scaling as well as permutation symmetries, making possible some of the manipulations allowed under usual assumptions of spatial isotropy. The scaling dimensions of the fields and of the “mass” term are determined numerically [2], as well as analytically. It should be noted that this approach is not based on decimating a certain fraction of the fields. Rescaling the effective Hamiltonian restores the original number of degrees of freedom, albeit with different coupling constants.

The  $\psi^4$  interaction terms are treated perturbatively, using the usual Feynman graph expansion. The RG fixed point is Gaussian to first order, as in the Wilson momentum-shell Renormalization Group [3]. Perturbation theory results will be presented up to second order. A drawback which we have to face is that the upper critical dimension is  $\beta_c = 1$ , so that an  $\epsilon$ -expansion in  $\epsilon = \beta_c - \beta$  is only asymptotic, with  $\epsilon = 1$ .

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### Connecting the physics of swarming animals and active matter

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Bird flocks, insect swarms and fish shoals resemble fluids made up of many individuals where the controlling interactions are social rather than physical in character [1]. Some progress has been made reverse-engineering candidates for these interactions that are local in space, either in a metric-based [2] or topological sense [3,4]. A question that has been largely overlooked is whether the interactions should be expected to be local at all. We discuss the evidence for them having a non-local character and, furthermore, that there is a natural choice for this that is consistent with the cognitive limitations of a bird’s vision. This leads us to propose a non-local hybrid-projection model. This has the physically satisfying feature that it involves an unusually

small number of control parameters, when compared with other swarming models. We study the global character of the flocks that emerge from this model and their various phenotypes. Most significantly, an emergent state arises in which the probability that a typical bird can see out (sky) in any direction divided by the probability that its view is blocked by other bird(s) is  $O(1)$ . We refer to this as being marginally opaque. We present experimental data on bird flocks that confirm this prediction and discuss how these models may naturally be associated with evolutionary fitness, as well as being physiologically plausible. In an attempt to make a connection with thermodynamic systems of so-called "active matter" we report on recent work on thermophoretic colloids that are heated by an external light source, extending on [5]. We show that these systems can undergo first order transitions from compact to disperse states as the light intensity is varied. Intriguingly, we find that the same state of marginal opacity emerges: no compact state with a density below marginal opacity is stable. This reveals a previously unidentified similarity between social and thermodynamic swarms.

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### Reaction-diffusion-taxis model for spatio-temporal dynamics of five picophytoplankton populations

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Recently new models were devised to study spatio-temporal dynamics of phytoplankton populations in view of obtaining more precise predictions of the vertical biomass distributions in marine ecosystems. These studies can be crucial from the point of view of fishery. Indeed the abundance of fish species is strictly connected with primary production, i.e. phytoplankton biomass, responsible for chlorophyll concentration. In this work a one-dimensional deterministic reaction-diffusion-taxis model is used to reproduce the spatio-temporal dynamics, along a water column, of five picophytoplankton populations sampled in a real ecosystem. In our analysis, to better reproduce the spatio-temporal behaviour of picophytoplankton populations we take into account the periodical changes of the light intensity and profiles of vertical turbulent diffusivity, obtaining the time evolution of the system over a period of five years. Moreover, the seasonal variations of both depth of thermocline and thickness of the upper mixed layer close to the water surface are considered. As a first step, the spatio-temporal behaviour of biomass concentration of each picophytoplankton population is calculated by numerically solving the equations of the model. Afterwards, the numerical results for biomass concentration, expressed in  $cell/m^3$ , are converted in total concentration of chlorophyll a (*chl a*) and divinyl-chlorophyll a (*Dvchl a*), obtaining the chlorophyll distributions along the whole water column. These theoretical profiles are compared with experimental

data for chlorophyll concentration collected in a site of the Tyrrhenian Sea in four different days of different seasons of the year. Statistical analysis, based on  $\chi^2$  goodness-of-fit test, shows that numerical results are in a good agreement with real chlorophyll distributions for all seasons investigated. In particular, numerical results indicate that the primary production of phytoplankton biomass is strongly influenced by the light intensity and vertical turbulent diffusivity, which take on different values along the water column, depending also on seasonal variations. These findings could contribute to predict future changes in phytoplankton distributions due to global warming, and to devise strategies which can prevent the decline of primary production and consequent decrease of fish abundance.

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### Crystal growth in confined spaces

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Recent progress in the synthesis of nanoporous materials with controlled structural properties made it possible to address various phenomena occurring in mesoscale systems. Among them, different aspects of phase transitions of confined fluids can now be related to the structural properties of the porous matrices. In this work, we present the results of our experimental nuclear magnetic resonance study of crystal growth processes in mesoporous silicon, a material with linear, macroscopically-long pores [1, 2]. In an ideal cylindrical pore, the freezing temperature is reduced in proportion to the inverse pore size. In the material under study, however, freezing is found to start at temperatures notably lower than the transition temperature determined by the average pore size. The kinetics of this process is found to be very slow and to depend on the temperature. In particular, power-law dependencies of the crystal growth process were observed. These findings we have associated with the fact that the material under study possesses a substantial degree of disorder, which can be represented as a distribution of the pore diameters along the pore axis. Taking into account that the phase transitions under confinement are activated processes with the pore size-dependent barriers in the free energy, the overall process of the crystal growth turns out thus to occur under the condition of disordered transition rates. The latter is shown to give rise to slow kinetics of the phase transition. The experimental results are further corroborated by using a simple Ising-like lattice-fluid model, which is capable to model the freezing and melting processes of fluids confined to pores with arbitrary organizations of their pore spaces [3]. By using dynamic Monte Carlo simulations we find that this model reproduces several experimental observations including, in particular, suppression of the transition temperatures, irreversibility between freezing and melting and strong effect of structural disorder upon the

pathways of the phase transitions. The mechanisms governing the both transitions are established. As an important point, this model shows the occurrence of the confinement size- and temperature-dependent crystal growth rates. Different regimes of the ice phase propagation, including, in particular, the observation of Sinai-like diffusion, are discussed.

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### Transition to sustained turbulence as a non-equilibrium phase transition

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Wall bounded shear flows such as pipe and plane-Couette systems, exhibit a sub-critical instability where a finite amplitude perturbation is needed to trigger turbulence. At values of Reynolds number (the control parameter) that are not too high, turbulence occurs as localized patches surrounded by laminar flow. For low values of the Reynolds number, these patches are transient, and the system returns to the laminar state. Due to the sub-critical nature of the transition, turbulence cannot spontaneously arise and the laminar state is an absorbing one for the system. However, turbulence can spread by invading adjacent laminar regions. At high enough values of the Reynolds number, the proliferation of turbulence can outweigh its decay, leading to sustained spatio-temporal chaos. This is analogous to contact processes (used to model the spreading of epidemics) that exhibit a non-equilibrium phase transition to sustained chaos. In particular, it is conjectured that, under fairly general conditions, systems with an unique absorbing state exhibit a phase transition in the universality class of directed percolation. In this contribution, we exploit this analogy and examine the phase transition to turbulence through experimental studies of pipe flow and direct numerical simulations of plane-Couette flow. In the case of pipe flow, the phase transition scenario has not been examined so far because of the long time scales involved ( $10^8$  advective units), though the critical point has been estimated indirectly. We show that it is possible to exploit the memoryless nature of the turbulence proliferation and decay processes close to the transition point, to construct a quasi-periodic pipe. Together with an accurate long-term control on the Reynolds number (better than 0.1%), it is possible for the first time to follow the intermittent dynamics for arbitrarily long times, and extract quantities such as the equilibrium turbulent fraction, which is the order parameter for the transition. In the plane-Couette system, direct numerical simulations were carried out in a long, narrow domain, unconfined in the direction normal to the turbulent stripes. A comparison of the spatial proliferation and decay

rates of the turbulent stripes allows us to estimate the critical point in this system. We examine the phase transition scenario for both these systems. We present estimates of the critical exponents, and compare them with those of the directed percolation universality class.

