Permutation group entropy: A new route to complexity for real-valued processes

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Generalized entropies can play a crucial role in the characterization of the complexity of a large class of deterministic and random processes. In the first part of this talk, we will propose the notion of group entropy as a class of information measures possessing an intrinsic group-theoretical structure. This structure emerges when imposing the requirement of composability of an entropy with respect to the union of two statistically independent systems. A generalization of the celebrated Shannon-Khinchin set of axioms is proposed, obtained by replacing the additivity axiom with that of composability. This formulation, which makes use of the theory of formal groups of algebraic topology, leads to the new, infinite family of non-trace-form entropies called group entropies. The first example of this class is represented by the standard Rényi entropy. We will show that complex systems can be classified into universality classes, each characterized by a specific phase space growth rate, and described by a suitable group entropy, representing the complexity measure associated to the given universality class.

As a nontrivial application, we shall present a generalization of the permutation entropy approach to data analysis by Bandt and Pompe, which has been very successful since its inception 20 years ago. Nevertheless, the theoretical aspects have remained somehow limited to noiseless deterministic time series and dynamical systems, the main obstacle being the super-exponential growth of allowed permutations with length when randomness (also in form of observational noise) is present in the data. To overcome this difficulty, we propose a new methodology based on complexity classes, which are precisely defined by the growth of allowed permutations with length, regardless of the deterministic or noisy nature of the data. We consider three major classes: exponential, sub-factorial and factorial. Our main results is a novel family of generalized permutation entropies which widely extend the standard notion of permutation entropy. They represent new complexity measures which possess many interesting group-theoretical and analytic properties. Besides, our information measures can be considered as the topological analogue of the family of group entropies. The conceptual framework proposed provides us with a unified approach to the ordinal analysis of deterministic and random processes, from dynamical systems to white noise.

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References

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