Kappa distributions and power-law spectra in space plasmas

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One of the first studies in the literature to provide a theoretical justification for the formation of kappa distributions in space plasmas was by Ma and Summers (1998). Their study involved a non-relativistic kinetic equation for the electron distribution function incorporating the effects of stochastic acceleration by whistler mode waves. Importantly, Coulomb collisions, which result in both friction and momentum diffusion, were included. The resulting kinetic equation was shown to permit an exact solution for a kappa distribution, under certain conditions, and the spectral index kappa was found to depend on the power of the wave turbulence. The Ma and Summers (1998) study was linked to the Earth's plasma sheet, Saturn and Jupiter.

Recently, Summers and Stone (2022) carried out an analysis of energy spectra of "killer" electrons in Earth's outer radiation belt. These highly energetic (relativistic) electrons can seriously damage orbiting satellites. Killer electron generation was modelled by means of chorus wave diffusion in a 1-D relativistic Fokker-Planck equation incorporating an electron loss term. Various forms of wave spectral density were used including Gaussian and power-law (with index q). The resulting model equations depend on the important controlling parameter k = DT where D is a diffusion parameter and T is the timescale for electron loss.We solve the equations numerically and demonstrate that net electron energization occurs when k exceeds a critical value.We use classical methods to obtain analytic solutions for the electron distribution f(E) that are valid for large energy E. Specifically,we obtain simple analytic forms for the electron spectra for the cases (a) a full-band whistler-mode spectrum, and (b) a lower-band chorus spectrum for both a Gaussian spectrum and a power-law spectrum. The analytic spectra are found to test well against the full numerical solutions. Typically, the spectra involve inverse exponentials in energy rather than inverse power-laws. Exceptionally, for a full-band whistler spectrum, in the case that q = 4, we obtain an exact inverse power-law spectrum in energy where the power is a specified function of the parameter k. We carry out comparisons of our model spectra with experimental satellite data. We show comparisons of the analytic and numerical solutions with four selected "events", namely Magnetic Storm 1 (October 9, 2012), Magnetic Storm 2 (March 17,2013), Non-Storm (February 23,2013), and Average Geosynchronous conditions. We find good agreement between both analytic and numerical solutions with the experimental spectra for all four events. Kappa distributions were not readily found in the relativistic Fokker-Planck models examined by Summers and Stone(2022). However, as a further project it would be interesting to generalize these models by including additional physical effects such as frictional or collisional terms to see if kappa distribution solutions then naturally occur.

References

[1] C. Ma, D. Summers, Geophys. Res. Lett., 25, 4099 (1998).
[2] D. Summers, S. Stone, J. Geophys. Res. Space Physics, 127, e2022JA030698 (2022).