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### Thermodynamics of string gas in space-time with horizon

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It is a well known fact that a string gas in flat space has a limiting temperature, the so called Hagedorn temperature. At high energy, the density of states of a single string grows exponentially as a function of the energy which means that the partition function is not defined above a certain critical temperature. Close to the Hagedorn temperature, it is entropically favourable for the strings to coalesce into a single string so that the thermodynamics of the string gas is dominated by a single long string. This temperature can be related to a certain string state on the thermal manifold, the thermal scalar, which becomes tachyonic. Because of the thermal manifold, strings can wind around the time circle. The state with winding number one becomes tachyonic at precisely the Hagedorn temperature. In Kruzenski et al. ("Random walks and the Hagedorn transition", *JHEP* 0607 (2006)) it was shown how this state can be recovered from the single string path integral which gets dimensionally reduced close to the Hagedorn temperature. The resulting path integral describes a random walk coinciding with the spatial configuration of the long string. In field theory language, the thermal scalar is a 3D complex scalar (corresponding with the winding number  $\pm 1$  excitations). We generalize this thermal scalar picture of the Hagedorn transition to curved spaces with a horizon and show how zero modes of the thermal scalar living closely to the horizon dominate thermodynamics. To do this, the path integral derivation of Kruzenski et al. is critically revised and a correction term which is crucial in curved space is identified. We apply the thermal scalar picture to large black holes which in the neighbourhood of the horizon can be approximated with a Rindler metric. We elaborate on higher order  $\alpha$  corrections for Rindler space and the link with the cigar CFT as recently discussed in Givone et al. (String theory at the tip of the cigar, *JHEP* 1309 (2013) 079). We analyse the thermal scalar spectrum in Rindler space for bosonic, type II and heterotic strings and determine the Hagedorn temperature from the infrared behavior of the free energy. We discuss the critical behavior of the string gas and interpret this as a random walk near the horizon. Combining field theory arguments with the random walk path integral picture, we realize (at genus one) the picture put forward by Susskind of the string gas as a long string surrounding black hole horizons. We find that thermodynamics is dominated by a long string living at string scale distance from the horizon. The Hagedorn temperature of the string gas turns out to be

equal to the Hawking temperature.

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### Brownian corrections to particle motion in the XY Hamiltonian Mean Field Model

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We study the dynamics of the  $N$ -particle system evolving according to the Hamiltonian

$$H = \sum_i^N \frac{p_i^2}{2} + \frac{K}{2N} \sum_{i,j}^N [1 - \cos(\theta_i - \theta_j)], \quad (1)$$

commonly known as the XY Hamiltonian Mean Field (XY-HMF) model.  $\theta_i$  is the position of the  $i$ -th particle on the circle  $S_{2\pi} = \mathbf{R}/2\pi$  and  $p_i$  its conjugate momentum. This model has two behaviours, viz. (i) for an attractive potential ( $K = 1$ ), a phase transition occurs from an inhomogeneous, clustered phase to a homogeneous phase for a given value of temperature ; (ii) for a repulsive potential ( $K = -1$ ), no phase transition occurs.

In the second case, particle motion may be approximated by the ballistic motion with small corrections. For particles with initial positions uniformly distributed in  $S_{2\pi}$ , while initial velocities are distributed in equally spaced (by  $\Delta v_0$ ) beams containing one particle each, i.e.,  $v_{j0} \sim (j/N - 1/2)\Delta v_0$  for the initial velocity of the  $j$ -th particle, it is shown that corrections to the ballistic velocities are in the form of independent Brownian noises. Moreover, we also estimate a time validity for this approximation. Molecular dynamics simulations of the XY-HMF model with the proposed “particles in monokinetic beams” initial conditions are presented to confirm our preliminary theoretical results.

For the attractive case, we model the system, in presence of the ordered phase, as composed of two sets of particles:  $N_p$  *passing particles* move according to a ballistic motion corrected by the presence of  $N_c$  *cluster particles*, that lay inside the cat’s eye and are assumed to have fixed positions. Thus, we focus on the dynamics of passing particles with the same strategy as above. The presence of cluster particles, however, no longer allows us to admit small corrections to a ballistic motion. Preliminary numerical simulations for this case show that these corrections diverge from a Brownian noise in very short times.

We are still carrying out the calculations for the attractive case. As a first order correction to the velocities, we expect to get Brownian noise and non-negligible corrections for higher orders.

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### Two-component Hamiltonian Mean Field Model

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We consider a system of  $N$  particles interacting in a ring of radius one. The system is composed of two species of particles:  $N_1$  particles of species 1 and  $N_2 = N - N_1$  of species 2. The interaction potential is that of the Hamiltonian Mean Field (HMF) model and the nature of the interaction is given by the combination of particle species, much like in a charged particles model. We compute the equilibrium condition for the system and, through this, determine the geometry of phase space. We present molecular dynamics simulations to prove our findings. Moreover, we show that our model presents a “bicluster” formation for low energies, similar to the findings for the repulsive HMF model.

The Hamiltonian describing the system is given by

$$H = \sum_{i=1}^N \frac{p_i^2}{2m_i} - \frac{1}{2N} \sum_{i,j=1}^N e_i e_j [1 - \cos(\theta_i - \theta_j)], \quad (1)$$

where

$$e_i = +1 \text{ if } 1 \leq i \leq N_1 \quad -1 \text{ if } N_1 < i \leq N, \quad (2)$$

$$m_i = M \text{ if } 1 \leq i \leq N_1 \quad m \text{ if } N_1 < i \leq N, \quad (3)$$

and the conjugated pair  $(\theta_i, p_i)$  represents, respectively the position of the  $i$ -th particle on the ring  $S_{2\pi}$  and its momentum. We assume there are two species of particles in the system:  $N_1$  particles of charge  $e_i = 1$  and  $N_2 = N - N_1$  particles of charge  $e_i = -1$ . Therefore, the equations of motion for the  $i$ -th particle are

$$\begin{aligned} \dot{\theta}_i &= \frac{p_i}{m_i} \\ \dot{p}_i &= \frac{1}{N} \sum_j e_i e_j \sin(\theta_i - \theta_j) \end{aligned} \quad (4)$$

We define the mean field quantities (we also use the term magnetization)

$$\mathbf{M}_1 = \left( \frac{1}{N_1} \sum_i^{N_1} \cos \theta_i, \frac{1}{N_1} \sum_i^{N_1} \sin \theta_i \right) \quad (5)$$

$$\mathbf{M}_2 = \left( \frac{1}{N_2} \sum_{i=N_1+1}^{N_2} \cos \theta_i, \frac{1}{N_2} \sum_{i=N_1+1}^{N_2} \sin \theta_i \right), \quad (6)$$

$$\mathbf{A} = c_1 \mathbf{M}_1 - c_2 \mathbf{M}_2, \quad (7)$$

with which we obtain

$$\dot{p}_i = e_i (\sin \theta_i A_x - \cos \theta_i A_y) \quad (8)$$

$$H = \sum_{i=1}^N \frac{p_i^2}{2m_i} - \frac{1}{2} \frac{(N_1 - N_2)^2}{N} + \frac{N}{2} (A_x^2 + A_y^2) \quad (9)$$

We see, thus, that vector  $\mathbf{A}$  behaves as a mean field acting on particles.

Equilibrium properties are determined using a maximization of entropy method assuming all particles to be statistically uncorrelated.

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## Knots in proteins and DNA

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Ever since Kelvin has conjectured that atoms are composed of knots in the ether, these peculiar objects have stimulated the imagination of mathematicians and natural scientists alike. In recent years the field went through a renaissance and progressed considerably, spurred by the discovery of knotted DNA and proteins, and the realization that topology may not only diversify structure but also have a profound impact on the function of biological macromolecules.

