Bak-Tang-Wiesenfeld sandpile as the mechanism that generates the 1/x power-law and the 1/f spectrum

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Bak, Tang, and Wiesenfeld introduced the phenomenon of self-organized criticality with a sandpile model and positioned their model as the key to understanding the flicker noise. However, the BTW sandpile and its modifications are primarily known for the power-laws describing state variables, where a few power-law exponents characterize a great many isotropic model modifications defined on the square lattice. The BTW sandpile itself generates the power-law size-frequency relationship with the exponent equaled to 1.20, and the models with the exponent located closer to 1 are not known. The spectrum of basic quantities in sandpiles exhibits a constant at low frequencies, the $1/x^2$ decay at high frequencies, and, for some models, the 1/x part in-between. This spectrum structure does not elucidate specific features underlying many processes including, for example, the superposition of pulses. Therefore, researchers created additional constructions to produce the "pure" 1/f spectrum with sandpiles.

In this talk, I'll show that the departure from the original BTW sandpile was premature, and the reciprocal function in both size-frequency distribution and spectrum is attained with the BTW mechanism. The 1/x size-frequency relationship is obtained with clustering of events in space and time in the BTW sandpile. The precise definition of the clustering, affecting the cluster volume, gives the tool to control the exponent of the size-frequency relationship. This clustering allows to fill in the range of exponents from 1.20 to 1 and, possibly, to values being smaller than 1.

In the case of spectrum, I'll focus on the dynamics of the average system stress in contrast to the sequence of event sizes, which is typically analyzed. The spectrum of the average system stress indeed exhibits a constant at low frequencies that turns to 1/x at moderate frequencies and finally to the $1/x^2$ decay at high frequencies. However in the thermodynamic limit, the role of the parts precisely matches the expectations of researchers. The 1/f part extends over all time scales that represent the dynamics at the critical level of stress. The 1/f spectrum transits to a constant at the time scales that correspond to extremely rare drops of the system to the subcritical state. Finally, the $1/x^2$ part is insignificant because it covers the time scales up to the system size, whereas only the scales related to the system area "survive" in the thermodynamic limit. I also note that this insignificant spectrum part is associated with the power-law segment of the size-frequency relationship, whereas the 1/x spectrum is associated with the tail of the event distribution.