A κ-generalized Wasserstein metric in the graph-space for seismic waveform inversion issues

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Full-Waveform Inversion (FWI) is an advanced technique used in imaging problems to estimate the physical attributes of a medium from indirect measurements. In several geophysical applications, FWI is based on analyzing and processing seismic waves recorded at different points on the Earth's surface to reconstruct an accurate subsurface model. FWI is a robust procedure that presents significant improvements over conventional data inversion methods, as it uses all the information in seismic waveforms rather than relying solely on wave amplitudes or travel-time arrivals. This means that the FWI considers the recorded seismic waves' phase, amplitude, and complete shape, allowing a better characterization of subsurface physical parameters (e.g., conductivity, sound speed, or density). The FWI process involves several steps. In the first one, in a seismic survey, seismic waves are generated by artificial sources through controlled explosions and registered by a network of seismic receivers, composing the observed data set. Then, an initial subsurface model is produced based on prior knowledge of the region under study or conventional seismic imaging techniques. This model is then used to generate synthetic simulations of the expected seismic waveforms by solving a wave equation. These synthetic waveforms are then compared to the observed ones employing an objective function. The next step consists of modifying the initial model to minimize the difference between the synthetic and observed data (residuals) through iterative algorithms that modify the model incrementally until the residuals are minimal. During this optimization process, phase, amplitude, and shape information from the waveforms are considered to refine subsurface physical parameter estimates. However, the so-called cycle-skipping (also known as phase ambiguity issues) poses a significant challenge in FWI applications. Such a phenomenon is mainly responsible for guiding the optimization process towards a non-informative local minimum, generating inaccurate subsurface models. This sensibility to the phase ambiguity occurs because the conventional objective functions measure pointwise (sample by sample) the misfit between synthetics and observed data, losing the convexity property regarding the time shifts between them. This work presents a robust objective function based on the κ-generalized Wasserstein metric to mitigate phase ambiguity problems. The proposed objective function is computed by solving a combinatorial optimization problem using a Kuhn-Munkres algorithm to minimize the difference between the observed waveforms and a permuted version of the synthetic waveforms. In this way, we consider the Kantorovich formulation of the optimal transport problem and the probabilistic maximum likelihood for obtaining a new κ-objective function. Since the Wasserstein metric is valid only for comparing probability functions and the seismic waveforms are not normalized quantities and not positive-defined quantities, we represent the waveforms in the graph space to satisfy the probability axioms. The results suggest that the κ-graph-space optimal transport FWI is an efficacious approach to alleviate the impact of cycle-skipping difficulties and to improve the objective function convergence, resulting in higher-resolution subsurface models when $\kappa = 0.6$.