In the first part of my presentation I will give an overview of knotted proteins. From an evolutionary point of view protein knots occur in all kingdoms of life and topology is typically preserved amongst homologs. Nowadays, knotted protein structures can even be created artificially, and knotted designs could be used in the future in the context of protein engineering. I will also demonstrate why the folding of knotted proteins may not be so difficult after all by explaining coarse-grained folding simulations of a rather complicated protein, which features six elementary crossings in a projection onto a plane. While knots in globular homopolymers are abundant, protein knots are rare and occur in less than one percent of all known structures. To address this conundrum I will present simulations of a coarse-grained heteropolymer model and argue that the addition of sequence may facilitate evolution towards unknotted proteins even though on average globular heteropolymers are just as knotted as homopolymers. In the second part I will discuss the occurrence and implications of knots in DNA. Viral DNA is known to be highly knotted in the capsid and shows a preference towards torus-type knots. These phenomena will be explained in the context of coarse-grained simulations of single semiflexible polymers in spherical confinement. I will also present a mechanism, which allows two knots on a polymer chain to pass through each other and swap positions along the strand. Associated "topological" free energy barriers only amount to a few kT, which may enable the interchange of knots on a single DNA molecule. This peculiar mechanism is not only interesting from an aesthetic point of view, but may also play a role in future technological applications such as nanopore sequencing once strand sizes exceed 100000 base pairs.

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## Trajectory statistics and turbulence evolution

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We present a statistical study of the strongly nonlinear phase in turbulence evolution, where both stochastic and quasi-coherent characteristics appear and compete. An interconnected study of test modes and test particles in turbulent plasmas and fluids permitted to analyze the strong interdependence between transport and the turbulence evolu-

tion. The aim is to understand the tendency of organization in fluids and plasmas. The dominant nonlinearity in turbulent fluids and plasmas is determined by the advection term. This determines eddying motion or particle trapping for turbulence in the nonlinear regime in magnetically confined plasmas or two-dimensional fluids. We have developed analytical methods (the decorrelation trajectory method and the nested subensemble approach [1]), which describe the statistical effects of trajectory trapping. We have shown that test particle transport is strongly modified in the presence of trapping and that weak perturbations produced by other components of the motion determine strong nonlinear effects. They are the consequence of the long memory of the Lagrangian velocity. Trapping also determines a large degree of coherence in the sense that bundles of trajectories that start from neighbouring points remain close for very long time compared with the eddying time (very strong clump effect). Trapped trajectories form quasi-coherent structures that are similar to fluid vortices.

The test modes on turbulent plasmas and fluids were studied by determining the renormalized (average) propagator using the statistics of the characteristics of the equations, which are particle trajectories. We have shown that the random motion, which leads to diffusion, has a stabilizing effect on turbulence while trapping, which generates quasi-coherent trajectory structures, leads to strong nonlinear effects.

In the case of drift type turbulence [2], the nonlinear effects consist of the increase of the correlation lengths, the nonlinear damping of the drift modes and generation of zonal flow modes. The strength of each of these processes depends on the stage in the evolution of turbulence (reflected in the parameters of the turbulence), and more precisely on the fraction of trapped trajectories [3]. In the case of the turbulence in two dimensional ideal fluids, the evolution strongly depends on the initial condition. Nonlinear effects appear for initial states with large amplitudes and small space correlation of the vorticity, and they lead to vorticity separation and to the increase of the correlation length.

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## Interdependencies and Causalities in Coupled Financial Networks

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This paper explores the relationships of coupled financial networks. We study the foreign exchange and stock market networks for 48 countries from 1999 until 2012. We examine the influences of one network on the dynamics of the other and propose a model, based on complex principal component analysis, for extracting significant relations between these networks. We identify important component (node) relationships within and between the two networks that cannot be detected by concurrent simple correlation analyses or equal-time principal component analysis. We find that in general the

foreign exchange markets have predictive power regarding the performance of the global stock markets. We also find that the US stock market, closely followed by the German and Canadian equity markets, can forecast performances of other world equity markets. We divide the entire time period into "mild crisis" (1999-2002), "calm" (2003-2006), and "severe crisis" (2007-2012) periods and find that the "severe crisis" period behavior dominates the relationships in the currency-equity interdependent network. We also explore formation of communities of synchronizing nodes in this financial coupled network system, and clearly identify four distinct communities dominated by the (i) stock markets for Europe, South America, USA, and Canada; (ii) currencies of European countries, and Canada; (iii) currencies for Russia, USA, South America, Asia, and the Middle East, and (iv) stock markets for Asia (including Japan) and the Middle East.

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### Phase behavior of randomly cross-linked diblock copolymers

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We study theoretically the intricate phase behavior of  $AB$  diblock copolymer melts able to undergo two phase transitions due to quenched random constraints: Localization of a fraction of the polymers at random positions (amorphous solidification or gelation) is driven by the number of permanent cross-links between random pairs of  $A$  blocks, formation of a periodic pattern of  $A/B$ -rich microdomains (ordering) is controlled by the  $AB$  incompatibility  $\chi$  inversely proportional to temperature. Our model aims to capture the system's essential microscopic features — the copolymers have Gaussian-chain connectivity, a binary variable records the type of each monomer (at arc-length  $s$ ), and the cross-links are entropic harmonic springs with tunable rigidity. For a first overview, we have focused on an incompressible melt of monodisperse diblock copolymers with equal contents of  $A$  and  $B$  (symmetric diblocks) and rigid (inextensible) cross-links. A Deam-Edwards distribution with cross-link density parameter  $\mu$  is employed to generate a fairly realistic ensemble of random network realizations that reflects spatial correlations at the instant of cross-linking. By identifying suitable order parameters —  $A$  density modulations in simple and in replicated Fourier space — and setting up a replica field theory to perform the generic disorder-average over all cross-link realizations, we derive a free-energy functional governing the interaction fields. An expansion of this functional in the spirit of Landau theory allows us to trace the instability lines of the mixed liquid state and of the ordered liquid state (lamellae for symmetric diblocks) in the plane of  $\mu$  and  $\chi$ .

Starting from the mixed liquid state, block-type-selective cross-links induce ordering at critical incompatibilities  $\chi_c$  smaller than in an uncross-linked diblock copolymer melt, as reported in [1]. Increasing the cross-link density in the mixed liquid state reproduces the known transition to an isotropic gel at a critical average polymer coordination number of one. At the saddle-point level, an intuitive relation between the gel order parameter in the two-fold replicated sector and the ordering amplitude in the ordered liquid state arises. Of particular interest are the conditions of appearance and the characteristics of the gel state with  $AB$  microstructure, which is expected to be stabilized by cross-links against mixing upon heating.

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### Cross-link induced stiffening of weakly bending biopolymers

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The partition function of two weakly bending, semiflexible polymers aligned in parallel by a longitudinal force and with  $N - 1$  cross-link sites regularly spaced along the polymer contours has been calculated analytically [1]. From this theoretically feasible model, we can extract physical properties characteristic of tension-aligned biopolymers, which are predominantly determined by bending rigidity and which undergo either permanent or reversible cross-linking. The first case is prominently exemplified by the connective-tissue protein collagen, the second case by double-stranded DNA, whose elasticity (without explicit cross-links) is often explained by the wormlike chain model [2]. The force-extension relation and the differential stiffness show that permanent cross-links effectively stiffen the polymer pair by reducing thermal bending fluctuations transverse to the force and alignment direction. At small tensile force  $f$ , the relative extra alignment by cross-linking is largest. Concomitantly, the linear elastic constant increases drastically due to even a small number of cross-links. The related differential stiffness increases as much as 100% for small  $f$  or for large numbers of cross-links. An increase in the additional alignment is observed for both growing number and growing strength of cross-links. Within our model, the strength or rigidity  $g$  is given by the inverse squared equilibrium length of the entropic harmonic springs that represent the cross-links. Asymptotically for large tensile force, the cross-link contribution to the extension decays differently for extensible and rigid cross-links, as well as for linking the polymers continuously along their contour,  $N \rightarrow \infty$  at finite total strength  $Ng$ , but faster than the global saturation as  $f^{-1/2}$  to the maximal (polymer contour) length. Only in the combined limit of continuous and rigid

cross-linking, the stiffening by attaching the chains to each other “survives” in the strong stretching regime. In this limit, the force-extension curve for one of the linked chains resembles that of one weakly bending wormlike chain without constraints, stiffened by a factor of 4; in the linear regime, the apparent stiffness is doubled. Complementary to these results for permanent cross-links, introducing reversible cross-links at the specified sites gives rise to a tension-induced binding cross-over, *i.e.*, a sharp, mostly discontinuous, increase of the average number of bound cross-links with force.

