Exponential increase of transition rates in metastable systems driven by non-Gaussian noise

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Noise-induced escape from metastable states governs a plethora of transition phenomena in physics, chemistry, and biology. While the escape problem in the presence of thermal Gaussian noise has been well understood since the seminal works of Arrhenius and Kramers, many systems, in particular living ones, are effectively driven by non-Gaussian noise for which the conventional theory does not apply. Here we present a theoretical framework based on path integrals that allows the calculation of both escape rates and optimal escape paths for a generic class of non-Gaussian noises. We find that non-Gaussian noise always leads to more efficient escape and can enhance escape rates by many orders of magnitude compared with thermal noise, highlighting that away from equilibrium escape rates cannot be reliably modelled based on the traditional Arrhenius–Kramers result. Our analysis also identifies a new universality class of non-Gaussian noises, for which escape paths are dominated by large jumps.

Our results demonstrate that non-Gaussian noise can induce qualitatively very different escape behaviours. The instantons (optimal escape paths) with jump sections indicate an escape strategy that is fundamentally different from the one we find in thermal equilibrium systems: instead of completing the entire escape using a rare sequence of small fluctuations, the system prefers to wait for a single rare fluctuation that is large enough to carry it across the steepest section of the potential barrier. Remarkably, the prefactor of the escape rate highlights the existence of two universality classes associated with these two types of escape: the Kramers prefactor, which also applies to non-Gaussian noise in the parameter range where the escape path is smooth; and a distinct prefactor for the jump case. The theoretical analysis shows that the exponential speed-up of transition rates can persist and in fact become more pronounced even in the regime where non-Gaussian noise kicks are so rare in relative terms that their contribution to the noise intensity is vanishingly small. It might be possible to exploit this effect to optimize switching behaviour in artificial systems driven by non-Gaussian noise such as colloids interacting with an active microbial heat bath on which thermodynamic cycles can be imposed; indeed, recent experiments have shown that non-Gaussian noise can be used to tune the performance of a colloidal Stirling engine by shifting the operating speed at which power is maximum.

Time permitting we will discuss applications to escape problems for highly persistent active matter, as well as extensions to non-Gaussian noise with memory.

References

[1] A. Baule, P. Sollich, Exponential increase of transition rates in metastable systems driven by non-Gaussian noise. Sci. Rep. 13, 3853 (2023); (and references therein)