Squeezing in Bose-Einstein condensate of weakly interacting atoms: non-Gaussian fluctuations of the order parameter and #p-complexity of the joint particle-occupation statistics

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The talk is devoted to new results on the statistics of Bose-Einstein condensate (BEC) in a weakly interacting gas confined in a mesoscopic trap in an equilibrium state. Detailed analysis of such statistics in presence of interparticle interactions constitutes a rather complicated problem, mainly due to the nontrivial relation between bare atoms (quantity of which is measured in experiments) and quasiparticles, which occupation numbers determine the probabilities of microstates. For a cold, partially condensed equilibrium gas, we find the universal analytic formula for a characteristic function (Fourier transform) describing the statistical properties of BEC. The analysis employs the Hartree-Fock-Bogoliubov-Popov approximation describing the condensate and quasiparticles by means of the nonlinear Gross-Pitaevskii and Bogoliubov - de Gennes equations treated in a self-consistent manner. It consistently takes into account the nontrivial Bogoliubov transformation between particles and quasiparticles which enables squeezing-of-quantum-state effects, and allows us to see how these effects are reflected in the BEC statistics. Via the method of characteristic function we, for the first time, describe an evolution of the statistics of the total number of condensed particles as the interparticle interaction increases from an ideal gas regime to the Thomas-Fermi limit. Such evolution is governed by the interplaying effects of squeezing of fluctuations and restructuring of the Bose-Einstein condensate and quasiparticle wave functions and spectrum. We find that there are two essentially different – the quantum-dominated and the thermally-dominated, – regimes of fluctuations, and specify a range of system parameters where the transition between these regimes happens. We clearly show that the thermally-dominated BEC statistics may be essentially non-Gaussian and dependent on boundary conditions, and explain why these effects are still present even in the thermodynamic limit and even in the Thomas-Fermi limit (where one may expect that the condensate screening should disable the influence of the boundary conditions on the macroscopic properties of the system) [1]. The proposed method also allows us to address a deeper problem of calculating a joint probability distribution for the particle occupation numbers in a set of different excited states of BEC [2]. Here we find interesting connections to quantum computations and simulations. We show, via the newly discovered combinatorial theorem [3], that the considered probabilities are proportional to matrix hafnians. In the general case, calculating a hafnian is a ♯P–hard problem, which means the corresponding runtime of a classical computer grows exponentially with the size of the matrix. This is a key property for the well-known and widely discussed concept of GaussianBosonSampling quantum simulators, where the outcome probabilities of multi-photon detecting events are also encoded via matrix hafnians. We elaborate on the discovered analogy and discuss how the squeezing effects induced by the Bogoliubov transition lead to a computational complexity of the excited particles' statistics calculations.

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References

[1] S.V. Tarasov, V.V. Kocharovsky, Vl.V. Kocharovsky, Phys. Rev. A, 102, 04331 (2020).
[2] V.V. Kocharovsky, Vl.V. Kocharovsky, S.V. Tarasov, Phys. Rev. A, 106, 063312 (2022).
[3] V.V. Kocharovsky, Vl.V. Kocharovsky, S.V. Tarasov, Linear Algebra and its Applications, 651, 144-161 (2022).