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### Simulation of Canonical Properties of quantum systems at finite temperatures by Path Integral Numerical Methods

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Path Integral Monte Carlo (PIMC) method proved to be a powerful tool for simulation of a wide variety of quantum systems. For a system of identical particles a sum in the partition function over particles permutations appears. For Bose-systems that does not create any problems, while for Fermi-systems the terms in the sum are antisymmetric and this leads to the fact that some contributions in partition function are negative. With the decrease of temperature positive and negative terms compensate each other and it is very difficult to extract any useful data. This problem is known as the sign problem. Proposed approach is based on the bead-approximation for PIs, but there is no restrictions to use any other approximations for the canonical density matrix. Within bead approximation quantum particle is replaced by a closed “polymer”, with beads representing position of a quantum particle in a moment of imaginary time. In this case we are free to use all methods, designed for simulation of classical systems. A new approach in obtaining the partition function of a system of several interacting particles (fermions) in external field within path integral Monte Carlo method based on direct averaging of exchange contributions over the positive weight determined by non-closed trajectories [1]. In the weight function we get an integral over non-closed trajectories, which we compute using expanded ensemble method [2] with Wang-Landau algorithm [3]. We obtain ratio of the integral over non-closed trajectories to the similar integral over non-closed trajectories written for the reference system, in our case system of noninteracting particles in harmonic field. For the latter analytical expressions are known. This way the complete partition function is obtained at finite temperatures down to their low values. The new approach allows us to reach observably lower temperatures, compared to the previously proposed path-integral Monte Carlo method [4], thus allowing significantly reduce the sign problem. The proposed approach yields an independent way to treat thermal properties of quantum systems, so the good agreement with previous data [4] permit us to test and state the validity of

both approaches. A quantum dot model of several interacting electrons confined in harmonic oscillator field was simulated. The interaction with environment (phonons, plasmons) was accounted for within harmonic heat bath approximation [5]. The presented approach allows one to carry out calculations for low temperatures that make it possible to estimate the ground-state energy and low-temperature thermodynamics.

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### Information geometry on the $\kappa$ -thermostatistics

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Information geometry is a powerful framework to study a family of probability distributions by applying the geometric tools developed in affine differential geometry. It has a dualistic structure of affine connections, which is useful and has been applied to many scientific fields such as information theory, statistics, neural networks, statistical physics, and so on. Information geometry was introduced in 1980s and has been mainly applied to the exponential family of probability distributions. Recently it has been attracted great attention for studying some deformed exponential families of probability distributions by using the method of information geometry. Indeed the  $\alpha$ -geometry invented by Amari is deeply related with the  $q$ -deformed exponential family, which plays fundamental role in Tsallis generalization of thermostatistics. The  $q$ -deformed relative entropy is related to the  $\alpha$ -geometry on the statistical manifold with a constant curvature. Ohara *et al.* have obtained a dually flat structure on the space of the escort probabilities by applying  $\pm 1$ -conformal transformation to the  $\alpha$ -geometry. Naudts has generalized Callen’s thermostatistics, and named it *generalized thermostatistics*. He introduced so called  $\phi$ -exponential function, which is his generalization of the standard exponential function. The  $\phi$ -exponential function includes the  $q$ - or  $\kappa$ -deformed exponential function as a special case. The  $\kappa$ -deformed function is given by

$$\exp_{\kappa}(x) \equiv \left[ \kappa x + \sqrt{1 + \kappa^2 x^2} \right]^{\frac{1}{\kappa}}, \quad (1)$$

for a real deformed parameter  $\kappa$ . Naudts also studied the information geometric structure for the  $\phi$ -exponential family. In this contribution, we explore the information geometric structures of the statistical manifold generated by the  $\kappa$ -deformed exponential family:

$$p(x_i, \theta) = \alpha \exp_{\kappa} \left[ \frac{1}{\lambda} \left( \sum_k \theta^k f_k(x_i) - \gamma(\theta) \right) \right], \quad (2)$$

where  $\alpha$  and  $\lambda$  are  $\kappa$ -dependent constants, and  $\gamma(\theta)$  is a potential function for the normalization of the probabilities.

A key point for constructing the  $\kappa$ -statistical manifold is to choose an appropriate conjugate representation, which is introduced by Zhang as a generalization of the  $\alpha$ -representation. The Legendre structures in both the information geometry and  $\kappa$ -thermostatistics are shown to be consistent each other, and play fundamental role. The associated canonical divergence is also derived.

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### Anomalous diffusion of particles in living breast cancer cells an indication to cancer-specific function

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Mechanics of cancer cells are directly linked to their metastatic potential (MP), or ability to produce a secondary tumor at a distant site. Metastatic cells can squeeze through blood vessel walls and tissue. Such considerable structural changes rely on rapid remodeling of internal cell structure and mechanics. We perform a comparative study, using particle-tracking to evaluate the intracellular mechanics of living epithelial breast cells with varying invasiveness. Probe-particle transport differs between the cell types, likely relating to their cytoskeleton network-structure and underlying transport. The basic analysis included evaluation of the time-dependent mean square displacement (MSD), the second power of the displacement. Particles in all the evaluated cell lines exhibit anomalous super-diffusion with an MSD scaling exponent of 1.4, at short lag times below 1 second. While indicating active transport within the cells, the MSD alone cannot reveal the underlying mechanisms. Hence, we analyze particle motion through a combination the MSD, other powers of the displacement, and various trajectory and displacement analysis procedures to identify structural and dynamic changes associated with metastatic capabilities of cells. The dynamic cytoskeleton and especially the molecular motors acting on it provide the cell with its remodeling capabilities and allow active transport within the cell. While active transport in living cells has been well-documented, the underlying mechanisms have not been determined. Here, we systematically target the cytoskeleton, molecular motors, and ATP energy related processes to determine their roles in particle transport. Our results show that particle motion is likely driven by different processes in each cell type. Intracellular transport in high MP cells is suggested to originate from fluctuations of microtubule filaments as well as from direct and indirect interactions between particles and microtubule-associated molecular motors. In the low MP cells we suggest that motion results from direct and indirect interactions between particles and microtubule-associated molecular motors, being transported by them or nudged by passing motors, respectively. The benign cells, however, reveal significant involvement of the acto-myosin network,

where particle motion was related to network contractions. Thus, we are able to provide insight into dynamic intracellular structure and mechanics that can support the unique function and invasive capabilities of highly metastatic cells.

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### Quasi-power laws in multiparticle production processes

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In multiparticle physics all large transverse momentum ( $p_T$ ) distributions exhibit power-like behavior. They are nowadays usually fitted by one of two recognized forms: either by original Tsallis distribution (with two parameters:  $q$  and  $T$ ),  $f(p_T) = C \cdot [1 - (1 - q) \frac{p_T}{T}]^{\frac{1}{1-q}}$  [1] or, by the so called "QCD inspired" Hagedorn form [2] (with parameters:  $m$  and  $T$ ),  $h(p_T) = C \left(1 + \frac{p_T}{m_T}\right)^{-m}$  with  $m = \frac{1}{q-1}$ . Historically, they are supposed to originate from, respectively, two different dynamical sources: from the purely statistical (or thermodynamical) approach or from hard collisions of constituents governed by Quantum Chromodynamics (QCD). In fact, they are equivalent (and can be used interchangeably). It also turns out that they can be derived from a number of dynamical pictures which can be used to characterize these processes. Among many possibilities the most interesting is the so called superstatistics, stochastic network approach, and connection with multiplicative noise (notice also that essentially all distributions of interest, including Tsallis, can be derived from information theory based on Shannon entropy [1]). Tsallis distribution represents the simplest way of describing the whole observed range of measured  $p_T$  distributions (successfully fits observed yield extending over fourteen decades) [2]. However, closer inspection of results obtained when fitting recent data at highest available energies at LHC reveals that this behavior is in fact decorated with some log-periodic oscillations. Assuming that this is a genuine effect and not an experimental artefact, it suggests that either the exponent of the power-like behavior is in reality complex, or that there is a scale parameter which exhibits specific log-periodic oscillations [2,3]. This problem is discussed using Tsallis distribution with scale parameter  $T$ . At this stage we consider both possibilities on equal footing because, at the present level of investigation, we are not able to indicate which of the two possible mechanisms presented here (complex  $q$  or oscillating  $T$ ) and resulting in log-periodic oscillations is the preferred one. This would demand more detailed studies on the possible connections with dynamical pictures as, for example, the one discussed long time ago by studying apparently similar effects in some exclusive reactions using the QCD Coulomb phase shift idea [3].

