A multifractal analysis of the rugged energy landscape of spin glasses.

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The roughness of the energy landscape of spin glasses is a widely recognised property. However, it seems to be insufficiently investigated, probably because approaching it analytically raises multiple difficulties. Therefore, we have approached it numerically by theoretically relying on the so-called thermodynamic formalism of multifractals and the concept of intermittency, first highlighted by Batchelor (1953) [1]. It has become an essential characteristic, because of the structuring it generates. This research has also made it possible to model and quantify this phenomenon in various fields of physics and other disciplines (Parisi, 2022). In awarding its 2021 prize to Giorgio Parisi, the Nobel Committee for Physics has recognised both the importance of this phenomenon and the work carried out, as highlighted by Schertzer and Nicolis, 2022 [2]. Using the Metropolis-Hastings algorithm, we seek to continuously improve, site by site, the current and local minimum of the energy. At each step, we can analyse the multifractality of the Bolzman coefficient field, which is the thermodynamical analog of the energy flux density of a turbulent cascade. The spectral analysis of this field is preliminary analysis and has confirmed a scale symmetry os the second order singularities. We are currently carrying out a so-called universal multifractal analysis [3], which allows to test scale symmetries of singularities of other orders. It furthermore enables to determine significant physical parameters such as the mean intermittency, more precisely its codimension, and the multifractality parameter α.

References

[1] G. K. Batchelor, The theory of Homogeneous Turbulence. Cambridge: Cambridge University Press (1953).
[2] D. Schertzer, C. Nicolis, Nobel recognition for the roles of complexity and intermittency, Eos, 103 (2022).
[3] D. Schertzer, S. Lovejoy, Physical modeling and analysis of rain and clouds by anisotropic scaling multiplicative processes, J. Geophys. Res. AGU, 92(D8), 9693-9714 (1987).
[4] G. Parisi, Comme un vol d’étourneaux, une introduction personnelle à la science de la complexité, 202, Flammarion, Paris (1922).
[5] D. Schertzer, I. Tchiguirinskaia, A century of turbulent cascades and the emergence of multifractal operators, Earth Space Sci., 7(3), e2019EA000608 (2020).
[6] G. Nicolis, C. Nicolis, Foundations of Complex Systems: Emergence, Information and Prediction, 2nd ed., 384 pp., World Sci., Singapore (2012).