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## Tsallis distribution decorated with log-periodic oscillation

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The two parameter Tsallis distribution,  $f(X) = C \cdot \left[1 + \frac{X}{mT}\right]^{-m}$  with scale parameter  $T$  (identified in thermodynamical applications with the usual temperature) and with real power index  $m = 1/(q - 1)$  ( $q$  being known as the parameter of nonextensivity in statistical mechanical approaches) are nowadays very well known and applied in vast variety of situations. Actually, Tsallis distribution can be regarded as generalizations to real power  $m$  (or  $q$ ) of such well known distributions as the Gosset-Student distribution ( $X = t^2$ ,  $m = (\nu + 1)/2$  with integer  $\nu$ , which for  $\nu \rightarrow \infty$  becomes a Gaussian distribution and for  $\nu = 1$  a Cauchy distribution). We discuss a Tsallis distribution with complex nonextensivity parameter  $q$ . In this case usual distribution is decorated with a log-periodic oscillating factor. The presence of log-periodic features signals the existence of important physical structures hidden in the fully scale invariant description [1]. The system considered and/or the underlying physical mechanisms have characteristic scale invariance behavior,  $f[(1 + \alpha)X + \alpha nT] = (1 - \alpha n)f(X)$ . This form follows quasi-power law (Tsallis distribution) with the complex power index  $m = [-\ln(1 - \alpha n) + 2\pi i]/\ln(1 + \alpha)$ . It can also be shown that discrete scale invariance and its associated complex exponents can appear spontaneously, without a pre-existing hierarchical structure. We illustrate our point by example of transverse momentum distributions obtained for the highest presently available energy of 7 TeV. Albeit Tsallis distribution fits look pretty good (large  $p_T$  transverse momentum distribution exhibit apparently a power-law behavior), closer inspection shows clearly visible log-periodic oscillations [2,3]. Finally, complex  $q$  also means complex heat capacity,  $C = 1/(q - 1)$ . In particular, it will be shown that results for heat capacity can be used to a new phenomenological interpretation of the complex  $q$  parameter. Namely, one can argue that  $q - 1 = \text{Var}(T)/\langle T \rangle^2 - iS(T)/\langle T \rangle^2$  where  $S(T) = \omega \int e^{-i\omega t} \langle \text{Cov}[T(0), T(t)] \rangle dt$  is the spectral density of temperature fluctuations (i.e., the Fourier transform of the covariance function averaging over the nonequilibrium density matrix). This can be regarded as a generalization of our old proposition for interpreting  $q$  as a measure of nonstatistical intrinsic fluctuations in the system (which corresponds to the real part of the above  $q$ ) by adding the effect of spectral density of such fluctuations (via its imaginary part).

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## Queuing model of a traffic bottleneck with bimodal arrival rate

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Recently, it has been investigated under which circumstances the traffic flow through a bottleneck can be optimized in

stochastic traffic models. The strategy in [1] was to reduce (enhance) the arrival rate, whenever the car density is above (below) a certain threshold while the exit rate is kept constant. This regulation, referred to in [1] as density-feedback control, serves as a traffic-management strategy that tries to prevent for high volume of traffic. One practical realization would be to demand a certain toll or to suggest different routings via the navigation system while the threshold density is exceeded. In this way the arrival rate could be reduced. The simplest idea to model the situation is to choose a bimodal arrival rate that switches between two constant values. The threshold density is typically the density where the flow becomes maximal. In [1] the problem was studied on the example of the totally asymmetric exclusion process (TASEP) [2] by solution of its mean-field equations which in turn are a discretization of the noisy Burgers equation with Diffusion constant  $D$ . This article revisits this problem of tuning the density in a traffic bottleneck by reduction of the arrival rate. It gives the main results of [1], especially the results from the mean-field theory for the TASEP that neglects correlations between neighboring sites. Beyond the results of [1] in the present approach a different approximation is applied that is even broader: instead of the TASEP, we think of the bottleneck containing one compact queue of cars and completely neglect the distribution of unoccupied sites in this queue. A related approach has been considered in [3] for the TASEP however there the queue length was considered to be unbound. Here we consider a maximum queue length  $L$  that represents the length of the bottleneck, a threshold  $N^*$  and a queue length  $N$ . There are mainly three different cases to consider. Arrival rate changed from large to small, Small arrival rate reduced further, Reduction to a still large arrival rate. We derive its phase diagram using a random walk argument. One observes one jammed region, one region of vanishing density, and one where the queue length is equilibrated at the threshold. Despite the simplicity of the model the physics is in accordance with the previous approach: The density is tuned at the threshold if the exit rate lies in between the two arrival rates.

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## Global Inequality in Energy Consumption from the Maximal Entropy Perspective

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First, I will briefly review the principle of maximal entropy and its applications to monetary and income inequality in economics. We found that income distribution in many countries, such as USA and European Union, exhibits a two-class structure. For the majority of population (about 97 percent), income distribution is roughly exponential, resembling the Boltzmann-Gibbs distribution of energy in physics. For the upper tail (about 3 percent of population), income distribution changes to a power law, known as the Pareto law. The tremendous increase in income inequality in the last 20 years originates completely from the increase of the upper-class income share relative to the lower-class share. Then, I will present our recent study of global inequality in

energy consumption per capita based on data from the U.S. Energy Information Administration (EIA) for 1980-2010. We find that the Lorenz curves have moved up during this time period, and the Gini coefficient  $G$  has decreased from 0.66 in 1980 to 0.55 in 2010, indicating a decrease in global inequality. The global probability distribution of energy consumption per capita in 2010 is close to the exponential distribution with  $G=0.5$ . We attribute this result to the globalization of the world economy, which mixes the world and brings it closer to the state of maximal entropy. We argue that global energy production is a limited resource that is partitioned among the world population. The most probable partition is the one that maximizes entropy, thus resulting in the exponential Boltzmann-Gibbs distribution function. A consequence of the latter is the law of 1/3: the top 1/3 of the world population consumes 2/3 of produced energy. We also find similar results for the global probability distribution of CO2 emissions per capita. Interestingly, all three Gini coefficients for global income inequality, energy consumption, and CO2 emissions are close to each other and exhibit very similar behavior, thus confirming effective equivalence between money, energy, and fossil fuels at the current (unsustainable) stage of human civilization.

More info at <http://physics.umd.edu/yakovenk/econophysics/>  
Time evolution of the Lorenz curves for global energy consumption in 1980-2010 is visualized in computer animation videos at <http://arxiv.org/src/1312.6443v1/anc>

A popular press release "The Entropy of Nations" based on the presented paper was picked by many online news outlets, e.g. <http://phys.org/news/2014-01-entropy-nations-global-energy-inequality.html>

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## Realistic order book model based on real data analysis

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We introduce a new order book model which reproduces all major stylized facts such as a power law distribution of price changes, abnormal diffusions and distribution of transaction intervals.

Maslov introduced a basic order book model[1] which describes the dynamics of an order book as a stochastic process. Namely in this model we give the dynamics of limit orders and market orders stochastically. The Maslov model reconstructs a power law distribution of price changes, however the market price produced by the Maslov model shows much greater oscillation than the real data and the diffusion properties of price are also different from the real data.

In order to construct a more realistic model, we analyzed the order book data of financial markets and give the base

model the characteristics observed from the data. We find an important rule for the position where a new limit order is placed, that is the place of a new limit order depends on the volatility. When the market is volatile, limit orders tend to be placed in a deep positions where the distance is away from the best price, while on the other hand when the market is stable limit orders tend to be placed in shallow positions. We also find the normalized distance by volatility follows the unique exponential distribution except for the special day such as government intervention. This feedback of volatility implies that dealers observe the volatility and they tend to extend their spread between bid and ask price when the market is volatile.

The revised model, which contains the properties of a new limit order position, reproduces the power law distribution of price changes. We can also find volatility clustering and temporal non-uniformity of dispersion in time series of price changes as seen in the real data.

We also give two more effects to the revised model in order to reproduce abnormal diffusion of the market price, potential properties[2] observed from the market price and the statistical properties of transaction intervals: first is the trend follow effect, i.e. feedback effect of price changes, and second is expansion and contraction effect of psychological time, i.e. feedback effect of transaction intervals. These effects are discussed in the dealer model[3].

From our analysis, three feedback effects are very important to describe real market fluctuations in the market prices and transaction intervals, and these endogenous feedback effects are caused by dealers' observation of trends and volatility in the market price and market activities. We can simulate various situations such as a government intervention in our new model by adding properties of a government intervention.

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## The Schottky Anomaly in Easily Synchronizable Oscillator-Networks

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In the last decade, much interest has been attracted to studies of complex networks consisting of dynamical elements involved in a set of interactions. Investigations focused on understanding the relationship between the topological structure of a network and its collective synchronous behavior.

In this study, we consider the following model equations

$$\frac{d\theta_i}{dt} = \omega_0 + \frac{\epsilon}{N} \sum_{j=1}^N w_{i,j} \sin(\theta_j - \theta_i) + \xi_i(t),$$

where  $\epsilon$  is coupling strength and  $\xi_i(t)$  are independent white noises, such that  $\langle \xi_i(t) \rangle = 0$  and  $\langle \xi_i(t) \xi_j(t') \rangle = D \delta_{i,j} \delta(t - t')$ . Interactions between the oscillators are specified by the matrix  $\mathbf{w}$  with the elements  $w_{i,j} = 1$ , if there is a connection, and  $w_{i,j} = 0$  otherwise. The Jacobian around complete synchronization solution or (Laplacian matrix for network  $w_{i,j}$ )

is

$$L_{i,j} = \begin{cases} w_{i,j} & (i \neq j) \\ -\sum_{j=1}^N w_{i,j} & (i = j). \end{cases}$$

Using the eigenvalues of the Laplacian matrix

$$0 = \lambda_0 \geq \lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_{N-1},$$

we estimate the Kuramoto order parameter by assuming the connection is symmetry and the intensities of noises are sufficiently small. And the sum of the inverse of eigenvalues, related to the Kirchhoff index

$$\Lambda = -\sum_{i=1}^{N-1} \frac{1}{\lambda_i}$$

determines the synchronization performance. The network having smaller value of  $\Lambda$  shows better synchronization performance. We define a “graph Hamiltonian”  $H$  of the network using the Kirchhoff index. Through the function, we sample networks from the canonical ensemble borrow from the statistical mechanics by MCMC method. Since the synchronization performance can be easily improved by increasing the number of links, we consider the synchronization performance for given networks with fixed number of links.

Statistical analysis of the ensemble reveals that the synchronization-optimized network shows the transition from star to core-periphery structure depending on the connectivity of the network, and which is characterized by the variance of degree for the synchronization-optimized ensemble. The density of states and the thermodynamic quantities are calculated by the weighted histogram analysis method, and thermodynamic properties such as heat capacity shows the Schottky anomaly for sparse connectivity. This anomaly comes from the gap between lower energy levels in synchronization-optimized star-like network.

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### The model of public transport metropolis in the form of a multilayer network

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For the study of public transport networks of cities comparable in scale to New York, London and Moscow in this report methods of tensor analysis of multilayer networks [1] on the basis of differential-geometric formalism are developed.

The public transport network of megacity is represented by a variety of routes of different transport types (bus, trolleybus, tram, underground) with stopover points for boarding and disembarkation of passengers. We propose a network model of public transport of a metropolis, which is represented as a single multilayer network, where each layer corresponds to one of the transport routes. It allows to study the global properties of the public transport network and consider this network as a whole, taking into account the transfer of passengers.

Geometric representation of the route network (route curve

of the road network, the route network and network differentiated manifold) is assigned to each public transport route. Similarly network and network differentiated manifolds are defined for individual types of transport and to all public transport. Tangent spaces of network manifolds are considered as the state spaces of networks. The relationships or links of the public transport networks are defined by linear mappings of the corresponding state spaces into themselves. Relations in the public transport networks are interpreted as elements (tensors of type (1,1)) of spaces obtained by the operation of tensor product of the state space and its dual space. The structure of differentiated manifolds of networks is that the tensor spaces of relations in the networks of individual modes of transport and in full public transport network are represented by direct sums of tensor products of spaces of route networks states and corresponding dual spaces.

Proposed mathematical construction allows to investigate public transport of a metropolis as multilayer network and, in particular, receive certain quantitative characteristics of networks via tensor algebra, such as, for example, degree centrality, betweenness centrality etc. [1]. Designed approach allows to introduce tensor fields on transport networks and study the dynamics of traffic flow in megacities.

An example of the application of the proposed design to the study of public transport network of Moscow will be given.

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### Kappa distribution of quiet-time solar wind electrons as turbulent quasi-equilibrium

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**Workshop:** 2

Space is filled with ionized gas, i.e., the plasma. The interaction among the ionized matter in space is mediated by electromagnetic interaction, i.e., collective effects. Electromagnetic fluctuations in plasmas often exhibit turbulent and nonlinear behavior. Modern kinetic theory of nonlinear and turbulent interactions in plasmas began in the 1960s, largely by the early pioneers in the former Soviet Union, but the development and application continue to this day. The general framework of nonlinear plasma theory begins with the equations of Klimontovich-Maxwell theory, but the most useful theory that describes nonlinear interactions among charged particles and weak electromagnetic turbulent fluctuations is the reduced theory, known as the weak turbulence theory. In this presentation, the weak turbulence theory is employed to discuss the origin of the non-equilibrium velocity distribution observed in space, known as the kappa distribution. Since the early days of space exploration that began in the 1960s, it was known from in-situ observation that the space plasma is almost never found in the state of thermal equilibrium, but rather, the space plasma features a non-equilibrium velocity distribution. Such a feature was simply modeled by the kappa distribution function. Until recently, however, the kappa model was largely regarded as a convenient phenomenological tool for fitting the observation, an exception being the early collisional transport model [Scudder, 1992]. One of the most

significant recent developments in understanding the kappa distribution is that such a state may indicate that the space plasma is in non-extensive thermo-statistical state [Livadiotis and McComas, 2013]. The present author takes a different approach in understanding the kappa distribution. By applying the weak turbulence theory, it was recently demonstrated that the space plasma (especially, the electrons) not only must be distributed according to the non-equilibrium velocity distribution known as the kappa distribution, but also, the specific kappa index was also calculated. This is based upon the notion of the quiet-time space plasma being in a dynamical steady state in which the weak electromagnetic turbulent fluctuations maintain quasi equilibrium state with the charged particles. The mathematical solution of the underlying equations of weak plasma turbulence theory reveals that the kappa model is the only acceptable solution [Yoon, 2012]. This finding represents a physics-based derivation of the same kappa model that the non-extensive thermo-statistical theory also predicts, and indicates that there must exist a profound inter-relationship between the two approaches.

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## Geometric Phase in Bose Einstein Condensates

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An eigenstate of a time dependent Hamiltonian follows its instantaneous eigenstate if the parameters of the Hamiltonian change slowly enough compared to the other time scales of the Hamiltonian. Adiabatic processes are desirable in many experiments on Bose-Einstein condensate (BEC). However lifetimes or coherence times of the system is shorter than the time required for adiabatic processes. A way to speed up the process to reach the same instantaneous adiabatic eigenstate through a non-adiabatic route was introduced in 2010 [1] (references therein). Since then, the expression shortcuts to adiabaticity has been attracted great attention. An experiment on shortcuts to adiabaticity was realized for harmonic trap and theoretically proposed for Fermi gases and Fermi-Bose mixtures and optical lattices [2]. The shortcuts to adiabaticity technique was extended to the system with varying number of particles changes in time.

The shortcuts to adiabaticity method allows the experimentalists to magnify the atomic cloud in a short time. Since the final and initial states are the same up to a constant phase difference, one can make measurements for larger atomic cloud. This method can also be used to measure geometric phase. Although it is desirable to measure geometric phase in BEC, it has never been measured in a BEC so far because of the following two reasons. Firstly, an adiabatic process takes long times compared to lifetimes of the condensate. Secondly, possible measurement of geometric phase can be carried out after time of flight expansion (TOF). However, we lost information on geometric phase after TOF. We propose an experiment to measure geometric phase [3]. One can use the technique shortcuts to adiabaticity to expand the condensate first so that the final extension is large enough to make observation. The final state is the adiabatic state from which one can turn on the periodical potential so that the system

makes a fast cyclic evolution. Therefore, the eigenfunction takes geometric phase at the end of the second non-adiabatic cyclic evolution. This geometric phase can be measured since the extension of the cloud is large enough. To compare the experimental results with the theoretical results, we need geometric phase expression for ultracold systems. In the present study, we will derive geometric phase expression for various systems such as harmonic oscillator and bright soliton in 1-D and non-dipolar and dipolar Bose-Einstein condensates in 3-D.

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## Memory effects in stock price dynamics: evidences of technical trading

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Technical trading represents a class of investment strategies for Financial Markets based on the analysis of trends and recurrent patterns in price time series. According standard economical theories, these strategies should not be used because they cannot be profitable. On the contrary, it is well-known that technical traders exist and operate on different time scales. We investigate if technical trading produces detectable signals in price time series and if some kind of memory effects are introduced in the price dynamics. In particular, we focus on a specific figure called *supports* and *resistances*, that are usually defined in a rather qualitative way: a support is a price level, local minimum of the price, where the price will bounce on other times afterward while a resistance is a price level, local maximum of the price, where the price will bounce on other times afterward. We first develop a quantitative definition and, as a consequence, a criterion to detect the potential values of supports and resistances. We notice that there are no preferred values, in contrast with the frequent opinion that price should bounce on round values. Then we show, using a dataset composed by the high frequency time series of the price of 9 stocks of the London Stock Exchange, that memory effects in the price dynamics are associated to these selected values. In fact we show that prices more likely re-bounce than cross these values, and that the conditional probability of a new bounce increases with the number of the previous bounces, that is, support and resistances tend to reinforce themselves. We verify that such an effect is not compatible with a random walk, even if one takes into account the subdiffusive features of the empirical price dynamics. As a further check, we shuffled the data, verifying that in this way we lose the signal. We study the time occurring between two consecutive bounces and the maximum distance of the price from the support or the resistance between two consecutive bounces. We find a quantitative evidence of the so-called *self-fulfilling prophecy*, that is the self-reinforcement of agents belief and sentiment about future stock prices behavior. In fact financial markets, in contrast with natural systems, are composed by agents

who may know, and may change, the rules of the game: as a consequence, if a large number of investors has the same expectations on the future value of the price, and if they react in the same way, they will operate in such a way to fulfill their own expectations. In this work we measure quantitatively the trust on one of the figures of technical analysis.

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### Local-time representation of one-dimensional Feynman path integral

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We derive a local-time path-integral representation for one-dimensional time-independent quantum-mechanical systems. This is an alternative to the traditional Feynman path-integral representation, or to the Schrödinger equation. Specifically, we express the matrix elements of the Gibbs operator  $e^{-\beta\hat{H}}$ , where  $\beta = 1/k_B T$ , as a path integral over  $x$ -dependent local time profiles, which is found to be tightly related to the Feynman path integral for radial harmonic oscillator. The new representation is also relevant to classical diffusion processes since the Gibbs operator is equivalent to the diffusion kernel.

Local times of stochastic processes have been studied in mathematics since their introduction by Lévy. They characterise the amount of time that a sample trajectory  $x(t)$  of a given stochastic process (typically the Brownian motion) spends in the vicinity of an arbitrary point  $x$ . This defines sample trajectory  $L(x)$  of a new stochastic process. Intuitively, one can think about diffusing Brownian particles and a laser beam shining at position  $x$ . The local time  $L(x)$  then returns the amount of radiation that a selected particle receives.

Our aim is to popularise the notion of local time within the physics community where it has not received much attention as yet. Therefore we use the language of path integrals and provide explicit formulas that allow one to calculate physical quantities, namely the Gibbs operator and the partition function of the system. The quantum evolution operator can also be obtain – via the Wick rotation.

Generalization of the local-time path-integral representation that includes arbitrary functionals of local time is also presented. We expect this result to provoke discussion about potential applications of our local time formula as the functional can be adjusted for a specific problem in mind. For instance, in the above example with diffusing particles, one can simply choose the functional to be the local time at some point  $x$  and thus calculate the average irradiation.

Furthermore, we discuss connections with the Sturm-Liouville theory, and asymptotic behavior of the Gibbs operator for large- and small- $\beta$ . We recall that the small- $\beta$ , i.e. large-temperature, asymptotics can be systematized in the so-called Wigner-Kirkwood expansion, which can be derived from both, local-time and Feynman, path integrals. Our local-time representation proves more convenient when tackling the large- $\beta$ , or low-temperature, regime where it correctly recovers the ground state contribution and promises to be a useful tool for calculating further corrections.

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### Adaptive Resolution Simulation of an Atomistic Protein in MARTINI Water

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Many of the multiscale approaches are concerned with interfacing of atomistic and the continuum models of liquids. In these hybrid methods, typically molecular dynamics (MD) or a similar approach is used for simulating dynamics on the atomistic scale, whereas the Navier-Stokes equation governs the fluid dynamics on the continuum scale. Alternatively, multiscale schemes have been introduced using particle-based models only, e.g., atomistic and physically simplified coarse-grained (CG) molecular models. The coupling can be achieved either by a fixed resolution approach, where different resolution domains interact with each other but do not exchange molecules, or the adaptive resolution approach, where molecules change their resolution according to their current positions. Among the most advanced methods for the latter kind of simulations is the Adaptive Resolution Scheme (AdResS) [1], which allows for concurrent coupling from quantum all the way to continuum length scales of molecular liquids and soft matter.

Here, we present an adaptive resolution simulation of a 56-residue protein G (pdb entry 1PGB, as an example of a well studied protein) in multiscale water [2]. We couple atomistic water around the protein with mesoscopic water, where four water molecules are represented with one coarse-grained bead, farther away by using AdResS. We circumvent the difficulties that arise from coupling to the coarse-grained model via a 4-to-1 molecule coarse-grain mapping by using a bundled version of the Simple Point Charge (SPC) water model [3], i.e., we restrict the relative movement of water molecules that are mapped to the same coarse grained bead employing harmonic springs. The water molecules change their resolution from four molecules to one coarse-grained particle and vice versa adaptively on-the-fly. We observe within our error bars no differences in structural (e.g., root-mean-squared deviation and fluctuations of backbone atoms, radius of gyration, the stability of native contacts and secondary structure, and the solvent accessible surface area) and dynamical properties of the protein between the adaptive resolution and fully atomistically solvated models. Our multiscale water model is compatible with the widely used MARTINI force field. This opens up a range of future applications involving the broad variety of solutes and solvents parameterized for this force field.

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## Temporal Asymmetries in Italian Interbank Market reproduced by a data-driven Agent-Based Model

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We study the changes in the topology of the structure of the Italian interbank market in the period from January 1st 1999 to September 1st 2009. We find temporal irreversibility in the growth of the largest component of the interbank trading network, a feature not common to any of the usual network growth models. More specifically we found that the largest component grows faster backward than forward in time. Such asymmetry, (also detected on the growth of the clustering and reciprocity coefficient) reveals that the trading mechanism changes along the day.

Temporal asymmetries are of special interest as one of the best signs of non-equilibrium systems. Exact knowledge of temporal asymmetries of certain system was previously used to classify those systems, to accept or reject models of such systems and to clarify micro-mechanisms that are driving the evolution of those systems. Although there is a huge corpus of economics literature describing economy as a non-equilibrium system we were not able to find any paper that would predict temporal asymmetry in trading patterns we have found in our data. Moreover the temporal asymmetries we have found are profoundly different from those one can find in the literature on temporal networks (or evolution of networks).

We are able to recover the statistical properties of the system by means of a simple Agent-Based Model (ABM) where the probability of matching between counterparties depends on a time varying vertex fitness (or attractiveness) describing banks liquidity needs. We show that temporal irreversibility is associated with heterogeneity in the banking system and emerges when the distribution of banks liquidity shocks is broad. We show that the asymmetry becomes more evident as the financial crisis approaches and we argue that it is correlated to the heterogeneity in the size of liquidity shocks banks experience. Furthermore although we are not modeling interest rates explicitly (a task that many financial economists are working on every day) our model implicitly point to relation between heterogeneity of shocks and the spread of interest rates and this surprising relation can be observed in data as well. Furthermore although we are not modeling interest rates explicitly (a task that many financial economists are working on every day) our model implicitly point to relation between heterogeneity of shocks and the spread of interest rates and this surprising relation can be observed in data as well.

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## Path Integral of Twisted and Bent DNA

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Complementary strands in DNA double helix show temporary fluctuational openings which are essential to biological functions such as transcription and replication of the genetic information. Such large amplitude fluctuations, known as the breathing of DNA, are generally localized and are due to the fact that the hydrogen bonds, linking the pair bases on the complementary strands, can break therefore exposing the bases for chemical reaction.

Theoretical approaches based on Hamiltonian methods are particularly advantageous in modeling the DNA dynamics as they can treat the system at the level of the base pairs incorporating nonlinear effects in heterogeneous sequences [1-3].

I apply finite temperature path integral techniques to a mesoscopic Hamiltonian which accounts for both the twisting and the bending of short circular DNA molecules [4]. The theoretical background for the application of the path integral formalism to predictive analysis of the molecule thermodynamical properties is discussed. The base pairs displacements with respect to the ground state are interpreted as time dependent paths. The portion of the paths configuration space contributing to the partition function is determined, at any temperature, by selecting the ensemble of path displacements whose amplitudes are consistent with the model potential for the hydrogen bonds.

Evaluating the equilibrium thermodynamics, we focus on the interplay between twisting of the complementary strands around the molecule axis and nonlinear stacking potential: it is shown that the latter affects the melting profiles only if the rotational degrees of freedom are included in the Hamiltonian. The use of ladder Hamiltonian models for the DNA complementary strands in the pre-melting regime is questioned.

Helix unwinding and bubble formation patterns are computed in circular sequences with variable radius. In agreement with the experimental findings, base pair disruptions are found with larger probability in the smallest minicircle of *66-bps* whose bending angle is  $\sim 6^\circ$ . Fluctuational openings appear along the helix to release the stress due to the bending of the molecule backbone. The interplay between molecule size and appearance of helical disruptions is analyzed. The bubble probability profiles are compared to experimental data available for some minicircles [5]. The method can be generalized to determine the bubble probability profiles of open ends linear sequences.

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## Finite temperature and magnetic field transport in 1D quantum magnets

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I'll present recent exact results on the finite temperature and magnetic field magneto-thermal transport in the one dimensional spin-1/2 Heisenberg model obtained using the Bethe ansatz (BA) method. In particular, I'll discuss the behavior of spin Drude weight as a function of magnetic field down to low temperatures. These new results are based on a previous analysis by the author of the spin Drude weight in zero magnetic field [1] that recently have been independently confirmed by state of the art numerical simulations and an open system Lindblad approach. The new data now allow for a complete evaluation of the thermal conductivity in the presence of a magnetic field and of the Seebeck coefficient.

Furthermore, and in relation to this development, (1) I'll discuss the thermodynamics, thermal transport and dynamics (ESR), of the spin S=1 easy-plane quasi-one dimensional quantum magnet NiCl<sub>2</sub>-4SC(NH<sub>2</sub>)<sub>2</sub> (DTN). The analysis is based on an effective spin-1/2 anisotropic ( $\Delta = 1/2$ ) Heisenberg model description that is put on a firm basis by comparing the thermodynamics of the S=1 model, obtained using TMRG, with exact BA specific heat and magnetization results for the s=1/2 Heisenberg model. For the thermal conductivity in a magnetic field, I'll compare numerical data on the S=1 model obtained using exact diagonalization techniques to exact results using the BA method. For the ESR data [2], using a recently developed BA technique [3], I'll show that the extremely sharp line observed in experiments, is due to a singular excitation to a single excited state. It is fascinating that this excited state, highly unusual within BA as it is described by a set of real pseudo-momenta plus one with imaginary part  $\pi/2$ , is exactly probed in ESR experiments - it corresponds to a uniform rotation of the total magnetisation.

(2) I'll discuss the thermal conductivity in a magnetic field of the quasi-one dimensional CuPzN compound that is well described by the isotropic spin-1/2 Heisenberg Hamiltonian. In particular, I'll analyze the presence of the magnetothermal correction and its relevance in the interpretation of experiments [4] and discuss the extent to which spin-orbit scattering suppresses this correction. Furthermore, I'll show that this correction becomes significant near the critical fields to the ferromagnetic state.

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## Entropy of frustrated Ising spin clusters

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A two-dimensional triangular lattice Ising antiferromagnet is a typical geometrically frustrated spin system with highly degenerate ground state and non-vanishing entropy, which results in lack of long-range ordering at any finite temperature [1]. However, in systems comprising only a finite number of spins located in domains of different shapes, it has been shown that the ground-state degeneracy can be strongly influenced by domain sizes and boundary conditions [2-4]. In the present study, we investigate geometrically frustrated Ising spin- $s$  ( $s = 1/2, 1$  and  $3/2$ ) clusters on a triangular lattice of various shapes and sizes with free boundary conditions by exact enumeration and Monte Carlo simulations, including finite temperatures and magnetic fields. We first study effects of the shape, size and the spin value on the ground-state energy and the residual entropy. In particular, we are interested in how these quantities can be influenced by minimal manipulation in the clusters' shapes, such as removal of a small number of vertices. We observe their behavior for different  $s$  on approach to the respective thermodynamic limits, which for  $s > 1/2$  are obtained by Monte Carlo simulations [5]. Depending on the cluster shape, the residual entropy density on approach to the thermodynamic limit can either vanish or remain finite and the dependence can be decreasing, increasing or non-monotonic. Nevertheless, the relative entropies normalized by the respective thermodynamic limit values turn out to be little sensitive to the spin value. In an applied external magnetic field, the ground state magnetization and the entropy show stepwise variations. The number of steps, their widths and heights depend on the cluster shape and size. While the character of the magnetization plateau heights is always increasing, the entropy is not necessarily nonincreasing function of the field, as one would expect. For some selected clusters showing interesting ground-state properties, the calculations are extended to finite temperatures by exact enumeration of densities of states in the energy-magnetization space. In zero field, the focus is laid on how the entropies of the respective clusters approach their residual values for various cluster sizes. In finite fields, particular attention is paid to the isothermal entropy changes either due to the field or the spin manipulation, focusing on cases showing an enhanced magnetocaloric effect. The exact results on the finite clusters are compared with the thermodynamic limit behavior obtained from Monte Carlo simulations and it is concluded that small triangular clusters can display better magnetocaloric properties than the infinite system.

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